COLLEGE PHYSICS

By JOHN A. ELDRIDGE

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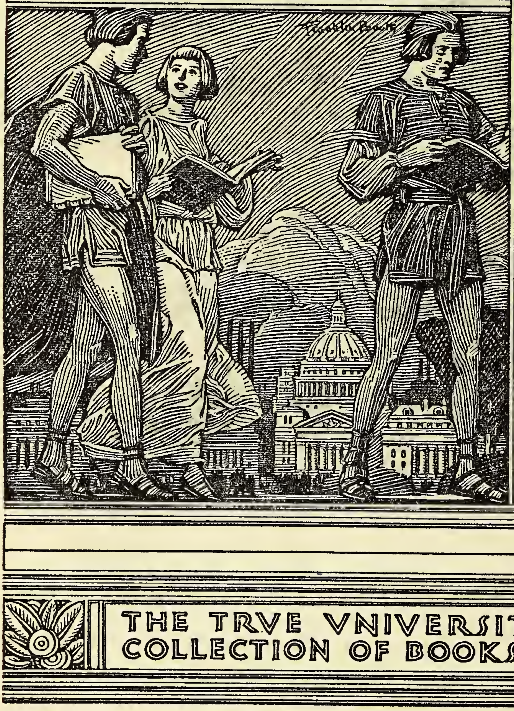

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COLLEGE PHYSICS

BY

JOHN A. ELDRIDGE
PROFESSOR OF PHYSICS
UNIVERSITY OF IOWA

SECOND EDITION

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PREFACE

This book is written in the belief that physics is interesting and useful for every student. Also it may be an inspiring adventure and a great deal of fun. Motors and gyroscopes and molecules, practical knowledge and philosophy, stories of pioneering adventure — how can it be otherwise? Yet there exists an opinion to the contrary, and, in a world largely founded on physical discovery, the majority of students leave college without ever having experienced, in high school or college, a course in physics. There is something wrong; physics has something very real to contribute to present-day education. For some years I have been watching these young college students when for one reason or another they have elected physics. They like to think but are usually not trained to think in mathematical symbols; they are interested in physical things, but more interested in the gyroscope and radio than in vernier calipers and the Wheatstone bridge; they are willing to work, but ask in return something which they themselves appraise as worth while.

I have also been observing young graduate students in physics. Much of their fundamental physics dates back to their earlier courses, and in this foundation there are some notable hiatuses. They know that $F = ma$, but are not clear about the distinction between the second and third laws of motion, not always clear about momentum and when it is conserved; they usually have a much more positive belief in the first than in the second law of thermodynamics; Boltzmann's constant is beyond their ken; they are not quite sure whether voltage is pressure, or energy per unit charge; they are a little hazy about the relation between electrostatics and electrodynamics; they know considerably more about conjugate foci than about resolution. For some years I have been making such mental notes of the elementary physics which might well be given greater emphasis to give a fair perspective of the physical scene.

I have tried to write a textbook which will appeal to the student with only a general interest in the subject — non-

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science majors — and a textbook which will at the same time give a solid foundation for the student with a direct professional interest in physics. Also a textbook which will be suitable for a student with no previous training in the subject, yet in point of view and selection of material will impress the student with high-school physics as in no sense a mere redoing of his earlier work. And a book in which the student will think through the subject for himself. Within these bounds the pre-medic student and the major in other sciences will surely find much of his own specific interest.

On paper this reads like a difficult order. (It is no more difficult than the task which faces each teacher in his classroom.) The reader will perceive where in the book I have addressed myself to these separate tasks (to remove any doubt, parts of less general interest are marked with asterisks). But I hope that there is a unity in the whole; I believe that the earlier chapters will not be wasted on the physics major and the starred sections will not be uninteresting and seldom difficult for the average student. These starred sections may be omitted without affecting the continuity of the course. . . . Proportion and the thinking through of problems are stressed in the earlier chapters even though it may be simpler to teach the filling in of formulas. . . . After each chapter are questions which I hope will encourage some physical thinking and possibly the formulation of other questions by the student himself. . . . The problems are for the most part simple and illustrative. (Is not the essential purpose of problems for the average student to present concrete illustration of the abstract concepts?)

In frank and kindly criticism students of past years have disillusioned me of some of my misconceptions: that physics is always an excellent discipline, that problems of themselves teach students to think, that laboratories necessarily teach practical application. But physics can be a thrilling adventure and a good deal of fun, and if the student becomes just a little excited about the subject, the moral and mental training and the practical values will come of themselves. When the student has learned to pass examinations and not to like physics (the case is not unknown), the teacher has failed.

This confession of faith appeared as the author's apology for the first edition. The second edition appears during a European war that threatens to become a World War. As I write this my radio brings words from the chief of the U. S. Navy, advising young men about training for their country's need: " Young man, learn about machines; become acquainted with the operation of your automobile, of your radio." Educational ideas have been changed overnight. Yesterday the dominant idea was untrammeled self-expression; today the word discipline has come back into the educational vocabulary. Discipline and training. If we believe all we hear there is to be a new emphasis upon the physical sciences. Read in these crucial days our first chapter seems to have a new and deeper meaning — or perhaps to some readers to have no meaning at all. For it would certainly seem that college students, future leaders of democracy, must learn to think — to think harder, more objectively, more incisively, more in anticipation of the future event — or we shall all be told not to think at all.

A word should be said about the problems which (in the new edition) cover a large range of physical principles and vary considerably in difficulty. With some care in selection the teacher will find problems of a level of difficulty suited to his class. Many teachers will wish to restrict the range to give greater problem-working skill in the more limited field. As to mathematical techniques, only the simplest algebra is required (linear equations). Essential background is given in Appendix I which is supposed to precede the course.

John A. Eldridge

July, 1940

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The data for Table 7 have been taken from Laurson and Cox: Properties and Mechanics of Materials (Wiley); for Fig. **12**–3 from findings of the National Safety Council; for Fig. **27**–3 from publications of the Bell Telephone Laboratories.

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COLLEGE PHYSICS

INTRODUCTION

CHAPTER 1

PHYSICS AND CIVILIZATION

The subject which we study deals with the laws of inanimate nature;[1] the laws of mechanics, of heat, of sound, of electricity, of light. It deals not with man himself but with the world which men touch and see and feel. It is the science basic to other sciences. Probably more than any other science, physics has modified the stage on which man's life is acted. And it is an ever-growing heritage from the past which, recognized or not, is deeply woven into our twentieth-century culture.

1. The Story of the Past. Man is a toolmaker. And man looks at nature and asks the meaning of things. And man is a dreamer.

Primitive man was born, learned to live, mated and had offspring, found or chipped stone tools, killed or found his food, crept to his cave or squatted about his fire — primitive man lived and died for some ten thousand generations before the dawn of history. Why was there so little change? Very slowly the old stone age turned into the new stone age, and then civilization came.

Some six or seven thousand years ago, we find civilization in the valleys of the great rivers, the Nile, the Tigris and the Euphrates, and the Indus. Egyptians and Sumerians and Indians have already, "when history opens," invented the city and the state. They have learned the arts of agriculture, they have domesticated animals; family feuds and tribal fights have enlarged into wars waged by national armies. The art of writing has been invented; a decimal system of counting and simple arithmetic has been discovered. This is the state of culture as we find it in those valley civilizations of some four hundred generations ago. One pyramid follows another, dynasties come and go, two hundred generations pass.

---

[1] The word *physics* and the adjective *physical* are derived from the Greek *physis*, meaning nature.

Then came Greek civilization, democracy of the city state, fostering the beautiful and true. The Greeks were artists; their philosophers puzzled about the behavior of people and things, about the stars and about isosceles triangles, for the sheer pleasure of knowing the world they lived in — and the pleasure of puzzling; yes, the Greeks were artists. Greek civilization was one of the bright epochs in history — too perfect to last. A few centuries, and the leadership passes to the practical-minded Romans: soldiers, politicians, engineers. Then come the decay of the Roman empire and the Dark Ages. The culture of Greece and Rome is nearly lost. A thousand years pass. Then slowly during the Renaissance a new civilization emerges, now not around the Mediterranean but farther north. First the old classical culture is rediscovered; then a few brave spirits begin to think for themselves. Science in the modern sense begins. And with science things move more rapidly. In the eighteenth century, steam begins to push pistons. The *tempo* increases; in the nineteenth century the sciences come of age. Steel and electricity! And now the hectic twentieth century: steel, steel everywhere; electric power in factory and home; telephones and radio and moving pictures; big business and advertising and general education.

This has been the progress of civilization. A man today is a part of a great organism; he differs greatly from that primitive man squatting before the fire. He is born, matures, mates, and dies as before, but life as a whole is a very different thing today from what it was a generation ago, vastly different from the life of our earliest human forefathers. A world war today, with perhaps a whole civilization in the balance, is greatly different from the tribal feud of the cave man. Factory production is different in all its implications from the home crafts of the past. Life with a whole world tied together in a radio broadcast is a different thing from the life when each village was isolated from others, free to develop its own provincial customs. Life in an age of science and rationalism is a different thing and demands a different level of thought from life in an age bound by superstition and custom. It demands a *critical* habit of thought, for without this our civilization bears within itself the seeds of its own destruction.

TOOLS

2. Muscles for Machinery. The most obvious single difference between a city of today and the Babylon of 2000 B.C., or the Athens of 500 B.C. or the Rome of 100 A.D. or the London of 1500 A.D., lies in the machinery available for doing man's work. Until about 1750 most of the work of the world was done with muscles — usually the muscles of men, but occasionally the muscles of horses or oxen or elephants or camels. A certain Caecillus in the days of Augustus had 4000 slaves — perhaps equivalent to 400 horsepower; but this is no more than we may find in a single airplane engine today. Occasionally wind power was used; very rarely, water power; but most of the world's work was done by human labor.

3. Steam for Power. Steam engines were first used in English coal mines about 1700. Shortly before, an expatriate Frenchman, Papin, who was demonstrator for the Royal Society in the time of Newton and Boyle and Hooke, had taken the " demons " out of steam by inventing the safety valve. He was experimenting with a pressure cooker (a bone digester as a demonstration for the physicians in the society) when he found that steam would make things move. So Papin's bone digester became one of the earliest steam engines. Thomas Newcomen invented the first practical engine and Newcomen engines were used for many years to pump water out of coal mines. Great clumsy things these early engines were, so wasteful of fuel that they could not be used outside the coal field.

About the time of our American Revolution, James Watt, an instrument-maker, improved this steam engine — and in the early years of the new century this new servant found its way into the factory, onto the rails and into water transportation. The Industrial Revolution had begun. Many of the early inventors were mechanics and blacksmiths and carpenters. But in the nineteenth century science asserted itself. Thermodynamics was developed, and chemists invented steel. The age of steel and the age of machinery began. The older push-pull engines made way for the smaller, faster steam turbines; and electricity was used.

4. Electricity for Convenience. Steam engines were smoky and clumsy. The steam engine could not take the place of the servant in the home — and in the factory the steam engine meant a wilderness of belts and rotating shafts. In 1831 Faraday discovered the principle of electromagnetic induction, and by the end of the century we had a new servant, cleaner and more flexible than the steam engine. Electrical power — the conversion of coal energy, oil energy, or water energy into electrical energy and into work — so that the average workman in an American factory has at his disposal 5 horsepower of available energy, so that he can produce ten thousand barrels of flour or make a thousand needles or a hundred bricks, while his predecessor of a few centuries ago was making one. A single large electrical generator will produce the power of 200,000 horses. Electricity carried the age of power into every nook of our life.

It is mechanical power and electricity — the steam engine, gasoline engine, electrical motors and appliances — which, more than anything else, have changed the material aspect of our civilization.

THE MEANINGS OF THINGS

5. Throughout all history we find men asking questions. At first man found in every phenomenon of nature the whim of some god or spirit or witch or fairy. The stars were points of light in the roof of the world; having no other obvious function they were supposed to be concerned with human destinies — hence the study of astrology.

In the Greek cities scattered about the eastern Mediterranean science was born. The Greeks had a real enthusiasm for truth. They asked themselves about the sun and stars, about the meaning of time and space, the nature of light and sound; they asked about the fundamental constitution of matter. Pythagoras and Democritus and Aristotle, philosopher-scientists; Euclid, the mathematician; Archimedes, mathematician and physicist; and Hipparchus, the astronomer — then, after them, fifteen hundred years of twilight.

The rediscovery of Greek learning, the discovery of printing, and the discovery of America did something to men's minds. A

Polish monk by the name of Kopernik (Copernicus) taught that the earth went around the sun. A great painter and sculptor and scientist, Leonardo da Vinci, saw things as they were. Galileo, studying the fall of a stone, looked to nature itself rather than to the dogmas of Aristotle and the ancients. And so in the sixteenth and seventeenth centuries science was born again in Europe. And these things did something more to men's minds — because of belief in the Copernican system men were imprisoned and burned at the stake.

Late in the seventeenth century groups of " natural philosophers " banded themselves together into societies. In England the Royal Society was founded. Among its members were Robert Boyle, pioneer in chemistry, and, one of the remarkable men of all time, Isaac Newton. Newton showed that the force which guided the planets in their orbits and the force that made the stone fall to the ground were one. And this did something more to men's minds. There was seen to be a great oneness in all nature. But a century later Voltaire in France was still fighting against prejudice for Newton's ideas.

During the eighteenth century physics seemed to be catching its breath after these great advances. The nineteenth century was the century in which physics matured. The laws of heat, sound, light, electricity were discovered and put to use. Unity was found everywhere. Heat was the mechanical energy of the molecules; light, a form of electric wave. Beneath all were the laws of *conservation of energy* and *degradation of energy.* The chemists discovered the elements, and the biologists discovered evolution. All nature seemed to yield to the theories of the scientist.

Scientific progress in the last forty years has been unprecedented. Scientists have multiplied tenfold in number. Einstein revolutionized concepts of space and time. Explorers in the atomic world have, in the last two decades, turned topsy-turvy our most fundamental notions about things, undermined our ideas of cause and effect. America, forty years ago almost unheard from in the field of pure science, bids fair to lead the world. And what all this does to our minds remains to be seen.

So far as we know, during the last eight thousand years man has not developed by biological evolution; if he acts and thinks differently it is as a result of his own progressive conquest over his environment. The superstitious beliefs, the customs of the past, victories and defeats on the battlefield, these have all been important as producing stability or haphazard change. But these have not led to the *progress* of civilization; what consistent development we can find has been due very largely to the increasing body of thought which we call science. Much of it is due to physical science. Physics has grown rapidly since the time of Galileo, and its advance closely parallels the changes in civilization. Out of physics and chemistry come most of our inventions. And physics (allied with astronomy) has told us the size of the heavens, and (with geology) the age of the earth, and (with chemistry) the mass of the atom.

DREAMS

6. It is not a dead world which we study; we study the dreams of men, dreams which have materialized into a magic world. Our study is inherited from the contemplations of a Greek philosopher, the vision of Newton, the experiments of Faraday — from generations of scientists inspired to seek truth in nature. I do not know what you will see in the workings of an electric motor, in the wonder of the snow crystal, in the story of the atom, in the unity of nature, in the meaning of space and time. Dreams are not things for words; the beauty which you find will depend upon the quality of your imagination; but unless you learn to see a great wonder in these things it is not physics which you are studying but a dreary subject which died aborning.

CHAPTER 2

THE PHYSICS OF COMMON KNOWLEDGE[1]

A large part of physics, and especially of mechanics, seems almost intuitive. One could scarcely live in our age without this knowledge. It is the purpose of this chapter to remind the reader — and particularly the reader who has had no previous course in physics — of some of this commonplace knowledge. Perhaps some of the words will be strange — the facts are the most ordinary. Let us begin with pushes and pulls.

7. Forces. Physics deals largely with forces and the motions which forces produce. A force is simply a push or a pull. There is the upward force that we must exert to hold a heavy weight; there is the force that the horse exerts as it pulls the wagon; there are the forces applied by our two hands, one force to the right the other to the left, as we break a piece of string. These are familiar forces. We can measure them in pounds or ounces or tons.

When I exert a force on an object, when a rubber band pulls, when a compressed spring pushes, the source of such forces seems quite obvious. But upon all things about us there acts one force which seems more mysterious in its action. All things — chairs, tables, houses, trees — everything about us — is pulled down toward the earth. It is almost as if each object were pulled toward the earth by invisible rubber bands. This earth-pull we call the *weight* of the body. A strong earth-pull acts on a piece of lead; a lesser one on a feather; still less upon air. But upon all things acts this "force of gravity" — everything has its weight.

We shall meet other, less familiar, forces in our study, but at present we deal only with forces of commonest knowledge — the pull of the rope, the thrust of the arm, the back push of friction tending to stop a moving body, the downward force of gravity upon everything on earth. Let us take a rather casual survey of these forces. What is the direction of the force applied to the ball when a pitcher sets the baseball in motion? What is the direction of the force on the ball when the catcher catches it or the batter strikes it? Not only does the bat exert a force *on the ball*, but the ball exerts a force *on the bat*. In what direction is this latter force? How does the force of the bat on the ball affect the latter's motion? What does the ball do to the motion of the bat? This force, acting for only a moment between bat and ball, is less easy to measure than the steady force exerted by a taut rope, or the force required to support a weight. (It sometimes breaks the bat — would you estimate it as a few ounces or pounds or tons?) . . . What is the direction of the force with which the oarsman pulls upon the oar; how does the oar push the oarlock and boat; how does it push the water? . . . What downward forces and what upward forces act upon a seesaw?

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[1] It is essential that the student studying physics have a working knowledge of proportion. He should at this point study the chapter on proportion given in the Appendix.

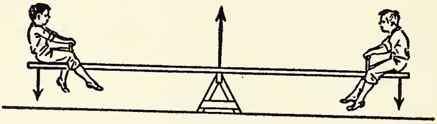

FIG. 1[1]

In analyzing such problems each force should be represented by an arrow. Draw figures for each of the above examples and represent the forces in this way.

8. Action and Reaction. Whenever one body acts on another two forces are involved: the force with which body A pushes on body B, and the force with which B pushes back on A. These forces are called, respectively, the force of *action* and of *reaction*; we shall find that they are always equal and always opposite in direction. Perhaps this is obvious. But unless he states his problem clearly the student is liable to confuse these two forces. The ball pushes forward (in the direction of motion) upon the catcher's glove, but the force on the ball (by the catcher) is in the opposite direction. What is the reaction to the force of the horse on the wagon; of your upward thrust when you hold a weight; of the force of the boy downward upon the seesaw?

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[1] When giving reference to figures outside a current chapter, the chapter number will be included in bold type; Fig. **4**–1 is Fig. 1 of Chapter 4.

9. Equilibrium. Forces act on everything about us — up, down, east, west, north, south. These pushes and pulls appear to be engaged in a continual tug of war. Usually the forces acting upon a body in one direction are balanced by equal forces acting upon it in the opposite direction, and the body remains at rest. The body is then said to be in equilibrium. I pull a box north and you pull it south, and if the forces are equal it remains at rest. . . At the same time gravity pulls the box down and the floor pushes it up. My force and your force and the earth down-pull and the floor force upward altogether exactly balance and produce equilibrium. . . The boys push down on the seesaw and the fulcrum pushes up — again equilibrium. . . What forces produce equilibrium upon a tree in a wind; upon an elevator; upon the lowest block of a pile of blocks?

When the forces upon a body do not balance, a change of motion is produced; the pitcher with his forward force starts the flight of the ball; the catcher with his opposing thrust stops it.

10. Effect of Non-Parallel Forces. Consider the case where two forces do not act in exactly the same or in exactly opposite directions. One may act north, the other east. The combined effect will then be that of a single force acting in some intermediate direction; this resulting force will be less than the sum of the two because the two do not exactly cooperate. In a later chapter we shall describe how the resultant effect of such oblique forces is found.

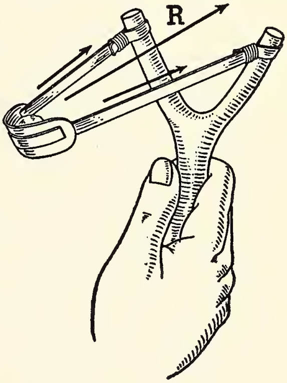

FIG. 2. Resultant of oblique forces.

11. Force-Pairs. If two equal forces act upon a body in opposite directions they produce an equilibrium of *forces* and there is no tendency for motion of the body as a whole. (Motion of a body as a whole is called *translation.*) Two separate cases of such force-pairs are to be considered:

(1) If the forces do not act along the same straight line they form a *couple* (forces A, B, Fig. 3). Quite clearly, a couple will produce turning if the body is free to move. If you and I pull

upon our box in this manner we shall rotate it. This tendency to produce rotation depends upon (*a*) the magnitude of the forces making up the pair, and (*b*) the distance between their lines of action (the " couple arm "). (Evidently it will be easier to turn the box if the forces act as *AB* rather than as *CD*.) Indeed, we define a couple as existing when two equal opposite forces act not in the same straight line, and the **magnitude of the couple**[1] as being the product of one of these equal, opposite forces by the couple arm (l).

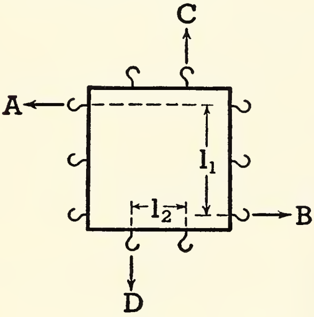

FIG. 3

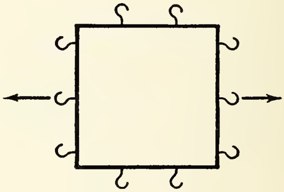

FIG. 4

(2) If the forces of the pair act in the same straight line there is no couple (Fig. 4). There is no tendency to produce translation or to produce rotation. The effect of such a force-pair is only to produce a stretching of the body.

12. Our present object is only to put into words the knowledge which the student already has. It may be that we do not recognize clearly the forces acting in the several problems; if not, it is a language difficulty. Perhaps we do not visualize the oar or the seesaw as we read the words. If not we shall have difficulty in learning of nature from any book. It is quite possible that we have never before heard of couples, but certainly we know that to turn a box requires a thrust of one arm and a pull by the other — surely all know how to twirl a pencil in the fingers. And my dog pushes with its paws and pulls with its teeth as it tries to tear the meat from its bone; it knows that a pair of opposite forces is required for such a task. This dog differs from a physicist in not being able to express its knowledge in language. And if we are to go beyond these very simplest instinctive processes it will be necessary to use language.

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[1] Bold-face type will occasionally be used to emphasize a term where it is being exactly defined.

There has been nothing numerical in the above examples. But a large amount of quantitative knowledge is also the heritage of our common experience. Even this dog of mine knows that the greater its speed the less time it takes to cover a certain distance — although it knows nothing of inverse proportion. And although perhaps we have never expressed the proportion:

$$\text{Distance} = \text{Speed} \times \text{Time},$$

surely we all know that the longer we travel (at a given speed) the farther we shall go. This is an equation of physics, but it represents only instinctive knowledge. We shall give one or two other examples of this commonplace physics which uses proportion.

13. Pressure in a Liquid. Anyone who has dived under water knows that the pressure which he can feel on his eardrums is greater at a greater depth. It is a matter of general knowledge that at great depths in the ocean the pressure becomes so great that a submarine will be crumpled in. As a matter of fact, *the pressure is directly proportional to the depth.*

A foot below the surface the pressure is 0.433 pound per square inch; 2 feet below it is 0.866 pound per square inch — what is the pressure 10 feet down, 100 feet down?

If the force at a certain depth in the ocean is 1 ton on a square yard, what will be the force at twice the depth on twice the area?

These are simple problems in proportion.

14. The Stretch of a Rod. Now consider a more complicated example. A rod (or a wire or a rubber band) is more or less stretched when it supports a load. If the load is not too great the rod resumes its original length when the load is removed. Everyone knows that of two rods the one with the greater cross-section will stretch the less. This assumes that the same force is applied to each[1] and that each rod is of the same material and each has the same length. Under these conditions a rod of twice the cross-sectional area will offer twice the resistance to stretching; and a given force will stretch it half as much. The doubly thick rod is equal to two of the smaller rods. So if L is the length, e the elongation, A the area, and r the radius[1]

---

[1] Of course equal and opposite forces must act on the two ends of the rod — a force-pair. Probably one end will be fastened and the force applied to the other.

$$\Delta L = e \propto \frac{1}{A},$$

or, since A varies as r^2, $$e \propto \frac{1}{r^2}.$$

(This inverse proportion can, of course, be written in other ways [App. I]; for example: $e_1/e_2 = r_2^2/r_1^2$ or $e = c/r^2$.)

If not only the cross-section of the rod but also its length and the applied force are varied, we have a case of joint proportionality. Under the same force, which will stretch more, a rod 1 foot long or 1 yard long? How will the elongation vary if the force is doubled? Will e vary as $1/F$ or as F^2 or as F?

15. Hooke's Law. The Puritan revolution ended in 1660 and Charles II came to the throne. Science was just awakening in England. In that year the famous Royal Society was founded ("The Royal Society for the improving of Natural Knowledge by experiments"). Here the "natural philosophers" met and discussed and sometimes quarreled. Robert Boyle, the "Sceptical Chemist," and "Mr. Isaac Newton, professor of the mathematicks at Cambridge," were members of the young society. Robert Hooke was the first "curator of experiments." His investigations covered a wide range; many of them are forgotten, but his name is preserved for every physics student in Hooke's Law:

"*Ut tensio sic vis.*"[2]
"As the extension, so the force."

$$e \propto F.$$

So Hooke answered the question of our last paragraph: the elongation is doubled when the force is doubled. To be sure, this exact proportionality is not directly obvious from our everyday experience, but it is certainly a not unexpected relation and is easily verified.

---

[1] The elongation is the change in length $L_2 - L_1$. It is often convenient to write this as ΔL (delta L, but read "the change in L"). We shall frequently use this symbol Δ to represent a difference or a change. Read this: "The change in length (or e) is inversely proportional to the area or varies inversely as r^2."

[2] The law was first given as an anagram, with letters of the Latin words scrambled: "ceiiinosssttuv." This was at the time a more or less common way of patenting an idea.

We may then summarize the laws of stretching: the elongation of a rod will be proportional to the applied stretching force (Hooke's Law) and will be proportional to the original length of the rod and will be inversely proportional to its cross-sectional area:

$$\Delta L \propto \frac{FL}{A}.$$

16. Analyzing Physical Equations. We are now not primarily interested in these physical laws. The laws of stretching will be studied in a later chapter. We wish to show how this proportionality, which is after all a fairly complicated relationship, can be obtained by the student for himself by rather simple analysis. The student should analyze physical problems for himself. Much of our earlier work will go little beyond such commonplace facts as these. The majority of our physical equations are only expressions of joint proportionality, showing how A depends on B and C and D. Always analyze such an equation before leaving it; break it apart. "If B and C are constant then A is proportional to D; is this reasonable?" "Other things being constant, A varies inversely as B^2, so if B is doubled A is divided by 4; how was that shown?" Equations may be soon forgotten; once these relative variations between physical quantities are understood they become a permanent part of our mental equipment.

UNITS

17. All know the common units of length (feet, inches, etc.) and of weight (pounds, tons, etc.) and of time (seconds, years, etc.).

From these, other units are derived: square feet or square inches, etc., for area; cubic feet or cubic inches for volume. (In common use are other volume units, such as quarts, bushels, barrels, etc., but these introduce needless confusion.) Speed is measured in miles per hour, or feet per second, etc. Pressure is measured in pounds per square inch, or ounces per square foot, etc.

18. The Metric System. The metric system, though not in common use in English-speaking countries, is generally used in science. Being a decimal system it has the same advantage that our dollars and cents have over the English pounds and shillings and pence. The system is based on the *meter* of length (slightly more than a yard) and the *gram* of mass (about one-thirtieth of an ounce). Other units larger and smaller than these are designated by the prefixes: *mega* (millionfold), *kilo* (thousandfold), *deci* (tenth), *centi* (hundredth), *milli* (thousandth), and *micro* (millionth).

TABLE 1

METRIC UNITS[1]

Length

Kilometer (km)	1000 meters	1.6 km = 1 mile (approx.)
Meter		39.37 inches
Centimeter (cm)	$\frac{1}{100}$ meter	2.54 cm = 1 inch 30.5 cm = 1 foot
Millimeter (mm)	$\frac{1}{1000}$ meter	

Also megameter, decimeter, micron ($\frac{1}{1000}$ mm) and angstrom unit (10^{-8} cm)

Volume

1000 cubic centimeters (cc) equals 1 liter (approximately a quart)[2]

Mass

Kilogram (kg)	1000 grams	2.2 lb.
Gram (gm)		454 gm = 1 lb.
Milligram	$\frac{1}{1000}$ gram	

1000 kg equals a "long ton" (2200 lb.)

The metric system of units was first established in France in the time of Napoleon. The meter was originally chosen as the ten-millionth part of the distance from the equator to the pole. More exact world measurements have shown that this relation is

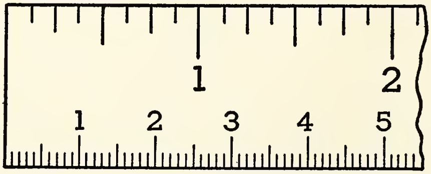

FIG. 5. One inch equals 2.54 centimeters.

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[1] The values given in this table should be memorized.

[2] In strict usage the *liter* is defined as the volume occupied by a kilogram of water at 4° C. It is thus practically but not exactly equal to 1000 cc.

only a close approximation. Today the meter is legally defined as the " distance at the temperature of melting ice between the centers of two lines traced on the platinum-iridium bar deposited at the International Bureau of Weights and Measures " at Sèvres (France). Copies of this prototype have been furnished to the Bureaus of Standards of the various countries.

The legal kilogram is the mass of a platinum-iridium cylinder preserved by the International Bureau. It was originally intended to be the mass of 1000 cubic centimeters of water at the temperature of its maximum density (4° C); the legal kilogram actually differs from this by about 0.003 per cent. A cubic centimeter of water weighs practically 1 gram.

19. Standard of Time. In addition to these length and mass standards we need a standard of time. A *solar* (" sun-time ") *day*, is the time from true noon (sun at meridian[1]) to true noon. The solar day varies slightly through the year; its average value is the *mean solar* (" clock-time ") *day*, and this is our time standard. Radio time signals are given out from the U. S. Naval Observatory at Washington each day at mean noon, marking the duration of the mean solar day. The 1/86,400 part of a mean solar day is a second.

The mean solar day is about 4 minutes longer than the *sidereal* (" star-time ") *day*, which is the time required for the earth to make exactly one rotation. Because of the motion of the earth about the sun the earth turns slightly more than 360° from noon to noon and makes 366¼ rotations in a year (365¼ solar days).

QUESTIONS

1. What kinds of bodies float in water? Certain bodies will float in salt water but not in fresh; why?

2. It is said that a body in Australia falls in the opposite direction from a body in America. Would you agree to this? If so, how would you account for it?

3. Why does not the moon fall down?

4. When used for a foot bridge, do four 1 by 8 planks tied together have the same strength as a 4 by 8 beam?

5. What is the function of a lever?

6. If a 2-lb. weight and a 1-lb. weight are on opposite ends of a yardstick, can the stick be balanced? How and why?

7. Is it possible for three men to pull, each with a force of 50 lb., upon ropes attached to a light box without moving it? If so, how?

8. Is it possible for a man to swim 2 miles an hour in a stream flowing 2 miles an hour and yet move only at the rate of 2 miles an hour?

9. If a man swims 3 miles an hour in a stream flowing 2 miles an hour, what is the greatest and least speed he can have?

---

[1] In other words, at its highest elevation, due south from the observer.

10. Make a rough estimate of the thickness of this book in centimeters; of the thickness of a single page in millimeters.

11. Estimate the weight of the book in grams and of a single page in milligrams.

12. What is your approximate weight in grams and your height in centimeters?

Vocabulary: Action and reaction, translation, couple, force-pair, metric system, solar (mean solar, sidereal) day.

PROBLEMS

1. What forces act, forward and backward, upon the baggage car of a train? Draw a figure.

2. Twisting. A certain wire is twisted through 180° by applying opposite forces of 10 lb. each at the opposite ends of a 6-in. cross bar. How much force would be required to do the same thing if the forces are applied only half as far out? Why? How much is the couple?

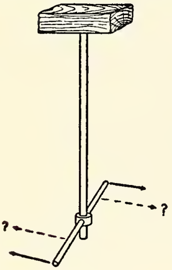

FIG. 6

3. If water flows 20 ft. per sec. through a pipe of $\frac{1}{2}$ sq. in. section how many cubic feet are delivered per minute?

4. If an elevator with a 1-in. cable can safely support 3000 lb. how many pounds can a 2-in. cable, twice as long, safely support?

5. A mile per hour is how many feet per second?

6. If a horse can lift 550 lb. a foot a second (or 1 lb. 550 ft. per sec.), how big a job is it in terms of horsepower-hours to lift 550 tons 20 ft.?

7. How far will a train going at 40 miles an hour go in 12 min.?

8. If a train, starting from rest, gains speed steadily until after 10 min. it is going at the rate of 30 miles an hour, what is its average speed, and what distance will it traverse in that time? What is its rate of "pick up" — i.e., how much did its speed increase per minute? If it continues to gain speed at this same rate, will it go twice as far in twice the time?

CHAPTER 3

MATTER

Physics treats of matter and energy. The ground, the trees, ourselves, the water we drink, the air we breathe, these are all things of substance, material bodies — matter.

Matter occurs in three obviously different forms — solid, liquid, and gas. When we make this classification, however, we must remember that it depends upon temperature. There is nothing so solid that it cannot, at sufficiently high temperature, be melted and evaporated.[1] There is no gas which cannot be liquefied and frozen.

Irrespective of any possible change in its form the total amount of matter in any closed system remains constant. This is the law of Conservation of Mass. Presumably the total amount of matter in the universe remains constant.

20. General Properties of Matter. There are three properties of matter which seem fundamental:

Matter occupies space. This means that where one material object is, another cannot be. The stone strikes the wall and is repelled; as the stone sinks into water it displaces an equal volume of water upwards; the wind blows against the sail and is reflected.[2] In virtue of this space-possessing property, matter has *volume.*

Matter has inertia. This means that matter resists a change in its motion. It is hard to stop a fast-thrown ball; it is hard to start a brick into sudden motion; kick it and we hurt our toe; a water jet exerts a force as it splashes against a wall. It is this property of inertia which, better than all else, measures the amount of matter — its *mass.*

---

[1] Not always, however, without chemical change. Complex chemical substances (for example, flesh and blood and vegetable matter) may decompose upon heating.

[2] If there seems to be an exception when one gas mixes with another or when a lump of sugar is dissolved in water, the exception is only apparent, for the molecules of one gas must find their places between those of the other. Two molecules cannot occupy the same position.

Matter has weight. Everything about us is pulled down toward the earth. All matter has *weight.*

21. Weight and Mass. This weight is the earth-pull on the body. Although nearly constant over the surface of our earth, weight decreases with altitude and at great distances from the earth becomes very small. On the other hand the inertia of a material body is an inalienable property of the body; inertia is a true measure of mass. However (and it is a fact of greatest significance[1]), in a given locality there is an absolutely exact proportionality between the mass and the earth-pull. Because of this proportionality masses can be compared by measuring their weights on a balance, and (since this is more convenient than measuring their inertias) masses are usually determined in this way. This fact sometimes leads the student into confusion between the mass itself and its weight.

22. Density. One of the most obvious differences between substances is that, volume for volume, some are heavier than others — iron than water, water than cork. Some are *denser* than others. **Density** is the mass per unit volume:

$$d = \frac{m}{V}. \qquad \text{(Definition)}$$

The density of water is 62.4 pounds per cubic foot, of iron 490 pounds per cubic foot — nearly 8 times heavier. Lead is 11 times, mercury 13.6 times, and osmium (heaviest substance) 22.5 times heavier than water. The ratio of the weight of a substance to that of an equal volume of water is called its **specific gravity**. (Since in the metric system the density of water is unity, the specific gravity is numerically equal to the density expressed as grams per cubic centimeter.)

23. The Structure of Matter. Let us now turn from these things of everyday sense perception. Seeking a cause for things, our long-ago forefathers placed in every tree a sprite, invented fairies to sprinkle the dewdrops, and heard the wrath of the gods in the thunder. Perhaps this old belief once seemed to be the theory of "common sense," but today the scientist has a very different view of the inner nature of matter. He sees all things as made up of countless numbers of small particles all in violent motion — molecules. In gases these molecules but thinly populate the space they occupy; they move rapidly hither and thither, striking one another and bouncing away. If we try to compress the gas, as in a bicycle pump, the molecules pound against the plunger and furnish the pressure that repels the intruder. The molecules all attract one another, sometimes slightly as in a gas, sometimes so effectively as to bind them together into the denser communities which we call liquids and solids. Here in solids and liquids there is little room; the molecules are crowded together; but still they move just as rapidly as in the gas, chattering back and forth, forever jostling their neighbors. This is the scientist's picture of matter — what led him to it will be part of the story of physics.

---

[1] So striking is this fact that, in his Theory of Relativity, Einstein finds the reason for this proportionality in the very nature of space itself.

TABLE 2

DENSITIES OF ELEMENTS AND COMMON SUBSTANCES

	gm. per cc.	lb. per cu. ft.
Alcohol	0.8	50
Water	1. *	62.4*
Sea water	1.025	64
Mercury	13.6*	850
Aluminum	2.7	170
Copper	8.3	520
Iron	7.9*	490
Lead	11.3	700
Gold	19.3	1200
Osmium	22.5	1400
Cork	0.25	15
Common woods	0.5–0.8	30–50
Glass	2.6	165
Marble	2.7	170

* These values will be frequently used and should be memorized.

24. Elements and Atoms.[1] There are very many different kinds of molecules — as many as there are different kinds of substances in the world. In his reference books the chemist lists some tens of thousands of, to him, familiar substances. But, about 150 years ago, when modern chemistry really started, the chemist began to realize that all these things are compounded out of a small number of elementary substances. Today it is known that there are 92 such *chemical elements* (Table 3). Some are common things like the nitrogen and oxygen in the air, the carbon in a charred stick and in a diamond and in all living things, the hydrogen gas which inflates our balloons, and the metals: iron, copper, gold, silver, etc. Others like sodium and calcium are common enough, but never found in nature except in compounds. Others like radium are very scarce. But of these 92 elements all things are made.

---

[1] The student unfamiliar with chemistry need not be disturbed if he does not attain from this brief introduction a thorough understanding of atoms and molecules.

This idea of the chemical element developed toward the end of the eighteenth century. A little later it became apparent that each such elementary substance was made up of small particles — they were called atoms (from the Greek, meaning *indivisible*). These atoms unite together into pairs or larger groups and these are the molecules. Two atoms of hydrogen and one of oxygen unite to form a molecule of water (H_2O); sodium (Na) and chlorine (Cl) unite to form common salt (NaCl). Out of such compounding of atoms come the uncounted numbers of different substances.

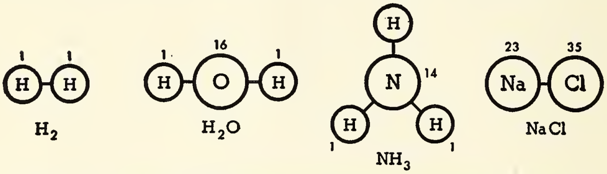

FIG. 1. Simple molecules (schematic): hydrogen, water, ammonia and common salt.

25. Atomic Weight and Atomic Number. It was another century before scientists found the actual sizes of these atoms. But out of his analyses the chemist found their *relative* sizes. He found that water was (approximately) two parts (by weight) of hydrogen and sixteen parts of oxygen. Hence (since there are two H atoms in the water molecule) the weights of the individual hydrogen and oxygen atoms must be in the ratio of 1 : 16 (approximately). In this way it has been possible to find the *relative* weights of all the atoms. These are given in Table 3. Hydrogen is the lightest, helium is four times heavier; then after some less

TABLE 3. PERIODIC TABLE OF ELEMENTS.

	I	II	III	IV	V	VI	VII	VIII	0
1	1 Hydrogen 1.0078								2 Helium 4.002
2	3 Lithium 6.940	4 Beryllium 9.02	5 Boron 10.82	6 Carbon 12.00	7 Nitrogen 14.008	8 Oxygen 16.00	9 Fluorine 19.00		10 Neon 20.183
3	11 Sodium 22.997	12 Magnesium 24.32	13 Aluminum 26.97	14 Silicon 28.06	15 Phosphorus 31.02	16 Sulfur 32.06	17 Chlorine 35.457		18 Argon 39.944
4	19 K 39.096	20 Ca 40.08	21 Sc 45.10	22 Ti 47.90	23 V 50.95	24 Cr 52.01	25 Mn 54.93	26 Fe 55.84 27 Co 58.94 28 Ni 58.69	
	29 Cu 63.57	30 Zn 65.38	31 Ga 69.72	32 Ge 72.60	33 As 74.93	34 Se 79.2	35 Br 79.916		36 Kr 83.7
5	37 Rb 85.44	38 Sr 87.63	39 Y 88.92	40 Zr 91.22	41 Cb 93.3	42 Mo 96.0	43 Ma?	44 Ru 101.7 45 Rh 102.91 46 Pd 106.7	
	47 Ag 107.88	48 Cd 112.41	49 In 114.8	50 Sn 118.70	51 Sb 121.76	52 Te 127.5	53 I 126.93		54 Xe 130.2
6	55 Cs 132.81	56 Ba 137.36	RARE EARTHS	72 Hf 178.6	73 Ta 181.4	74 W 184.0	75 Re 186.31	76 Os 190.8 77 Ir 193.1 78 Pt 195.23	
	79 Au 197.2	80 Hg 200.61	81 Tl 204.39	82 Pb 207.22	83 Bi 209.00	84 Po (210.0)	85*		86 Rn 222.0
7	87*	88 Ra 225.97	89	90 Th 232.12	91	92 U 238.14			

Rare Earths	57 La 138.90	58 Ce 140.13	59 Pr 140.92	60 Nd 144.27	61 Il?	62 Sm 150.43	63 Eu 152.0	64 Gd 157.3
	65 Tb 159.2	66 Dy 162.46	67 Ho 163.5	68 Er 167.64	69 Tm 169.4	70 Yb 173.5	71 Lu 175.0	

familiar elements comes carbon, three times as heavy as helium, then nitrogen, oxygen, and so through the table to uranium, the heaviest element, whose weight is to that of oxygen as 238 : 16. These relative weights are called *atomic weights*.[1]

The elements are numbered from lightest to heaviest; these are the *atomic numbers*.

26. Molecular Weights. Using this same relative scale, in which the weight of the oxygen atom is taken as 16, the relative weights of molecules can be expressed. So the *molecular weight* of water (H_2O) is $1 + 1 + 16 = 18$; for common salt (NaCl) it is $23 + 35.5 = 58.5$. Some molecules are larger and heavier. Sugar, for instance, is $C_{12}H_{22}O_{11}$, and its molecular weight is 342. The complex molecule of hemoglobin (which gives the red color to the blood) has a molecular weight of nearly 70,000. Some molecules consist of only single atoms (monatomic). This is true for the rare gases (helium with molecular weight 4; neon with molecular weight 20.2; etc.) and metal vapors. Although the common gases hydrogen, oxygen, nitrogen are elements, their molecules are not monatomic but consist of two similar atoms. The hydrogen molecule is H_2 (molecular weight approximately 2); the oxygen molecule is O_2 (molecular weight 32). The molecules in these gases split up into single atoms at very high temperatures, but as we find them their atoms are always paired.

27. Density of Solids and Atomic Number. A striking relation exists between density and atomic number. Figure 2 shows how, as we go through the sequence of elements, the density increases and decreases in regular fashion. (The density in each case is for the element in its solid state.) In fact, the elements are classified by the chemist into different " periods " with 2, 8, 8, 18, 18, and 32 elements in them. Many other properties behave like density; the elements gradually change as we go through a period and then the properties repeat. Taking cognizance of this periodicity, in Table 3 elements of similar properties are placed below each other. To accommodate the longer periods certain elements are shown boxed. These are known as transition elements. They resemble one another more or less and are not to be considered as belonging (in chemical properties) to the column in which they appear.

---

[1] It is often convenient to consider definite quantities: 1.008 grams of hydrogen, 16 grams of oxygen, etc. These quantities are *gram-atomic weights*. Similarly, a quantity of a substance which in grams is numerically equal to its molecular weight is called a *gram-molecular weight* (or a *gram molecule*).

Were atoms of different kinds similarly spaced in solids there would always be the same number of atoms per cubic centimeter and the density would be proportional to the atomic weights. Figure 2 shows that this is not so — some atoms are closely packed as in copper (Cu), rhodium, osmium, and platinum; some (as in sodium [11], potassium [19], etc., called the *alkalis*) are loosely packed. To explain this would take us very far into the realm of physics. For the present this periodic recurrence of properties can be taken to show that beneath all the manifold forms of nature lies a fundamental unity, an underlying law which transcends anything which common experience would lead us to anticipate.

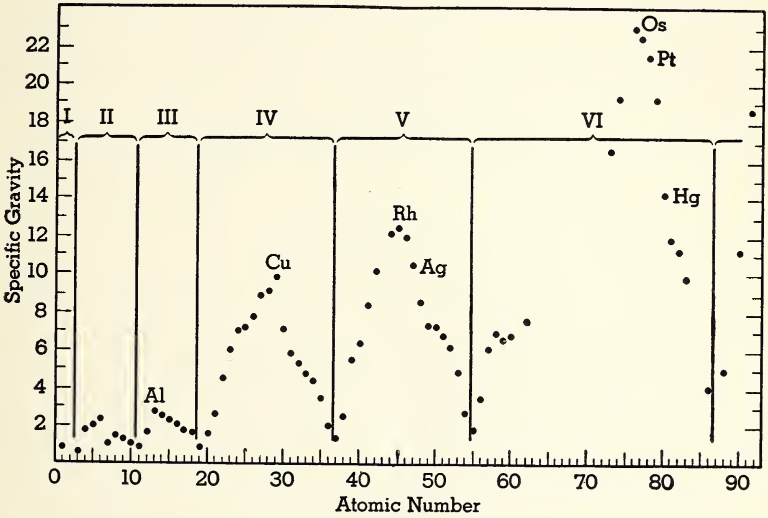

FIG. 2. Density and atomic number of elements in the solid state.

28. The Range of Physical Measurement. One must be greatly impressed by the vast range of magnitudes accessible to physical measurement — the bigness and smallness of measurable things.

A small marble has a diameter of about 1 cm; reduce this 10,000 times and we have the diameter of the smallest bodies clearly visible in the microscope (10^{-4} cm); 10,000 times smaller still (10^{-8} cm) are the approximate diameters of the atoms. But the atom itself is composed of electrons moving about a nucleus, somewhat as the planets in the solar system move about the sun. This atomic solar system has a diameter of about 10^{-8} cm, but the diameter of the nucleus is usually about 10,000 times smaller (10^{-12} cm). And the nucleus itself is composed of protons and neutrons, smaller still!

In water the distance between molecules[1] is about 3.1×10^{-8} cm (i.e., 0.000000031 cm). That is, in a centimeter cube of water there will be 3.2×10^{7} (32,000,000) molecules along one edge or 33×10^{21} molecules in the cubic centimeter — 33 thousand million million million. If the whole population of the United States started counting molecules, counting one per second day and night, it would take them nearly ten million years to count this number. Or, to use another illustration, such a number of grains of sand would cover the surface of the earth with a layer one grain thick. This is for a gram of water. In a gram molecule of water (18 grams) or a gram molecule of any other substance there are 6.02×10^{23} molecules.

Contrasted with these atomic magnitudes are the quantities measured by the astronomers. The sun is (roughly) 100 times greater than the earth in diameter, and 1,000,000 times greater in volume. Our sun is an average star. A hundred billion stars make up our milky way, our galaxy, and millions or billions of galaxies (nebulae) make up the universe. It has been estimated that there are some 10^{79} or 10^{80} electrons in the universe.

Thinking of the world of the microscope, and beyond, Dean Swift ("Gulliver's Travels") wrote:

> "Great fleas have little fleas upon their backs to bite 'em,
> And little fleas have lesser fleas, and so ad infinitum."

To which a distinguished mathematician (De Morgan) added:

> "And the great fleas themselves, in turn, have greater fleas to go on;
> While these have greater still, and greater still, and greater still,
> and so on."

29. We add then to the fundamental properties of matter:

Matter is not infinitely divisible but consists of discrete particles (atoms).

There are 92 kinds of atoms differing in chemical properties and countless numbers of different kinds of molecules formed from them.

There is an underlying relationship between the properties of the different atoms.

It is the chemist who stresses the difference between different forms of matter; the physicist usually deals with those properties which all matter has in common. In the following chapters we treat of the forces acting on material bodies — this is mechanics. Mechanics deals with commonplace things, and there is continual opportunity for the play of the student's common-sense experience. Then we study the various forms of energy — heat, sound, electricity, and light — and their effects on material bodies. And always there are two aspects of physics: physics as the practical guide to our mechanical civilization, and physics as the guide to the significant realities in nature — the physics of the engineer and the physics of culture. The two cannot be sharply divided, but some will appreciate physics for the one reason and some for the other. But on whichever side his interest lies, the student must see nature exactly and logically and with self-reliance. The author suggests that the student use technical words carefully, find original examples of each concept and keep the eyes upon the relations of things — which usually means to watch the proportions.

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[1] The student should take this opportunity to familiarize himself with the use of powers of 10: 3.2×10^{7} is the reciprocal of 3.1×10^{-8}, and $(3.2 \times 10^{7})^{3} = 33 \times 10^{21}$. See Appendix 2 (7).

QUESTIONS

1. Does air have inertia and weight?

2. Are you interested in mass or weight in buying a pound of sugar? In a paper weight? A hammer?

3. Distinguish between density and specific gravity.

4. The specific gravity of cork is 0.25; how much does a cubic foot weigh? A cubic centimeter?

5. Compare the weights of a molecule of hydrogen (H_2), helium (He), oxygen (O_2), and water (H_2O).

6. Find (and name) three elements in Fig. 2 which will float on water; several elements which will sink in mercury.

7. Criticize and correct: There are about 6×10^{23} hydrogen atoms in a gram of hydrogen (N_H). For other elements the number of atoms (N) in a gram is proportional to the atomic weight (A); $N : N_H = A : 1.008$.

Vocabulary: Mass, inertia, density, specific gravity, molecule, atom, element, atomic weight, atomic number, molecular weight, gram-molecular weight, periodic table, alkali.

PROBLEMS

1. Compare the mass of a 1-ft. cube of iron and a 1-meter cube of water.

2. Can you lift a cork sphere 1 meter in diameter? What is its weight?

3. What is the weight of a cubic inch of lead?

4. What is the weight of a column of mercury 1 sq. cm in cross-section and 76 cm long?

5. A cylindrical barrel is 1 meter tall and 60 cm across, made of 5 mm sheet iron. Find the weight of the barrel when half full of alcohol.

6. If there are (approximately) 6×10^{23} atoms in a gram of hydrogen, how much does each atom weigh? Each molecule?

7. What is your approximate age in seconds? Express this using powers of 10.

GENERAL PROPERTIES OF LIQUIDS, GASES, AND SOLIDS

CHAPTER 4

LIQUIDS

A *fluid* is distinguished from a solid by having no rigidity. It assumes the shape of any vessel in which it is placed.

Fluids are divided into liquids and gases. The liquid has a definite volume; it usually has a free surface. A gas completely fills the vessel in which it is placed.

This chapter treats of the properties of a liquid at rest (hydrostatics).

30. Pressure. The water in a tank exerts a pressure on the bottom and sides. **Pressure** is the force per unit area. It may be measured in pounds per square inch or in grams-weight per square centimeter. In everyday conversation the word pressure is sometimes used as meaning the total force on a surface. This is incorrect. If water stands 10 feet deep in a tank the pressure is the same, whether the base has an area of 6 square inches or 60 square feet. Of course the total force on the base is proportional to its area.

$$F = P \cdot A.$$

Pressure is the total force divided by the area. So, for a given force the pressure varies inversely as the area. A man puts on snowshoes to decrease the pressure. If a woman's foot is smaller than a man's in the same ratio as her weight, she exerts the same pressure on the ground. If I push a thumbtack with a force of a few pounds the pressure exerted by the point must be many tons per square inch, enough to rupture the wood.

$$P = \frac{F}{A}. \qquad \text{(Definition)}$$

31. General Properties of Fluids. We can call on everyday experience to justify the following facts about pressure in fluids:

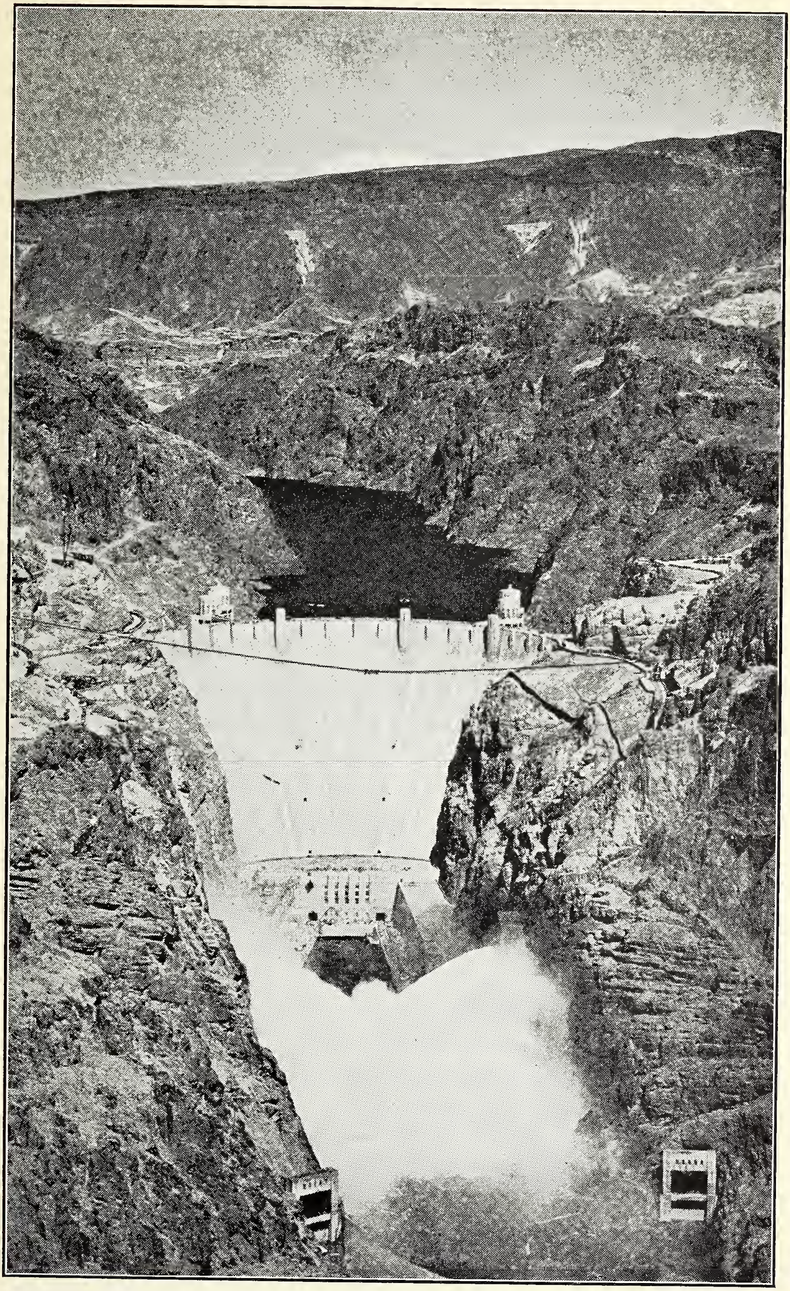

FIG. 1. Boulder Dam (Colorado River), showing power plant and jets from canyon wall outlets. Elevation from lower river level: Crest, 590 feet; mean lake level, 530 feet; outlets, 180 feet.

(1) The pressure always acts perpendicularly against any surface. Water pushes outward, not downward, on the side walls of the tank; it pushes down on the bottom. Water pushes directly upward on the bottom of a flatboat. The air in a tire pushes outward against the surface.

(2) At any point in the fluid the pressure is the same in any direction. If one has felt the pressure against his eardrums in swimming under water he will realize that the pressure will be the same regardless of the way he turns his head.

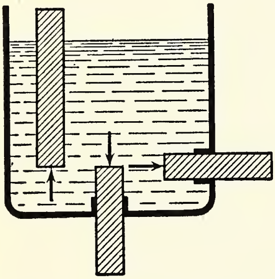

FIG. 2. Pressure is equal in each direction.

(3) The pressure becomes greater with depth. When a submarine sinks to more than a few hundred feet under water it is crushed.

(4) For all points at the same level the pressure is the same. It might at first seem that if a diver entered a submerged cave he would be sheltered from the pressure of the water above, but not so — at the same level the pressure is the same.

(5) (Pascal's Principle.) When pressure is increased (or decreased) at one point in a fluid all points experience the same change. When we increase the pressure in the tire valve the greater pressure is exerted over the whole of the inner tire surface.

32. Pressure and Depth in Liquids. A liquid usually has a free surface. As one descends into the liquid the pressure increases in proportion to the depth. One foot under water the pressure is 0.433 pound per square inch; 2 feet, 0.866 pound per square inch; 10 feet, 4.33 pounds per square inch; 100 feet down, 43.3 pounds per square inch.

$$P = 0.433\, h \text{ lb. per sq. in.,}$$

where h is the depth of water below the free surface.

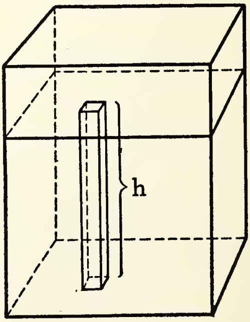

FIG. 3

Proof. Consider a tank with water 1 foot deep. Each square inch of the bottom must support the water directly above it, a column 1 foot by 1 inch by 1 inch. This is 1/144 of a cubic foot and weighs 62.4/144 = 0.433 pound. If the water depth is h feet, the weight over a square inch will be 0.433 h pounds, or in other words the pressure is 0.433 h pounds per square inch.

This is for water. In another liquid, the weight supported and the pressure are proportional to the specific gravity (§22).

$$P = 0.433\ h \times \text{sp. gr. lb. per sq. in.} \quad (h \text{ in feet.}) \qquad (1')$$

Example. Find the pressure at the bottom of a beaker containing 3 in. of mercury.

$$\text{sp. gr.} = 13.6; \qquad h = \tfrac{1}{4} \text{ ft.}$$
$$P = 0.433 \times \tfrac{1}{4} \times 13.6 = 1.47 \text{ lb. per sq. in.}$$

Observe, however, that this and every problem in this chapter can be worked step by step by proportion. The pressure $\frac{1}{4}$ ft. deep in water is 0.433 ÷ 4 = 0.108 lb. per sq. in.; in mercury it is 13.6 times greater.

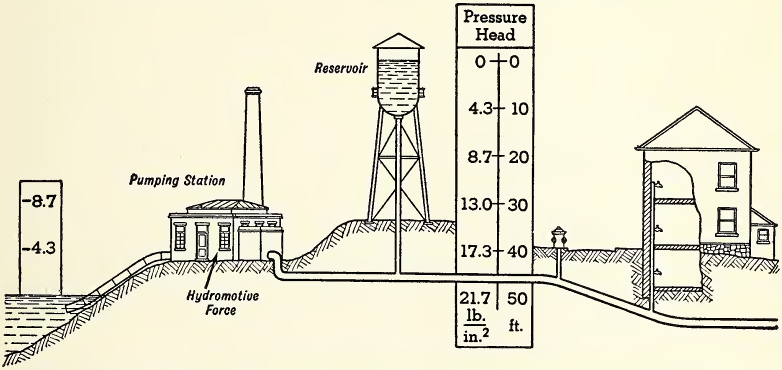

FIG. 4. Pressure depends on distance below the free surface. (Negative pressure, less than atmospheric [Chapter 5], shown at intake.)

In grams per square centimeter the pressure under water *is equal to the height in centimeters.* For 1 cubic centimeter of water weighs 1 gram; at a depth h centimeters below the surface there are h cubic centimeters over each unit area and the column weighs h grams.

$$P = h \text{ grams per sq. cm.} \quad \text{(Water.)}$$

For another liquid, weighing d' grams per cubic centimeter, the pressure is[1]

$$P = hd' \text{ grams per sq. cm.} \quad (h \text{ in centimeters.}) \qquad (1'')$$

---

[1] It is not necessary at present to stress the distinction between d' (the gram-*weight* per unit volume) and d (the gram-*mass* per unit volume). In the unit system which we shall later adopt weight is measured in *dynes*, and in these units $d' = d \cdot g$ and equation (1″) becomes

$$P = hdg \text{ dynes per sq. cm.} \qquad (1)$$

The same equation holds in British units with d in *slugs* per cubic foot (§109).

The comparative complexity of eq. (1′) (with its constant 0.433) is due to the fact that it involves mixed units: depth in *feet* and pressure in pounds per square *inch*. Eq. (1″) (or 1) holds, not only for metric units, but for any consistent units. For example, the pressure 10 feet under water is 10 × 62.4 = 624 lb. per sq. ft.

33. The Hydrostatic Paradox. All points on a level have the same pressure; the free surface need not be directly over the point in question. This leads to the "hydrostatic paradox." If the vessel in Fig. 5 is filled with water, the downward force on the bottom plate is as great as would be necessary to support a cylindrical column of equal height! Suppose that the height is 10 feet and the area 100 square inches — then the pressure on the bottom is 4.33 pounds per square inch and the total force 433 pounds. Yet the weight of all the water may be only a few pounds or ounces. How can this be? And why should the plate under the flange be forced to support more than the small column of water directly above it? Can you find the answer to the paradox? (Observe that the flange pushes down on the liquid.)

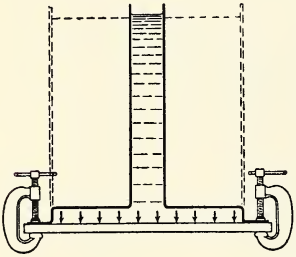
FIG. 5. Hydrostatic paradox.

The relation (1′, 1″) can be used to find the pressure in the pipes of a water system miles away from the free surface at the reservoir. The pressure depends only on the difference in levels. But these relations apply only to liquids at rest. It will develop (Chapter 14) that the pressure can be very different from this when the liquid is flowing.

34. The Hydraulic Press. If the pressure on A (Fig. 6) is increased by adding weights, an increased pressure will be acting against each wall. Pascal (French philosopher, a generation before Newton) said: A change of pressure at one point of a confined fluid is communicated undiminished to all parts of the fluid (Pascal's Principle, §31). In particular, when the weights are increased in Fig. 6 the same increased force acts on each unit area of B as of A. Hence the forces acting on these two surfaces are proportional to the areas

$$\frac{F_A}{F_B} = \frac{A_A}{A_B}.$$

The hydraulic press utilizes this principle. A force on the piston in the smaller cylinder of Fig. 7 produces (in virtue of the small area) a large pressure. This same pressure acting over the larger area of the second piston gives a large force — the force is increased in the ratio A_B/A_A. Such presses have many uses — they are used to compress hay, to compress paper into tough strong materials. . . Everyone has seen the large hydraulic jacks used for hoisting automobiles. . . The giant machines used in testing materials — crushing, bending, shearing large beams — are often hydraulic presses.

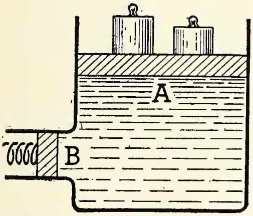

Fig. 6

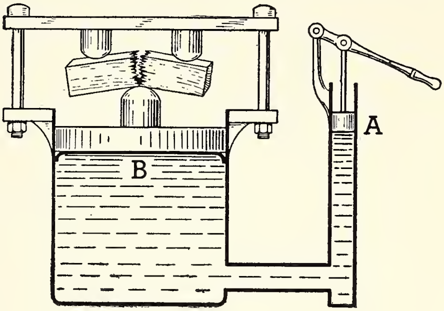

Fig. 7

35. The pressures obtainable with the hydraulic press are limited by the strength of the metal parts. Ultimately the strongest steel piston crumples. Pressures above 1,000,000 lb. per sq. in. have been reached. At such pressures properties of familiar things are greatly changed. Oils may become as viscous as wax. Water freezes above 100° C. The elements appear in new crystalline forms. Liquids penetrate solids. Solids bend or flow. Such experiments throw some light on physical behavior deep within the earth where much greater pressures exist (probably about 50,000,000 lb. per sq. in. near the center).

PROBLEMS

1. What is the pressure 100 ft. deep in a lake? In the ocean? 10 cm under mercury?

2. Find the pressure 76 cm under mercury. This is equivalent to what depth of water?

3. What is the pressure at the power plant at the lower river level at Boulder Dam (Fig. 1)?

4. The record for deep-sea divers is about 300 ft. Find the pressure at this depth.

5. A city reservoir is on a hill 200 ft. above the street level. What is the pressure at the top floor of an average 4-story building?

6. What force acts on the end of a 1-in. pole thrust 5 ft. under water?

7. Find the force acting on the end of a piece of chalk, 1 cm in diameter, thrust 10 cm under mercury.

8. In a specially constructed sphere (" bathysphere ") man has descended a half mile into the ocean. The sphere had a 6-in. circular window. Compute the force of the water against the window.

9. Find the average pressure and the total force on a dam 40 ft. high and 100 ft. across.

10. What are the pressure and total force on the bottom of a tank 6 ft. by 6 ft. with water 5 ft. deep? What are the average pressure and the force on one side of the tank?

11. A 10-cm hollow cube is submerged in water, its upper face 20 cm below the surface. Find the force exerted by the water against each face.

12. In Fig. 6 assume the cross-section of A 100 sq. cm, of B 5 sq. cm, the water 10 cm deep, and B 3 cm from the bottom (average). Assume the weight on A (including piston) to be 1500 grams. What is the force on the spring?

13. If the cylinders of a hydraulic jack are $\frac{1}{2}$ in. and 1 ft. in diameter, what force is required to lift a 1-ton automobile?

14. Problem 13 assumes that both pistons are at the same level. Answer the problem assuming that the input force (pump) is applied to a column of oil (sp. gr. 0.85) extending 5 ft. above the level of the large piston.

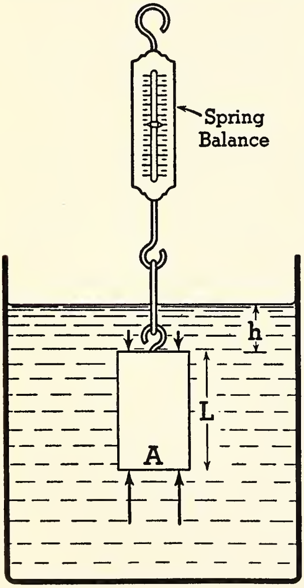

Fig. 8. Archimedes' Principle.

36. Buoyant Force. Wood floats; a stone sinks, but it weighs less in water than in air. All things are buoyed up by a greater or less force when immersed in water. Archimedes' Principle is: *A body immersed in a fluid is buoyed up with a force equal to the weight of the fluid displaced.* The student should carefully analyze this important principle: What is meant by " displaced "? Will the buoyant force depend on the volume of the body, upon the density of the liquid? Will it be the same for a brick and a wood block of the same size pushed under the surface? Will it vary with depth? Does it apply to gases as well as liquids? Does the principle appeal to your common sense?

Proof. If, into a vessel filled brimful of water, a stone is lowered, the displaced water overflows. The stone may be weighed with a spring balance, first in air and then in the water. Its loss of weight will be found equal to the weight of the spilled water.

This is an experimental proof, but it is more interesting to derive the principle logically. A rectangular block immersed in a liquid is subjected to pressure on all sides. The lateral forces exactly cancel. Not so those on top and bottom; the downward pressure on the top is

$$P = h \times d',$$

and the force is

$$F_{\downarrow} = h \times d' \times A.$$

On the bottom the pressure is $(h + L) \times d'$, and the force is

$$F_{\uparrow} = (h + L) \times d' \times A.$$

Subtracting, the net upward force is

$$\text{Buoyant force} = F_{\uparrow} - F_{\downarrow} = L \times A \times d'.$$

LA is the volume of the displaced liquid; multiplying by d' (its weight per unit volume) gives its weight.

Examples. 1. Weights of bodies in liquid. Given a stone whose weight is 10 lb., sp. gr. 3. How much will it weigh in water? Water is a third as heavy; its volume of water will weigh $3\frac{1}{3}$ lb. The net weight of the stone by Archimedes' Principle is $6\frac{2}{3}$ lb.

2. Density of solid. A solid weighs 10 grams in air and 8 grams in water. Find its specific gravity. By Archimedes' Principle its volume of water weighs 2 grams. Hence the object is 5 times heavier than water.

3. Density of a liquid. A solid weighs 10 grams in air, 8 grams in water, and 8.4 grams in alcohol. Find the density of alcohol. By Archimedes' Principle the water displaced weighed 2 grams; the alcohol displaced weighed 1.6 grams; therefore the specific gravity of alcohol is $1.6 \div 2 = 0.8$, and its density is 0.8 gram per cc, or 48.9 lb. per cu. ft.

King Hieron, according to the pretty story that has been handed down, suspecting that the gold in his crown had been alloyed with baser metal, gave Archimedes the problem of testing its purity. Archimedes wished to test its specific gravity. But how to do it without melting down the crown? He was in his bath when the solution came to him. Shouted he: "*Eureka*" ("I have found it") — and so his principle was born.

37. The Cartesian Diver. Fill a flask completely full of water. Invert in it a pill bottle with air enough just to float. If now the flask is stoppered the small bottle can be made to dive to the bottom by pressing the sides of the flask. At first the bottle with its bubble displaces slightly more than its weight of water; it floats. Squeezing the flask compresses the bubble, less water is displaced and the buoyant force is insufficient to float the bottle. (This *Cartesian diver* is named after Descartes, French philosopher, mathematician, and scientist. Descartes and Pascal belong to the generation between Galileo and Newton.)

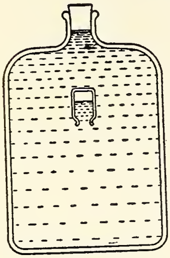

FIG. 9. Cartesian diver.

38. Floating Bodies. Before the body is completely submerged the buoyant force may equal its weight; the body then floats. By Archimedes' Principle the buoyant force equals the weight of the liquid displaced. Thus for a floating body the *weight* of the liquid displaced equals the *weight* of the body.[1] A ship settles down in the water until it displaces its own weight. We speak of a ship of 2000 tons displacement; this is the weight of the water displaced; it is also the weight of the ship.

A cork (specific gravity 0.25) floats on water with only one-quarter of its volume submerged. The weight of the displaced water equals the weight of the cork. The specific gravity of ice is 0.9; nine-tenths of the iceberg is under water.

The commonest way of measuring the specific gravity of a liquid is by means of the floating hydrometer (Fig. 10). This is in the form of a weighted glass tube which sinks to varying depths in liquids of different densities — always until it has displaced its own weight. The student has used it (or seen it used) in measuring the density of the liquid in an automobile storage battery. Evidently the density is inversely proportional to the displaced volume.

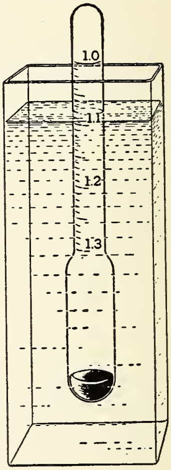

FIG. 10. Hydrometer.

With air in the lungs, the human body floats; exhale, and it sinks. Clearly the weight of the body differs little from an equal volume of water. In Great Salt Lake a man floats with head and shoulders above water. On a pool of mercury he could crawl with only arms and knees submerged. But in each liquid he displaces his own weight.

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[1] This is in contrast to the completely submerged body where the displaced *volume* equals the volume of the body.

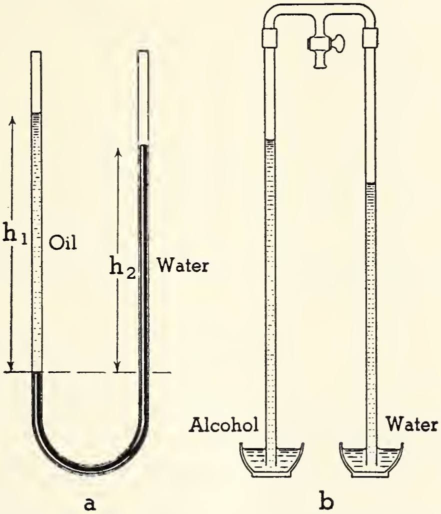

FIG. 11. Density by balanced columns.

39. Relative Densities of Balancing Columns. If two non-miscible liquids are placed in the arms of a U-tube, the heights to which they rise above the surface of contact are inversely proportional to their densities. For the pressure at this level is the same in both tubes. The pressure is produced by a depth h_1 of the one liquid, h_2 of the other. Hence from (1″)

$$h_1d_1 = h_2d_2$$

or

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}.$$

This is sometimes a convenient method for comparing densities. The method used for miscible liquids is also shown in the figure; by sucking through the stopcock the liquids have been drawn up in the two tubes and the elevations are inversely proportional to the densities.

40. The density of a body can be measured in the following ways:

(1) For a regular body: by direct measurement of weight and volume, obtaining dimensions by calipers.

(2) For a body heavier than water: by weighing, first in air, next in water.

(3) For regular body lighter than water: by floating it and observing proportion submerged.

(4) For liquids:

- (*a*) Volume measured directly.
- (*b*) Buoyant force on solid observed (hydrometer).
- (*c*) Balanced columns.

The student will probably have a chance to use some of these methods in the laboratory.

QUESTIONS

1. Give an illustration of each of the general properties of fluids. Do these properties apply to gases? Do they apply to wheat stored in a grain elevator? Do they apply to wax?

2. Why point a stake, why sharpen an axe? What is the principle of the parachute?

3. If the apparatus shown in Fig. 5 were placed on a balance would the balance register the total force on the bottom? Why?

4. Is the pumping of a flat tire on an automobile an example of the hydraulic press?

5. How much is the buoyant force on a liter of iron submerged in water? On a liter of stone? On a gram of iron? On a 1-oz. cork floating in mercury?

6. When a ship sinks does it go all the way to the bottom?

7. A substance is just heavier than water; suppose it to be (1) more compressible, (2) less compressible, than water. May it sink only part way to the bottom?

8. A boy floats in water when he inhales, sinks when he exhales. Approximately what is the volume of a 125-lb. boy?

9. A vessel of water is on a balance. Does the weight increase when you dip your hand in it? When you place a fish in it? How much does the weight increase if you place a 2-lb. rock in it (sp. gr. 2)? What force does the rock itself exert on the bottom?

10. By how much does the weight of this vessel increase when a 10-cc cork is thrust under the surface? Would the answer be different if the cork were tied to the bottom?

11. A small balloon is weighted until it just floats in a pond. If submerged some distance it sinks all the way to the bottom and remains there. This is because near the bottom: (*a*) the water is slightly heavier; (*b*) the volume of the balloon is decreased; (*c*) the weight of the water above it exceeds its buoyant force.[1]

12. Suppose that a bottle is completely full of water and a cork is pounded in. According to Pascal's Principle: (*a*) the pressure at any point is the same in any direction; (*b*) the pressure is equal at every point; (*c*) everywhere the pressure increases equally.[1]

Vocabulary: Fluid, pressure, Pascal's Principle, hydrostatics, hydraulic press, buoyant force, hydrometer, displacement (of fluids).

PROBLEMS

1. What is the loss of weight when a cubic foot of material is submerged in water?

2. What buoyant force acts on a 6-in. cube submerged in water? On a 5-gal. (approx. 20-liter) can? On a 7.9-gram iron ring? What buoyant force acts on a 1-oz. cork floating in alcohol?

3. How much does a 100-lb. anchor weigh in water?

4. A brick weighs 2 kilograms in air and 800 grams in water. Find its density.

5. If the above brick weighs 950 grams in oil, find the density of the oil.

6. A stone weighs 20 lb. in air, 14 lb. in water, and 15 lb. in oil. Find the specific gravity of the stone and of the oil.

7. We wish to identify a certain metal by density measurement. It weighs 45 grams in air and 17.8 grams in mercury. What is the metal?

8. Suppose that Archimedes found that Hieron's crown weighed 20 shekels in air and 18 shekels in water. Was it pure gold?

9. If the block in Fig. 8 is aluminum, volume 300 cc, and the liquid is alcohol, what will the spring balance read?

10. A liter flask weighs 200 grams and contains 50 cc of mercury. Will it float in alcohol?

11. How many cubic feet of water are displaced by a 200-lb. rowboat?

12. A flatboat is 20 by 30 ft. in area. How much will it be lowered when carrying a 1-ton automobile?

13. Large ore freighters are 500 ft. long and 60 ft. wide. (The sides are parallel over most of the length.) They carry 12,000 tons of iron ore. Approximately how much does the cargo raise the water line on the boat?

14. What force is required to submerge a 30-lb. oak plank (sp. gr. 0.6)?

15. A 100-lb. iron weight is suspended in water from a keg which weighs 35 lb. and has a volume of 2 cu. ft. Show that the keg will just float.

16. A fisherman uses a cork float weighing 20 grams. How heavy a lead sinker is required to submerge it?

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[1] These are multiple-choice propositions, with the correct statement (*a*, *b* or *c*) to be selected. Similar questions will be given after each chapter; the student may wish to answer these and keep his score of success. There is one rule in this game: the questions are to be answered after the chapter is studied and without further consultation with the text. Like all such questions, some are easy, some difficult, some tricky.

CHAPTER 5

GASES

THE ATMOSPHERE

41. Weight of Air. Air has weight. This can be shown by comparing the weight of a hollow spherical vessel when open to the air and when evacuated. The experiment can be made more striking by filling the vessel with air under pressure. Tanks of gases compressed under a pressure of several thousand pounds per square inch are used in the laboratory and in industry. The density of the compressed oxygen or acetylene or carbon dioxide is several pounds per cubic foot.

We live at the bottom of a sea of air. The air down here around us is about 800 times lighter than water; at 0° C and under standard barometric pressure it weighs 1.28 grams per liter. By Archimedes' Principle everything around us is buoyed up with a force of 1.28 grams for each liter of its volume. In making accurate mass determinations, particularly with substances of low specific gravity, correction must be made for this buoyancy. On a man the force is only 3 or 4 ounces — we cannot swim in air, though we have invented airplanes which will.

Things lighter than air float. Balloons are filled with hydrogen or helium gas. Hydrogen is $14\frac{1}{2}$ times lighter than air. So for each pound (or gram) of hydrogen gas there is a buoyant force from the displaced air of $14\frac{1}{2}$ pounds (grams): a net upward force of $13\frac{1}{2}$ pounds (grams).

42. The Magdeburg Hemispheres. The material nature of air has not always been recognized. Gas means geist, ghost, mysterious.[1] Up through the Middle Ages the science of pneumatics was summed up in the philosophical dictum: Nature abhors a vacuum. The first steps toward a real science were made by the Italian Torricelli and the German von Guericke.

Von Guericke — mayor of Magdeburg who studied nature in his spare time — invented the air pump. He made himself two large close-fitting hemispheres; he evacuated this sphere and showed that (so great was the pressure due to the weight of air above) a force-pair of several tons was needed to separate its halves. He demonstrated this before the king. A team of eight horses was attached to each hemisphere. But all these king's horses could not pull the hemispheres apart. This was about 1650, just before the age of Newton and Boyle and Hooke.

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[1] This is the derivation sanctified by tradition. However, van Helmont — last of the alchemists — who claims to have invented the word, asserts that he derived it from the word *chaos*.

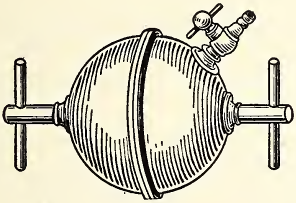

FIG. 1. Magdeburg hemispheres.

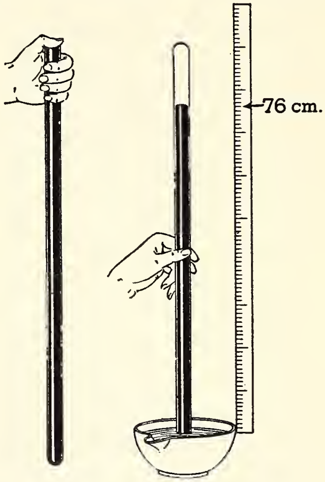

FIG. 2. Mercury barometer.

43. The Barometer. A less spectacular but more significant experiment had been performed by Torricelli some years earlier. He filled a long tube with mercury and inverted this over a dish of mercury. The mercury dropped down from the closed end until it stood about 76 centimeters (about 30 inches) above the level of the mercury in the dish. Above this was a vacuum — evidently nature's abhorrence of a vacuum had a very definite limit!

This was the first barometer — atmospheric pressure meter — and the standard barometer today is of the same type. Air, many miles of it, pushing down produces a pressure on the surface of the mercury in the dish. At this same level in the tube the pressure is due to 76 centimeters of mercury. (Compare the balanced liquids, §39.) This column of mercury then measures the pressure of the atmosphere. In grams per square centimeter the pressure 76 centimeters deep in mercury is (from equation 1″)

$$P = hd' = 76 \times 13.6 = 1033 \text{ grams per sq. cm.}$$

44. Atmospheric Pressure. The pressure of the atmosphere changes from day to day, varying with the amount of air above us and with its humidity (for water vapor is lighter than air). Quite evidently the pressure must vary with altitude. Seventy-six centimeters (very nearly 30 inches) is an average reading for the barometer at sea level. This is taken as the *standard atmosphere.* This standard atmosphere thus corresponds to a little more than a kilogram per square centimeter, or 14.7 pounds per square inch. A column of water 34 feet high would exert the same pressure. (Water is 13.6 times lighter than mercury; 13.6 × 30 inches = 34 feet.)

Standard atmosphere = pressure of 76 cm or 30 in. of mercury or of 34 ft. of water.
= 14.7 lb. per sq. in. = 1033 grams per sq. cm.
= approximately 1 million dynes per sq. cm (§110).

45. Effects of Atmospheric Pressure. The pressure of the atmosphere is nearly 15 pounds per square inch, more than a ton per square foot. Von Guericke's hemispheres had a diameter of 22 inches; the force on them was nearly 3 tons. Large glass vessels, unless thick, will collapse when evacuated. But for the most part, atmospheric pressure is not noticed. Air at the

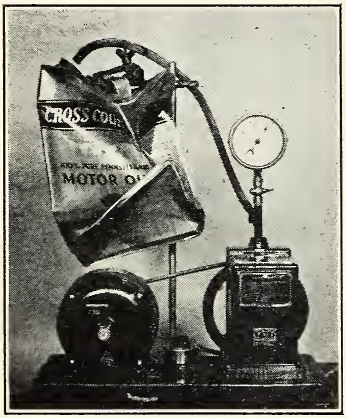

FIG. 3. Evacuated can is crushed by atmospheric pressure.

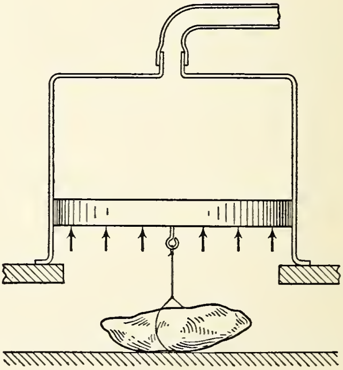

FIG. 4.

same pressure is pushing outward and inward on the walls of our houses; our bodies are adjusted to the pressure in which we live. There is still a pressure of 15 pounds per square inch in a flat tire. When we ask for 30 or 40 " pounds pressure " in our tires we mean 30 or 40 pounds per square inch, not in total, but in excess of atmospheric pressure.

In the previous chapter, when we computed the pressure under water, the atmospheric pressure on the surface was disregarded; this must be added to the values there found to get the *total pressure.*[1] When we estimate the pressure in a closed tube at points above the exposed surface, the pressure (hd') is to be subtracted from atmospheric, since h is here negative.

Examples. The total pressure 1 cm below the surface of the mercury in the dish in Fig. 2 is $1033 + 13.6$ grams per sq. cm; 1 cm up the tube it is $1033 - 13.6$ grams per sq. cm. . . Note the intake pressures in Fig. 4–4.

The pressure of the atmosphere can be demonstrated by means of the cylinder and tight-fitting piston illustrated in Fig. 4. When air is pumped out of the cylinder the piston is pushed up with a force of 15 pounds per square inch. (This was indeed the type of the earliest steam engines used in water pumps in English mines. The steam was used only to obtain a vacuum. The steam pushed the air from the cylinder, the steam was condensed leaving a vacuum, and the air did the work.) These are of course instances of "suction" — but suction is not in itself a force. We produce suction by removing the compensating pressure from within a pipe and then allowing the atmospheric force to operate.

46. The Water Pump. Water pumps are of either the suction or the pressure type. In the suction pump the plunger acts as a vacuum pump, removing the air from the pipe; atmospheric pressure on the well raises the water (Fig. 5*a*). The suction pump cannot raise water more than 34 feet.

In the force pump the plunger pushes the water. In Fig. 5*b* the lower valve opens on the up stroke of the piston and water enters the barrel; on the down stroke this valve closes and the upper valve opens and the water is forced into the pipe. The action of the pump is improved by the presence of an air drum; during the expulsion stroke some of the water is pushed into this chamber, compressing the air; during the return stroke this water is fed to the pipe. The air drum acts as a ballast giving a fairly continuous flow and makes the task of pumping easier.

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[1] When working problems by proportion it is often convenient to express the atmospheric pressure as an additional 34 ft. of water (or proportionately more or less if the barometer is above or below standard).

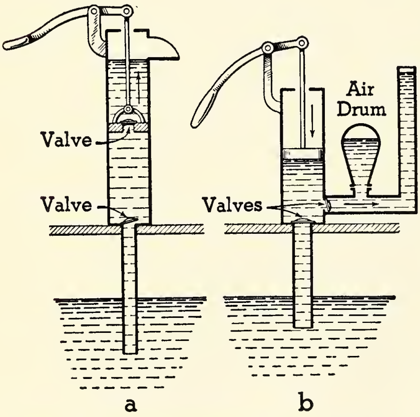

FIG. 5. Suction and force pumps.

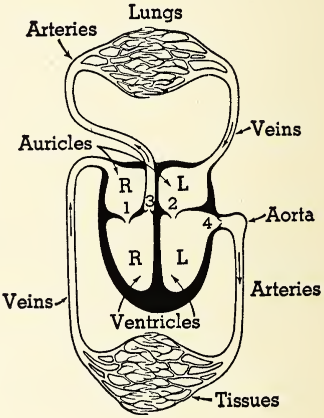

FIG. 6. Heart and vascular system.

47. The heart is a two-stage force pump, circulating blood in the arteries and capillaries and veins through the lungs and the tissues of the body. The pumping chambers are the ventricles which rhythmically expand and contract. As the ventricles expand, the flap valves 1, 2 open and blood enters from the auricles; as the ventricles contract these valves close, valves 3, 4 open, and blood is forced from the right ventricle through the arteries and capillaries of the lungs, from the left ventricle into the *aorta* and thence to the other arteries and capillaries of the body. The aorta has flexible walls and takes the place of the air drum in Fig. 5*b*. The sudden rush of blood from the left ventricle distends the aorta, and, instead of being shot out in sudden spurts, the blood flows continuously from this elastic bag during the whole heart cycle. This lessens the sharpness of the pulse and decreases the strain on the heart. For the right ventricle (where the pressures are much smaller) the flexibility of the arteries serves the same purpose.

48. The Siphon. Probably most persons are familiar with the siphon, by means of which water can be lifted somewhat and then transferred from a higher to lower level. In Fig. 7 the lower end of the siphon is closed. The pressure at *a*, *a′* is atmospheric, at *b*, *b′* less than atmospheric, at *c* greater than atmospheric. Therefore, if the stopper at *c* is removed water will rush out rather than air rush in. It might be supposed that water would flow both ways away from the bend, emptying the siphon; the pressure of the air prevents this. Evidently the siphon cannot raise water more than 34 feet.

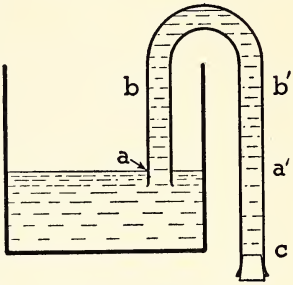

Fig. 7. Siphon.

49. Variation of Atmospheric Pressure in the Lower Atmosphere. Atmospheric pressure is equal to the weight of the column of air over unit area; it decreases with altitude. The difference in pressure between two levels is equal to the weight of the air column (of unit cross-section) between the levels. At sea level the pressure decrease is about 1.28 grams per square centimeter for each 10 meters of rise. (*Proof:* 10 meters × 1 square centimeter = a liter volume; this weighs 1.28 grams under standard conditions.) The barometer drops 1 inch for 900 feet of rise. As we rise quickly in an elevator we feel a pressure change on our eardrums.

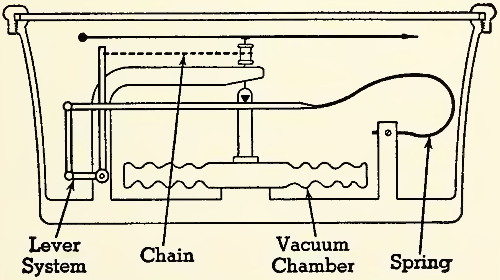

Fig. 8. Aneroid barometer.

Barometers are used for measuring altitudes in mountain climbing and in airplanes. For this purpose the compact aneroid barometer is used. Here the actuating mechanism is a small evacuated chamber with a flexible metal wall; this chamber is compressed, now more, now less, as the pressure changes, and the slight motion of the wall operates a pointer on the scale.

PROBLEMS

1. Change a pressure of 70 cm of mercury to inches of mercury, to feet of water, to grams per square centimeter, and to pounds per square inch.

2. The standard atmosphere is equivalent to how many inches of mercury and to how many feet of water? (The values which have been given are only approximate.)

3. The diameter of the Magdeburg hemispheres was 22 in. Find the force upon them, pushing them together. (For area use the cross-section; why?)

4. Tires with " 45-lb. pressure " are used on a 3-ton truck. How big an area of each tire will be in contact with the ground?

5. If the stone and piston of Fig. 4 together weigh 20 kilograms and if the piston is 100 sq. cm in cross-section, how far must the internal pressure be reduced to lift the stone?

6. What is the ratio of the pressure 40 ft. under water to that at the surface?

7. Above a mercury surface stands 3 ft. of water. The barometer reads 29 in. Find the total pressure 2 in. under the mercury.

8. If the alcohol column in Fig. 4–11*b* is 1 meter long, what is the pressure of the air above it? What is the pressure halfway up the water column?

9. What is the difference of pressure (" hydromotive force ") maintained by the pumping station in Fig. 4–4.

10. The fourth (top) floor of a house is 40 ft. above the basement. (*a*) Compare the water pressure at an attic and a basement faucet. (*b*) The main valve to the water system is now closed. Explain why no water flows out when a second-floor faucet is opened. (*c*) What is the pressure at the top floor if it is a basement faucet which is opened? Will water flow out at the basement?

COMPRESSIBILITY OF GASES

50. Density of Gases. Table 4 gives the densities of several common gases under standard conditions (standard atmosphere pressure, 0° C). Air is a mixture of nitrogen and oxygen in the ratio of about 4 to 1, with a little argon and water vapor, and a trace of other gases.

51. Avogadro's Hypothesis. Table 4 also gives the molecular weights (§26). The student, as he compares these molecular weights (which are in the ratio 1 : 2 : 10 : 14, etc., as we read down in the table) with the densities (0.09, 0.18, 0.9, etc.), will certainly see the relation between them. The Italian physicist Avogadro first saw this relation about 1810. It was not so obvious in that day because atomic weights were rather indefinite and molecular weights more so. (Avogadro was first to recognize that in gases like hydrogen and nitrogen the molecule is composed of a *pair* of atoms.) Indeed the relation was not generally accepted for many years — it became known as Avogadro's Hypothesis.[1]

Avogadro's Hypothesis: Volume for volume, the weights of different gases are proportional to the weights of their individual molecules; this means that the equal volumes contain the same number of molecules. Actually one gram molecule of any gas contains 6.02×10^{23} *molecules and occupies* 22.4 *liters (under standard conditions of temperature and pressure).*

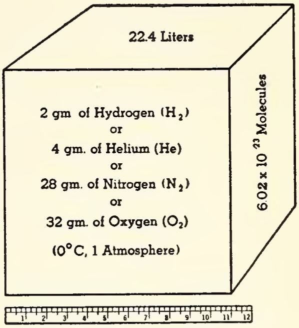

FIG. 9.

TABLE 4

	Density Grams per Liter	No. of Atoms in Molecule	Approximate Molecular Weight
Hydrogen........	0.09	2	2
Helium..........	0.18	1	4
Neon............	0.90	1	20
Nitrogen.........	1.25	2	28
Oxygen..........	1.43	2	32
Carbon dioxide....	1.97	3	44
NH_3............	0.77	4	17
Air (at 20° C.)	1.28		

52. Compressibility of Gases. Gases are much more compressible than solids or liquids. Robert Boyle had heard of von Guericke's experiments, and he and Hooke proceeded to make an air pump. Boyle investigated the properties of gases. (Charles II attended a meeting of the Royal Society and returning chortled over the earnestness of these fussy gentlemen who spent their time " only in weighing of air and doing nothing else since they sat.") Consider a quantity of gas as enclosed in a cylinder with a piston. When we push on the piston the gas volume decreases. Boyle found that if he added (approximately) 15 pounds pressure to air (doubling the pressure) its volume was halved. Tripling the pressure gave one-third the volume, and so on. And so for other gases. This was all done at a fixed temperature. Boyle's Law is: At constant temperature the volume of a gas varies inversely as the pressure. Or

---

[1] An hypothesis is an unproved proposition; a scientific hypothesis is proposed as a basis for experimental investigation. If subsequently confirmed by many and all possible tests it becomes a " law " or a " fact." This might more properly be called Avogadro's Law.

$$P_1 : P_2 = V_2 : V_1$$

or

$$PV = \text{constant.} \qquad \text{(Boyle's Law.)} \qquad (2)$$

53. Examples of Boyle's Law. When the barometric pressure drops, the air about us expands; the volume increases 1 per cent when the pressure falls 1 per cent. At higher altitudes (lower pressures) the volume of a given mass of air increases; it is approximately doubled every $3\frac{1}{2}$ miles. Mountain climbers find breathing difficult. When gases are compressed into steel drums at a pressure of several hundred atmospheres the volume shrinks several hundredfold. By means of Boyle's Law the density of a gas, given in Table 4 for standard conditions, can be found for any other pressure (still at standard temperature).

Examples. 1. Suppose that a bubble 1 cc large rises from a depth of 17 ft. in water. What is its volume as it reaches the surface? In applying Boyle's Law the total pressure must be used. Consider the atmospheric pressure as equivalent of 34 ft. of water. Then

$$\frac{V_{\text{top}}}{V_{\text{bottom}}} = \frac{P_{\text{bottom}}}{P_{\text{top}}} \quad \text{or} \quad \frac{x}{1} = \frac{51}{34}\,.$$

Hence the volume increases in the ratio of 3 : 2. (Bubble $1\frac{1}{2}$ cc at surface.)

2. What proportion of air leaves the house when the barometer sinks from 76 to 73 cm? If V_{76} and V_{73} represent the volume of a given amount of gas at the two pressures, we have

$$\frac{V_{73}}{V_{76}} = \frac{76}{73} = 1.04.$$

Hence the volume increases by 4 per cent — this proportion of the air leaves the house.

One readily sees the reason for Boyle's Law in terms of molecules and their motions. Compressing the gas does not affect the speed of the molecules but does increase their number per unit volume. So if the volume is halved the number of molecules per cubic centimeter will be twice as many, there are twice as many collisions against the walls and hence there is twice the pressure.

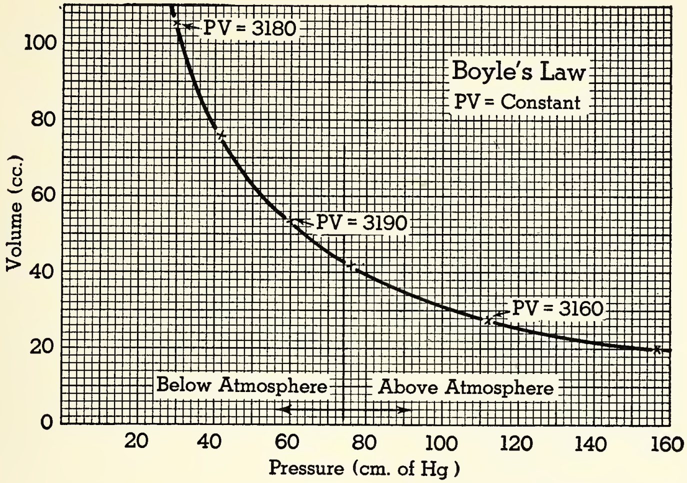

FIG. 10. Boyle's Law. (Data from student's experiment.)

54. The "Negative Compound Interest Law." If air, like water, had only a negligible compressibility the height of the atmosphere could be computed by a simple proportion. The atmosphere is equivalent in pressure to a layer of water 34 feet deep. Since air density at sea level is about 800 times less than that of water, its height would be a little more than 5 miles (800 × 34 = 27,200 feet). If air were incompressible we should rise 5 miles through air of constant density — and above that nothing.

But air does not behave in this fashion. The air expands with decreasing pressure, its density becomes less, in consequence the change in pressure with elevation becomes slower: $3\frac{1}{2}$ miles high the pressure is halved, in the next $3\frac{1}{2}$ miles it becomes half of that, then ($3\frac{1}{2}$ miles higher) half of that, and so on. This is shown in Fig. 11. This relation is not exact, largely because of the decrease of temperature with altitude. About 6 miles above the earth the outer atmosphere or " stratosphere " begins, in which the temperature remains constant (−60° centigrade or −75° Fahrenheit, approximately). Here the pressure halves in about 3 miles, decreases about tenfold every 10 miles.

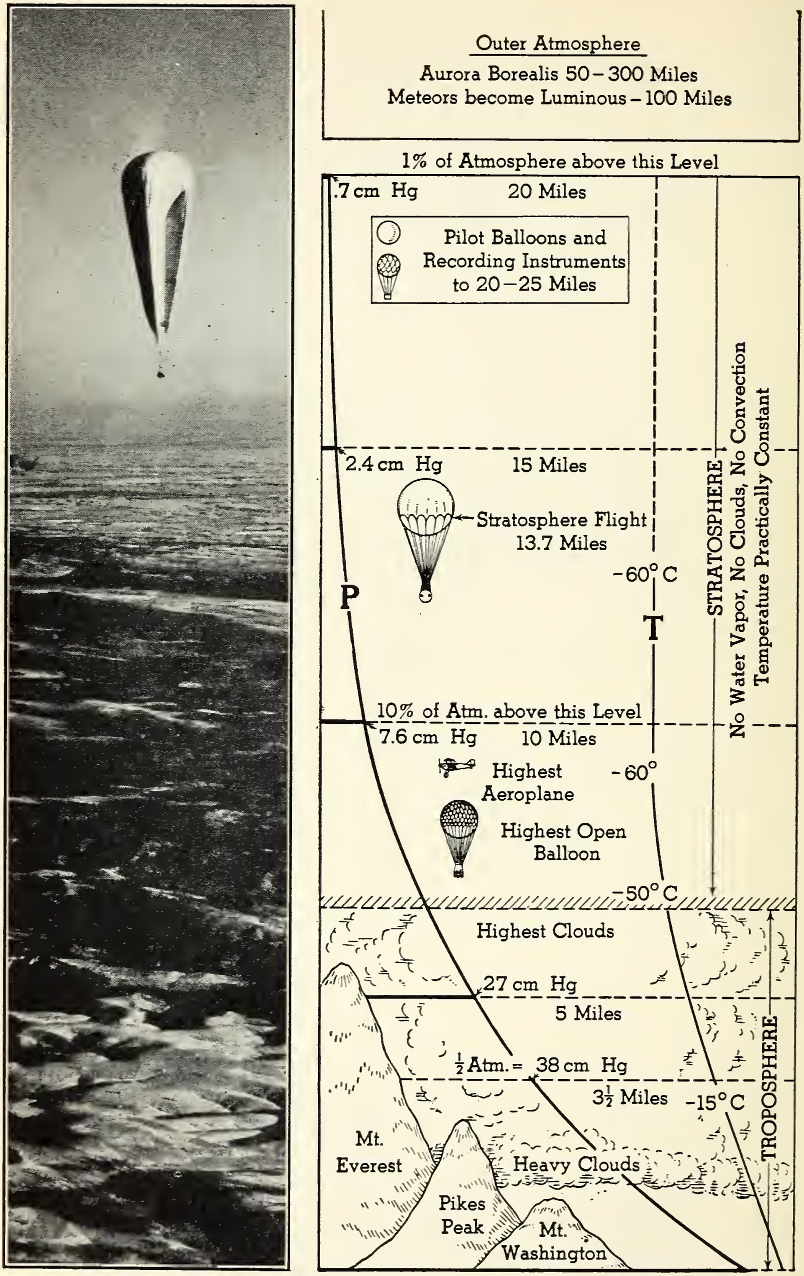
Outer Atmosphere
Aurora Borealis 50 – 300 Miles
Meteors become Luminous – 100 Miles
1% of Atmosphere above this Level
.7 cm Hg
20 Miles
Pilot Balloons and Recording Instruments to 20 – 25 Miles
2.4 cm Hg
15 Miles
Stratosphere Flight 13.7 Miles
-60° C
P
T
STRATOSPHERE
No Water Vapor, No Clouds, No Convection
Temperature Practically Constant
10% of Atm. above this Level
7.6 cm Hg
10 Miles
Highest Aeroplane
-60°
Highest Open Balloon
-50° C
Highest Clouds
27 cm Hg
5 Miles
½ Atm. = 38 cm Hg
3½ Miles
-15° C
TROPOSPHERE
Mt. Everest
Pikes Peak
Heavy Clouds
Mt. Washington

FIG. 11. Atmospheric pressure and temperature. (*Left*) Beginning of 1936 stratosphere flight.

Many quantities in physics decrease in this way. It reminds one of the compounding of money, except that when money is invested at compound interest there is an increase instead of a decrease. At 6 per cent compound interest, money doubles in 12 years; that money doubles in 12 more, and so on. If, instead of paying interest, the banker should charge compound interest for keeping money safe, the principal would halve in a certain period, and then, paying interest on a smaller amount, halve again and again and so on — never quite disappearing. This law of variation is sometimes called the "negative compound interest law."

55. Departures from Boyle's Law: Molecular Attraction. At ordinary temperatures and with pressure of only a few atmospheres, gases like hydrogen, nitrogen, and oxygen follow Boyle's Law very closely. But at very high pressures, when the distance between molecules is greatly reduced, the law no longer holds and further compression becomes very difficult (Fig. 12). The molecules themselves are now practically in contact (as in a liquid or solid).

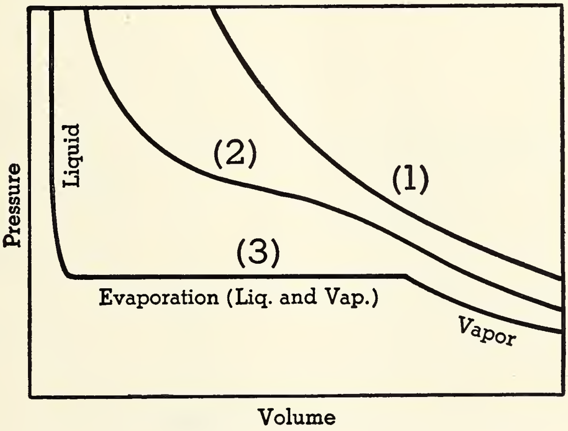

FIG. 12. Deviations from Boyle's Law: (1) high temperature, deviations inappreciable; (2) slightly above critical temperature (§276), marked deviation; (3) below critical temperature, condensation.

Before a gas reaches this state of abnormally small compressibility it passes through a stage of abnormally large compressibility (see curve 2, Fig. 12). The reason for this is the attraction which the molecules have for one another. This attractive force is not very great between hydrogen or oxygen molecules; it is very small indeed for helium. It is greater for carbon dioxide molecules, and quite large for water molecules or sulfur molecules. At a certain stage in the compression this attractive force may offset the tendency of molecules to fly apart; the molecules then tend to gather into condensed aggregates which we know as liquids and solids. We shall find that all gases can be liquefied at low temperatures.

Substances which can be condensed in this way at ordinary temperatures are called *vapors*, and for them we may expect serious departures from Boyle's Law. For other gases the law can be considered as very nearly valid for ordinary pressures and temperatures — for air even at 3000 pounds per square inch (200 atmospheres) the departure from the inverse proportion is not more than 1 per cent!

56. Vacuum Pumps. Not long ago hand-operated pumps, of a design not unlike that of von Guericke and Boyle, were used. Today, for obtaining a fairly good vacuum, the motor-driven rotary oil pump (Fig. 3) is employed. In one common type the rotor is mounted eccentrically (Fig. 13*a*); in the course of a revolution it traps a volume of low-pressure gas before it, compresses it, and pushes it out the outlet valve. To eliminate leaks in the valves and bearings these working parts are submerged in oil.

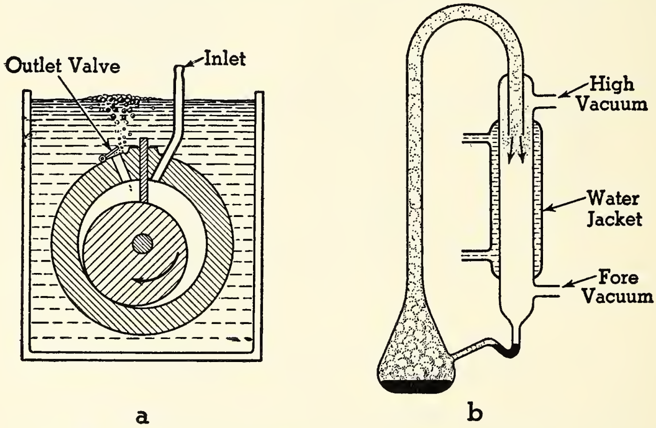

Fig. 13. (*a*) Rotary oil pump. (*b*) Mercury diffusion pump.

For obtaining the very highest vacuum, mercury or oil vapor pumps are common. The mercury or oil is heated; a strong stream of vapor passes up and out from an orifice, is then condensed and returned to the boiler. As these heavy molecules rush out they strike against the occasional air molecules in their way and drive them downward. Very quickly such a pump carries away the "last trace" of gas from a vessel. A vapor pump cannot pump directly into the atmosphere. A rotary pump must be used in conjunction with the vapor pump to give a rough fore-vacuum.

Most students are familiar with the common water aspirator. This will reduce the pressure to about 2 centimeters of mercury. A rotary oil pump will reduce it to about 0.001 millimeter; a mercury vapor pump will reduce it a hundred thousand times further. For most practical purposes this may be considered a perfect vacuum — although there still remain several hundred million molecules in each cubic centimeter.

QUESTIONS

1. What is the net buoyancy per pound of gas of a helium-filled balloon? Helium is twice as heavy as hydrogen.

2. The fragments of a broken lamp bulb weigh more than the original (evacuated) bulb — why?

3. If a mass of iron is weighed on a balance with brass weights will the correction for buoyancy be serious? Why?

4. Does a football bladder weigh more when inflated?[1]

5. Automobile tires are pumped to "30-lb." pressure. Will their pressure change at high altitudes?

6. What is the pressure in the siphon of Fig. 7, 2 ft. above the free surface? If a small hole could be made in the tube at this point would water flow out? Why will a siphon not work in a vacuum?

7. What is Avogadro's Hypothesis?

8. How does gas density vary with pressure? What does a liter of air weigh at 100 atmospheres pressure?

9. When does a balloon stop rising? (Contrast a hypothetical balloon with gas confined to a fixed volume and the actual balloon of Fig. 11 where room is left for expansion and the gas is always at atmospheric pressure.)

10. Under standard conditions, the densities of different gases vary directly as: (*a*) the weight of the individual molecule; (*b*) the number of atoms in the molecule; (*c*) the atomic weight.

11. As a balloon (such as Fig. 11, with room for expansion) rises the buoyant force will: (*a*) remain constant because a constant weight of air is displaced; (*b*) decrease because the density of the air is decreasing; (*c*) increase because the hydrogen is expanding.

Vocabulary: Standard atmosphere, standard conditions, suction (force) pump, air drum, siphon, aneroid barometer, Avogadro's Hypothesis, Boyle's Law, stratosphere, vapor, vacuum (mercury-vapor) pump.

[1] It is said that Aristotle weighed a bladder full of air and deflated, observed no difference, and thus concluded that air had no weight.

PROBLEMS

1. How many molecules are in a cubic centimeter of gas at standard temperature and pressure?

2. What are the molecular weights of krypton (element number 36, monatomic), carbon dioxide (CO_2), and ethane (C_2H_6)? Compute the densities of these gases under standard conditions.

3. A bubble of air rises from a depth of 40 ft. below the surface of a lake. How many fold does its volume increase?

4. A toy balloon, inflated with 200 cu. in. of hydrogen, rises 5 miles in the atmosphere (Fig. 11). What is its volume at this altitude? (Neglect the change in temperature.) What is the volume of this same balloon submerged 20 ft. under water?

5. A tube a meter long closed at the top is lowered 100 ft. into a lake. How high will water rise in the tube? (This device has been used for deep-sea sounding.)

6. At what pressure will the gas represented in Fig. 10 have a volume of 1 liter? What volume will it occupy at 15 atmospheres' pressure?

7. A gas tank has a capacity of 2 cu. ft. and is filled with hydrogen at " 1500-lb. pressure." How many such tanks are required to fill a dirigible balloon of 7,000,000 cu. ft. displacement?

8. A closed tank, used for hot-water storage in the home, has a 10 cu. ft. capacity. Suppose that it is originally full of air; how much water can flow into the tank from the city mains, pressure 60 lb. per sq. in. (above atmospheric)?

9. A closed tank is 10 ft. high with valve at top and bottom. It contains 8 ft. of water and above that air at atmospheric pressure. If the upper valve is closed and the lower valve opened to the air, how many inches will the water level drop? (Final pressure in tank is approximately 26 ft. of water.)

10. How much does the air weigh in a cubic-foot tire at a pressure of " 45 lb."?

11. What is the change in weight of a liter of air when the barometer changes from 76 to 74 cm (Hg)?

12. What is the volume of a kilogram of hydrogen gas at a pressure of 3 atmospheres (0° C)?

13. What is the weight of gas in a 2 cu. ft. tank filled with oxygen at " 2000-lb. pressure "?

14. The German dirigibles, used in trans-Atlantic transportation, contained 7,000,000 cu. ft. of hydrogen. Find their lifting power.

CHAPTER 6

ELASTICITY

A solid body has a definite size and shape. Usually it is made up of crystals — sometimes of a single large crystal, often (as in metals) of many microscopic crystals. In each crystal the atoms have a regular arrangement, perhaps like the corners of a cube, perhaps like the corners of a tetrahedron (as in a pile of shot) — there are a great many different crystalline forms. But, whatever the arrangement, the atoms in a solid exert forces on one another which resist a change of size or shape.

TABLE 5

COEFFICIENTS OF LINEAR EXTENSION AND BREAKING STRESS

	Relative Change in Length (Strain) Caused by a Stress of			Breaking Stress	
	$1\ \frac{\text{kg}}{\text{sq. cm}}$	$1\ \frac{\text{kg}}{\text{sq. mm}}$	$1\ \frac{\text{lb.}}{\text{sq. in.}}$	$\frac{\text{kg}}{\text{sq. cm}}$	$\frac{\text{lb.}}{\text{sq. in.}}$
	(In parts per million)				
Aluminum......	1.4	140	0.1	1,500	21,000
Duralumin.....	1.4	140	0.1	4,000	60,000
Brass..........	1.0	100	0.07	3,000	42,000
Copper (hard) ..	0.8	80	0.06	2,500	35,000
Iron (wrought)..	0.5	50	0.03	6,000	85,000
Steel (drawn)...	0.5	50	0.03	10,000	140,000
Lead..........	6.	600	0.4	300	4,200
Glass..........	2.	200	0.14		
Wood.........	10.	1000	0.7		
Rubber (band)..	100,000				

57. Elastic Stretching. In Chapter 2 the "common-sense" laws of stretching were briefly considered. The first column of figures in Table 5 gives the relative change in length ($\Delta L/L$) of rods of 1 square centimeter cross-section under a stretching force of 1 kilogram. Study this table and understand exactly what it means. By simple proportion and without further discussion the student should be able to solve simple problems.

Examples. How much will a brass rod 1 sq. cm in cross-section and 1000 cm long stretch under a load of 10 kilograms? How many parts in a million will a steel rod of 5 sq. cm cross-section stretch under a load of a ton (1000 kilograms)?

The elongation is proportional to the force and the length, and inversely proportional to the cross-sectional area (§15).

$$\Delta L = c_L \frac{LF}{A},$$

or transposing

$$\frac{\Delta L}{L} = c_L \frac{F}{A}. \qquad (3')$$

The relative change in length ($\Delta L/L$) is called the strain; the force applied per unit area (F/A) is called the stress. *The strain is proportional to the stress.* This is a more general statement of Hooke's Law (§15).

58. Stress and Strain. Stress and strain are technical expressions. **Stress** is the force per unit area. By fixing one end of a rod and pulling on the other we apply a force-pair. Suppose that in this way we apply a stretching force of 100 pounds to a rod of 1 square inch cross-section, or suppose we pull with a force of 10 pounds on a wire with $\frac{1}{10}$ square inch section. In either case the stress is 100 pounds per square inch.[1]

$$\text{Stress} = \frac{F}{A}. \qquad \text{(Def.)}$$

A different kind of stress is involved when we attempt to compress a substance. Here the stress, the compressional force per unit area, is called *pressure* (§30). A third kind of stress (shearing stress) produces a change of shape without change of volume.

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[1] This is the "applied stress." In strictest usage stress is a property existing in the body. The atoms in a metal cohere because of the attractive forces between them. The stress which exists throughout the metal is a measure of the cohesive forces which act across each square inch or square centimeter opposing the stretching force-pair. For the cases which we consider this internal stress is equal to the applied stress.

A stress produces a strain. By **strain** is meant the *relative* change in size (or sometimes in shape).[1] If a cord a meter long is stretched a centimeter the strain is $\frac{1}{100}$ (i.e., 1 per cent). This is a *linear strain.* If ΔL is the elongation and L the original length:

$$\text{Linear strain} = \frac{\Delta L}{L}. \qquad \text{(Def.)}$$

When subjected to pressure of 25 kilograms per square centimeter a liter of water is compressed to 999 cubic centimeters; this is a *volume* (or bulk) *strain* of 1 part in a thousand.

$$\text{Volume strain} = \frac{\Delta V}{V}. \qquad \text{(Def.)}$$

A third type of strain is the shear. A shear is a change in shape such as is produced in a book when its two covers are moved in opposite directions, the pages sliding slightly over one another. When a shearing strain is produced in a rigid body the neighboring layers of molecules must be slightly displaced relative to one another.

For any kind of elastic distortion Hooke's Law applies: The strain is proportional to the stress. Equation (3′) is the expression for Hooke's Law as applied to stretching. The constant of proportionality (c_L) is the strain per unit stress. It is given (in millionths) in Table 5.

The reciprocal of this constant is often used; it is called *Young's Modulus* (§64).

59. Elastic Limit and Tensile Strength. When a wire is loaded with successively greater forces, its length at first increases proportionally, following Hooke's Law. If the load is now removed the wire resumes its original length. Such a body, which springs back to its original condition, is said to be *elastic.* Glass and steel are elastic; putty is not. But for each substance there is an *elastic limit,* and for stresses exceeding this limit the material is permanently deformed. Substances with low elastic limit are said to be *plastic.* Lead and gold and aluminum are notably plastic; they can be rolled or hammered out into thin foil.

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[1] Notice that this is quite different from the common use of the word. When one says that he has a feeling of strain, he does not mean that he feels a little taller than usual!

After the elastic limit is exceeded the substance usually "gives" — stretches with only a slight increase in stress. Ultimately it breaks. The breaking stress is called the *tensile strength.* Breaking stresses are given in Table 5; for drawn steel wire the breaking stress is 70 tons per square inch.

60. Bending. One of the common types of elastic deformations is the bend. When one steps on a plank supported at the ends it gives under his tread; it is stiffer and stronger if placed edgewise. In order for a beam to sag in this way, clearly the side on the outside of the bend must stretch or the inner side must contract. Actually something of each takes place, and along the center runs a "neutral surface" where there is no change in length. In consequence, the stiffness depends upon the coefficient of linear extension.

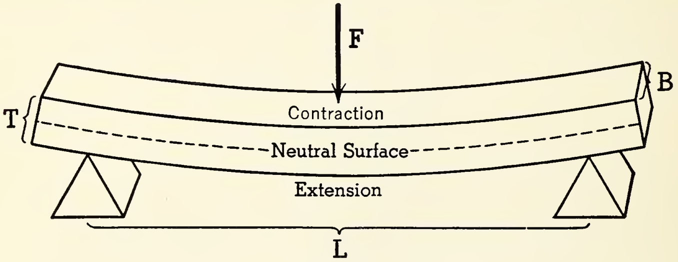

FIG. 1

The stiffness also depends, of course, upon the dimensions of the beam. It can be shown that the sag (S) varies directly as the load (F) and as the cube of the length (L), inversely as the breadth (B), and inversely as the cube of the thickness (T):

$$S \propto \frac{FL^3}{BT^3}.$$

How much stiffer is a beam twice as wide and twice as thick and half as long? Can you show that a two-by-four is four times as stiff when laid edgewise as when laid flatwise?

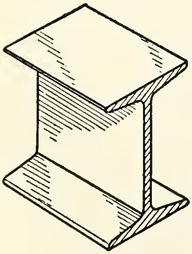

FIG. 2

In steel construction the "I" beam is the most common form of girder. It consists of a vertical *web* stiffened at top and bottom by *flanges.* The web is made wider than the flanges because thickness contributes more to the rigidity than does the breadth. The flanges resist bends from lateral forces which cannot be entirely absent. Most of the metal is in the flanges, in the upper which must resist compression and in the lower which resists extension; little is needed in the web near the neutral plane. In steel construction a variety of beam shapes are used. The "I" beam is used mostly in flooring where a vertical bending force can be counted on.

61. Compression. Stretching is not the simplest type of strain since it involves a change both of size and of shape. (A rod shrinks somewhat in cross-section when it is stretched in length.) We now discuss two strains, one (compression) a change of volume alone and the other (shear) a change of shape alone.

A liquid compresses when placed under pressure. (This is the only type of stress which can be applied to a liquid, since it has no definite shape.) By Hooke's Law the relative change in volume (strain) is proportional to the pressure (stress):

$$\frac{\Delta V}{V} = c_V P. \tag{3''}$$

c_V is called the *compressibility.* A solid can likewise be subjected to a uniform pressure on all sides by immersing it in a liquid under pressure; its volume decreases in proportion to the pressure, and its compressibility can be measured (Table 6).

TABLE 6

COMPRESSIBILITY

Relative change in volume (given in parts per million) for pressure of 1 kilogram per sq. cm.

Water	40
Mercury	4
Aluminum	1.5
Copper	0.7
Steel	0.5

Example. Find the change in volume of a liter (1000 cc) of water when subjected to a pressure of 25 kilograms per sq. cm.

Relative change in volume = 1000 parts per million or 0.1 per cent.

Hence 1000 cc decrease by 1 cc.

Compressibility of substances has been studied up to pressures of hundreds of tons per square inch. Although Hooke's Law ultimately breaks down, and, as the molecules become more closely crowded, the compressibility becomes less, substances are usually elastic under compression — they resume their original size when the stress is removed. This is true only when the pressure is applied uniformly on all sides. A column subjected to large compressional forces on its two ends is ultimately crushed.

62. Shear. A solid is distinguished from a liquid by its tendency to preserve its shape; this property is called rigidity. Suppose a block of rubber to be fastened to the table top. Pushing upon its upper face parallel to the table we distort it through an angle θ. This is a shear, a change of shape with no change of volume. θ is a measure of the shearing strain;[1] the stress is equal to the applied force (F) divided by the area (A) of the face. By Hooke's Law

$$\theta = c_\theta \frac{F}{A},$$

where the constant of proportionality c_θ is called the coefficient of shear. The reciprocal of c_θ is called the rigidity modulus.

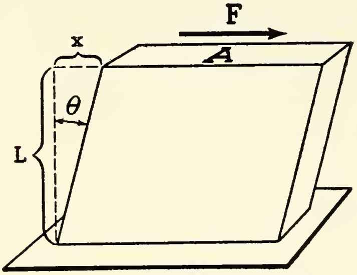

FIG. 3

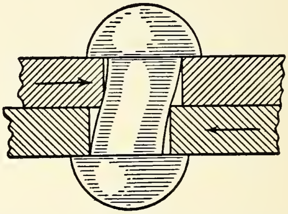

FIG. 4. Shearing stress on a rivet.

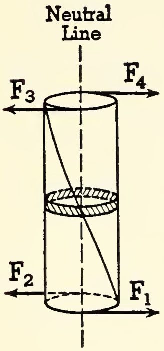

FIG. 5
Torsion.

A similar but much smaller shear is produced in other solids. For soft rubber a shearing stress of 1 kilogram per square centimeter will produce a shear of several degrees. Other solids are thousands of times more rigid.[2] In general the coefficients of shear are between 2 and 3 times greater than the coefficients of extension given in Table 5.

---

[1] If x is the displacement of the upper face and L the thickness of the block, x/L is the strain. For small distortions x/L is equal to the angle θ measured in radians (§170).

[2] This large coefficient of shear (small rigidity) is the notable characteristic of rubber; rubber is not very compressible — only a little more so than water. When a rubber band is stretched its cross-section varies almost inversely with its length; its volume remains almost constant.

63. Torsion. In Fig. 5 opposite couples are applied to the ends of a rod; the rod is twisted. If the rod is thought of as composed of a great many discs, these discs, as they turn, slide around over each other. Here again is a shearing stress. Down the center runs a neutral line with no shear. The angle of twist (or, more technically, of *torsion*) varies with (*a*) the length of the rod and (*b*) the strength of the applied couples; it varies (*c*) inversely as the rigidity modulus and (*d*) inversely as the fourth power of the radius of the rod. The same couple which will twist a 2-mm wire 1° will twist a 1-mm wire 16°.

64. Moduli of Elasticity. In introducing the subject it has seemed convenient to consider the strain as dependent upon the stress. More often one finds these relations stated the other way:

$$\text{Stress} = k \text{ Strain};$$

the coefficient of proportionality (k) is called the modulus of elasticity. For example, for stretching

$$\frac{F}{A} = k_L \frac{\Delta L}{L},$$

where k_L is called Young's Modulus (§58). It is the reciprocal of the coefficient of extension: $k_L = 1/c_L$. For the soft rubber given in Table 5 $k_L = 1/0.1 =$ 10 kilograms (weight) per sq. cm. For brass $k_L = 1/0.000001 = 1{,}000{,}000$ kilograms per sq. cm. Young's Modulus may be interpreted as the stress (F/A) which would be required for a doubling of length ($\Delta L/L = 1$) if this were possible without breaking.[1]

For compression

$$P = k_V \frac{\Delta V}{V},$$

and k_V is called the volume modulus (or bulk modulus). k_V is the reciprocal of c_V given in Table 6.

For shear

$$\frac{F}{A} = k_\theta \theta,$$

where k_θ is the rigidity modulus. k_θ is the reciprocal of the coefficient of shear c_θ. The value of the rigidity modulus can be found from tables; in general its value is between 2 and 3 times smaller than Young's Modulus (see §62).

65. The Nature of a Metal. When a section of metal is examined in a microscope it is found to consist of many crystals. The elastic properties of the metal are closely related to the shape and size of these crystals. Probably everyone has tried to break a piece of iron by repeated bendings — at each bend the elastic limit is exceeded, the minute crystals in the iron change in size, the metal becomes stiffer and finally breaks.

---

[1] The numerical values naturally depend upon the units used. In "absolute c.g.s. units" (§101) the modulus is given in dynes (instead of kilograms) per square centimeter and the value of the modulus is increased about 1,000,000-fold (more exactly, 980,000-fold). Young's Modulus for brass is approximately 10^{12} dynes per sq. cm. In absolute units the extension coefficients given in Table 5 (first column) are reduced by the same factor.

To find the fundamental properties of the substance itself a single crystal must be studied; recently techniques have been developed for growing large single crystals of many of the common metals; the consequence has been a rapid growth in our knowledge of metallic properties.

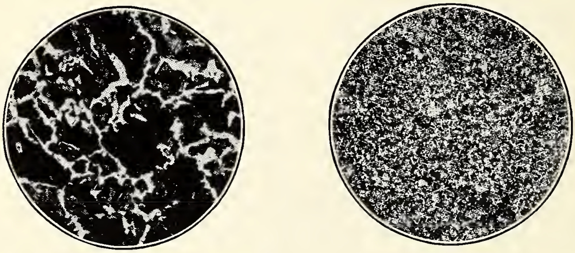

Fig. 6. Structure of steel: before and after heat treatment. (Photomicrograph.)

A question which is of great interest to the engineer concerned with the permanence of his structures is whether continued or repeated stress almost up to the elastic limit can in time cause a permanent strain in the material. Probably in a perfectly homogeneous metal there would be no such strain, but in practice "fatiguing" of the metal does occur. Possibly the student has met examples of it (breaking of valve springs, shafts, springs on automobiles).[1]

66. Hardening, Annealing, Tempering. After a piece of iron or steel or brass or copper has been drawn or hammered or bent, it hardens. In bending or stamping metals the work must occasionally be resoftened by heating to a red heat (and for iron or steel cooled slowly). This is *annealing*.

When red-hot steel is cooled suddenly (*quenched*) it is hardened. Usually the steel is then *tempered*, softened somewhat and rendered less brittle, by reheating to an appropriate temperature (indicated by the surface color) and requenching.

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[1] The classical example is Oliver Wendell Holmes' "One Hoss Shay," which "ran a hundred years to a day" and then instantaneously disintegrated ("Autocrat of the Breakfast Table").

TABLE 7

Properties of Steels and Steel Alloys

Chief Non-Ferrous Constituent	Characteristics	Use
0.1% carbon	Soft, ductile	Nails, chains
0.4% carbon	Medium	Beams, rails
1% carbon	Hard, brittle	Files, razors
3% nickel	Strong, hard	Machine steel
12% manganese	Ductile	Rock crushers
20% tungsten	Hard even at red heat	Lathe tools
7% molybdenum	Hard even at red heat	Lathe tools
1% chromium	Very hard	Files, armor plate
17% chromium	" Stainless steel "; non-corrosive	Engine valves, pocket knives, etc.
36% nickel	" Invar "; small thermal expansion	Surveyors' tapes, clock pendulums
78% nickel	" Permalloy "; high permeability (§347)	Magnetic shielding
4% silicon	High electrical resistance and permeability	Electromagnets (" transformer iron ")

Diamond and Carborundum are the hardest substances, but they are brittle. (They are used in grinding.) The discovery, in the middle of the last century, of steel, with its hardness, toughness, high elastic limit, and high tensile strength, greatly influenced our new civilization. Steel alloys — chromium steel, tungsten steel, vanadium steel, etc. — have been developed with still greater hardness and toughness, and these have played an important part in speeding up the machines of modern life. Recently duralumin, an aluminum-copper alloy, has been developed, which has tensile strength comparable with that of steel, much greater strength in proportion to its weight. It is used in the construction of dirigibles and airplanes.

67. Strength of Materials. It is essential that the architect and engineer know the strength of his materials. Surface hardness can be tested very easily; one method is to drop a hardened-steel ball from a definite height upon the sample and to measure the height of the rebound. Large machines are necessary for measuring the tensile strength, rigidity, torsional strength, crushing strength of structural beams. Testing laboratories have been established by the government, engineering schools, and large industries.

Everyone has a certain rough appreciation of the size of post required to resist crumpling, the size of wire or rod to hold a given weight. He knows that wires stretch, bulks compress, rods twist, under proper application of force. When Hooke announced his law he challenged scientists to determine *exactly* the strength of materials. Without this, construction of modern bridges, skyscrapers, and airplanes would be impossible. For steel, concrete, and all building materials, Young's Modulus, the bulk modulus, the rigidity modulus, crushing strength, and tensile strength are determined. The engineer uses these, computes allowable strains, adds a factor of safety of 100 per cent or so, and builds his bridge.

68. Solutions of Problems by Formula. Each of the equations in this chapter is simply the expression of a joint proportion, and it is suggested that the student consider problems in elasticity as problems in proportion. They can, however, be solved by the direct use of the formula. The values of c_L are given in Table 5 (to be multiplied by 10^{-6}) for kilogram weights per square centimeter or square millimeter and for pounds per square inch. Values of c_V (to be multiplied by 10^{-6}) are given in Table 6.

Example. Find the stretch of a 100-in. steel rod, ½ in. in diameter, under a load of 1 ton.

$$c_L = 0.03 \times 10^{-6} \qquad F = 2000 \text{ lb.}$$
$$L = 100 \text{ in.} \qquad A = \pi r^2 = 0.2 \text{ sq. in. approx.}$$

From (3′): $$\Delta L = \frac{c_L L F}{A} = \frac{0.03 \times 10^{-6} \times 100 \times 2000}{0.2} = 0.03 \text{ in.}$$

QUESTIONS

1. Upon what factors does the stretch of a wire depend?
2. What is Young's Modulus? May it be defined as the "stress required to produce a 100 per cent strain"?
3. Compare the sag of a plank under a load (*a*) when the load is doubled; when (*b*) the length, (*c*) the breadth, (*d*) the thickness, is doubled.
4. What kind of material is used for a file, a hammer, a spring, a horseshoe, a surveyor's chain?
5. Rubber is remarkable for its: (*a*) great compressibility; (*b*) great plasticity; (*c*) low modulus of rigidity.
6. If a body is perfectly elastic it necessarily: (*a*) obeys Hooke's Law; (*b*) stretches a great deal under an applied force; (*c*) resumes its original size when the stress is removed.

Vocabulary: Crystal, stress, strain, coefficient of linear extension, cohesive force, shear, elastic limit, tensile strength, plastic, hardening, annealing, tempering, quenching, duralumin, alloy, rigidity, surface hardness, compressibility, neutral surface, I-beam, flange, angle of deformation, torsion, Young's Modulus, bulk modulus, rigidity modulus.

PROBLEMS

1. If a 1-oz. weight stretches a rubber band 1 mm what will be the stretch when a 1-lb. weight is supported jointly by three chains of bands each consisting of 6 bands?

2. What stress is required to double the length of a rubber band (Table 5)?

3. A rubber band (two strands) is 1 mm thick, 5 mm wide, and 10 cm long. How much will a kilogram weight stretch it?

4. What force is required to stretch a $\frac{1}{4}$-in. steel rod by 0.1 per cent?

5. How much will a steel wire 1 mm in diameter stretch under a load of 11 lb.?

6. How much will a brass wire stretch (*a*) if it is 1 sq. mm in cross-section, 1 meter long, and supports a load of 1 kilogram? (*b*) If it supports a load of 15 kilograms? (*c*) As in *b*, except that the diameter of the wire is 2 mm (cross-section, 3.14 sq. mm)? (*d*) As in *c* except that the length of the wire is 350 cm?

7. Find the stretch of a steel rod 3 meters long, 3 sq. cm in cross-section, under a load of 3 tons.

8. A 150-ton locomotive is lifted by a pulley system with eight 1-in. (diameter) steel cables supporting the load. Assuming the cable lengths to be 50 ft., how much will they stretch before the locomotive is raised from the ground? Is there any danger that the cable will break?

9. (*a*) An aluminum bar is 1 sq. cm in cross-section. What force is required to stretch it by 1 mm for each meter of its length? (*b*) What force is required to stretch by 20 mm an aluminum wire 20 meters long and $\frac{1}{4}$ sq. cm in cross-section?

10. Find the tension in a steel wire 2 meters long, 1 mm in diameter, when it is stretched 1 mm.

11. Allowing a factor of safety of 200 per cent, how large a steel cable should be necessary to support a ton load?

12. How much weight will a brass wire 10 microns thick support?

13. By what per cent will water contract when subjected to a pressure of 100 kilograms per sq. cm? 100 lb. per sq. in.?

14. A plank sags 1 in. under a given load. Compare the sag of another plank twice as long, wide, and thick.

15. Compare the sag of two planks under their own weights when each dimension is doubled.

16. A man walks out on a springboard. Compare the bend when he is 1 yard, 2 yards, and 3 yards from the base. (Proportional relations the same as for a beam supported at each end.)

MECHANICS

CHAPTER 7

STATICS: BALANCED FORCES

Mechanics deals with forces and with the motions of bodies which forces produce. The subject is usually divided into statics and dynamics. Statics, which treats of bodies at rest, is the oldest branch of mechanics — not until the time of machinery was it necessary to know much of dynamics, but men had to know about equilibrium at rest as soon as they attempted to build houses and bridges and arches. The Egyptians who built the pyramids knew their statics — the pyramid is one of the most stable of structures.

69. Vectors. The forces of our commonest experience are the pushes and pulls which we exert on surrounding objects by muscular effort. We are accustomed to measure these pulls and pushes in pounds or ounces or tons; and we may measure them in gram-weights and kilogram-weights. But more than this statement of *magnitude* is necessary in the specification of a force, for a force has, in its very nature, *direction*. A 100-pound force upward or downward or eastward or northward — these are entirely different quantities; they produce quite different results.

A force is an example of a vector quantity. A quantity which has only magnitude is called a **scalar.** The number of oranges, or a duration of time, or an amount of money — such quantities are scalars. But many of the quantities of physics involve the concept of direction as well as magnitude, and such quantities are called **vectors.** Displacements[1] and velocities and forces are examples of such quantities.

It is an entirely different thing to walk a mile north and to walk a mile northeast or to ascend a mile upward. Similarly, velocity has direction — 10 miles an hour north and 10 miles an hour east are considered as different velocities. And so it is with forces. If two men push on a box each with a force of 10 pounds, the total force may vary from 20 pounds (as when the forces are in the same direction) down to zero (as when the forces are exactly opposite). A vector must always have its direction as well as its magnitude given. What is the direction of weight? In what direction does wind push on a sail? On a picture hung by two sloping wires what three forces act and in what directions?

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[1] Displacement: the change of place, as when a body is displaced (lifted) 3 ft. upward or displaced 3 miles northward. Displacement of the body as a whole is called *translation* (§11) (as distinguished from *rotation*).

In this chapter we discuss equilibrium as to translation; in the next chapter, rotational equilibrium.

We know how two scalars can be combined (as 2 and 3 give 5) and how a single scalar can be broken up into two parts (as 5 can be considered as made up of 2 and 3, or of 4 and 1). In this chapter we must consider the combination and "resolution" (breaking up) of vectors.

ADDITION OF VECTORS

70. Combining Vectors. A vector is represented by an arrow. Its length (to a chosen scale) represents the magnitude; the arrow point gives the direction. The arrows in Fig. 1 represent the successive displacements of a man who walks 10 miles east, 5 miles north, then 3 miles southeast. His total displacement is now approximately 13 miles, a little north of east. This, the combined result of several vectors, is called their *resultant*.

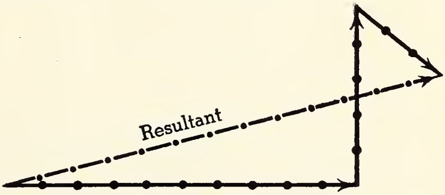

FIG. 1. Addition of vectors.

To find the resultant of several vectors, join them *tail to head* one after another; finally draw an arrow pointing from tail of the first to head of the last (the broken line in Fig. 1). This is the resultant. (Polygon of force method.)

Example. *A*, *B*, and *C* pull on a box with forces of 20, 20, and 15 lb. The force directions make angles of 120° with each other. Find the resultant force.

There is an alternative method of combining forces (or other vectors) when only two of them are involved. The vector arrows are drawn tail to tail; then a parallelogram is completed with these vectors as two sides; the diagonal (drawn in the angle between the two vectors) represents the resultant. This procedure using the *parallelogram of forces* is obviously equivalent to the other method. But since it can be applied to forces taken only two at a time we shall generally use the polygon method. Remember, in this latter method, the vector arrows are to be placed *tail to head.*

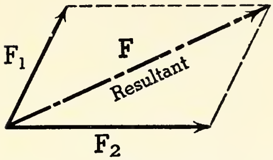

FIG. 2. Parallelogram of forces.

71. Equilibrium. First law of equilibrium: *If a body remains at rest the resultant of the forces acting on it is zero.* This is quite obvious for parallel forces, say all up and down. A 5-pound board rests on two trestles and two 10-pound weights rest on the board — 25 pounds down. The two trestles must, together, exert an equal upward force. The total force on the board is zero.

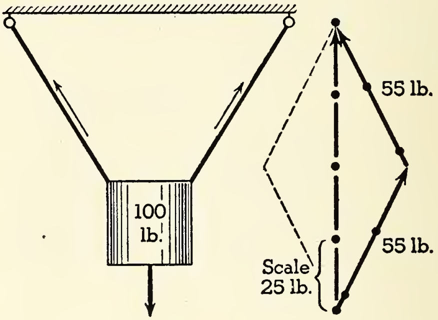

FIG. 3

When the forces acting are not parallel, vector addition is required. In Fig. 3 the forces in the two ropes must give an upward resultant of 100 pounds. But the *three* forces (the ropes and the weight) have a resultant of zero. *In equilibrium the vector diagram of all the forces acting forms a closed figure.*

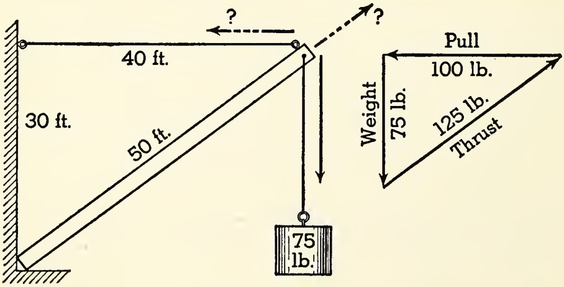

FIG. 4. The crane.

Illustrations: (1) *The crane.* Find the tension in the rope and the thrust of the beam in Fig. 4. The three forces must make a closed triangle, and (since the forces act in the directions of the ropes and beam) this force triangle is similar to the rope-beam triangle. Since the weight is 75 pounds, the thrust in the beam is 125 pounds (by proportion); the pull in the horizontal rope is 100 pounds.

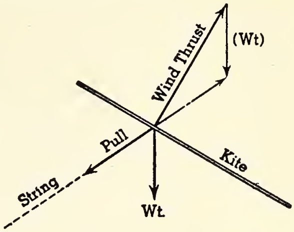

Fig. 5. Pull is anti-resultant of wind and weight forces.

(2) *The kite.* Upon a kite three forces act: there is (*a*) the weight vertically downward; (*b*) the pressure of the wind perpendicular to the kite; just opposite in direction and magnitude to the resultant of these two forces must be (*c*) the pull of the string.

(5) *The automobile.* We shall find that a similar equilibrium of forces occurs when a body is moving at constant velocity. Consider the automobile. The following forces act upon it: (1) its weight, (2) the support and forward thrust of the ground on the drive wheels, (3) the support and frictional retardation on the front wheels, (4) the backward force of wind resistance. The total force equals zero.

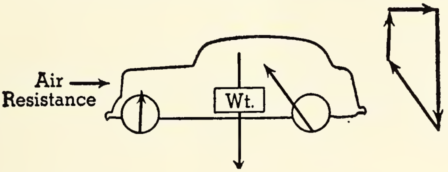

Fig. 6. Equilibrium of forces acting on automobile.

72. Resultant and Anti-Resultant. If the resultant of several forces is not zero, equilibrium can be produced by adding one other force exactly opposite to this resultant; this is called the anti-resultant. What is the resultant of the weight force and the rope force in Fig. 4? What is the anti-resultant?

PROBLEMS

(To be worked graphically)

1. A river flows 3 miles per hour north; a steamboat pointing east goes 4 miles per hour; a man on the boat runs exactly southwest 5 miles per hour. Is he at rest?

2. Find the resultant of a horizontal force of 1 kilogram and an upward force of 1500 grams. What is the anti-resultant of these two forces?

3. Four men push on a box. One pushes with a force of 20 kilograms north, another 30 kilograms east, another 30 kilograms southwest, another 20 kilograms northwest. In what direction does the box move?

4. Suppose that the wind exerts against a 100-gram kite a force of 300 grams in a direction 60° above horizontal. Find the direction of the kite string.

5. A balloon is tied to the ground by a long rope. Its weight (including gas) is 175 lb.; it displaces 250 lb. of air. The wind exerts a side thrust of 50 lb. Find the tension in the rope.

6. An airplane weighs 1000 lb. The propeller exerts a thrust of 100 lb. forward. Find the magnitude and direction of the supporting-resisting force of the air on the plane.

7. What force is required to balance two forces of 50 kilograms which make an angle of 160° with each other?

8. A 12-ft. hammock is tied 6 ft. from the ground, with enough slack so that a boy sits in it 3 ft. from the ground. The tension in each rope is 100 lb. What is the weight of the boy?

9. Suppose that the beam of the crane in Fig. 4 is raised to an angle of elevation of 60°, being tied by a horizontal rope 25 ft. long to a ring 46 ft. above the base. Redraw the figure, and compute the tension in the tie rope and the thrust of the beam.

RESOLUTION OF VECTORS

73. Resolution into Components. Two vectors combined give a single resultant; conversely, a single vector can be broken up (resolved) into two *components*. There are an unlimited number of ways in which this can be done (just as the number 5 can be broken into two parts [2 and 3; 7 and −2, etc.] in an infinite number of ways). But usually we wish to find *perpendicular components in definite directions*. A man wishes to go 10 miles northeast. Roads go only north and east. How far in each direction must he go? The student can complete Fig. 7, to show graphically the northward and eastward components of his journey. Evidently the triangle must be completed with the two components as the missing sides.[1]

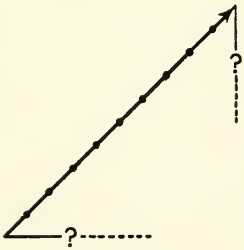

FIG. 7

74. Examples of Perpendicular Components. (1) A 10-foot flag-pole is elevated at an angle of 30°. How many feet high does it extend? How far horizontally? Certainly the student can solve this graphically; probably he can solve it exactly by the sine-cosine table (Table 52).

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[1] Or the parallelogram method may be used; the given vector is now the diagonal and the components are the adjacent sides.

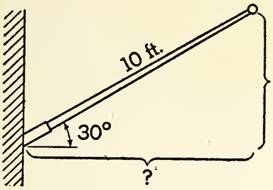

FIG. 8

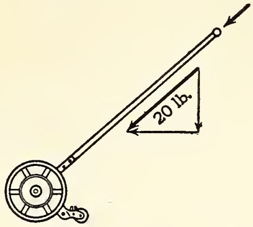

FIG. 9

(2) *The lawn mower.* The handle of a lawn mower makes an angle of 40° with the ground. What part of a 20-pound thrust is effective in moving the mower? The force is to be resolved into a horizontal component (effective) and vertical (non-effective). From the diagram it is evident that the effective component is approximately 15 pounds. (Find it exactly:[1] Effective force = $20 \times \cos 40°$). The vertical component presses the mower against the ground. How much is this component?

(3) *The inclined plane.* What force is required to pull a 100-pound cart up a 10-foot plank to a platform 3 feet high? The weight (W) is vertically downward; the problem is to find how much of this force acts along the plane (effective component, F). (The component perpendicular to the plane is non-effective.) The vector diagram here is a triangle similar to the triangle ABC.[2] Hence

$$\frac{F}{W} = \frac{H}{L}, \qquad (1)$$

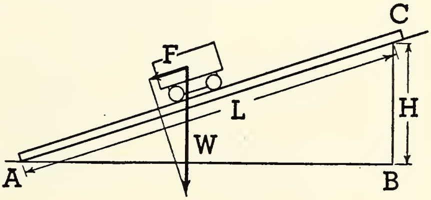

FIG. 10. Inclined plane. $F : W = H : L.$

and the force down the plane is 30 pounds. And this is the force which must be balanced if we are to push the box up the plane. (Eq. 1 has rather wide application and should be remembered.)

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[1] Though these components can be found exactly and most conveniently by means of the sine-cosine tables, the student may if he wishes solve all such problems graphically by a figure carefully drawn to scale.

[2] Because (as the student can show) corresponding angles in the two triangles are equal.

(4) *The hammock.* A 100-pound boy sits in a hammock. What is the tension in the ropes (Fig. 11)? Let T be the force in the rope. One component (n, not effective) merely pulls against the other rope; the vertical component (e) is $T \sin \theta$, and this must support half the weight. Hence

$$T = \frac{50}{\sin \theta}.$$

For small angles the tension may be very large. (Tightly stretched telegraph wires may be broken by a comparatively small weight of ice.)

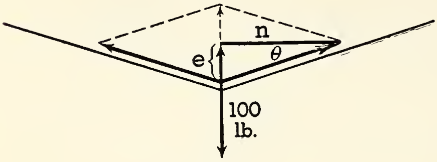

FIG. 11

This kind of resolution into two perpendicular components, one effective, one non-effective, has many applications.

75.* The Bridge Truss. The structural engineer must compute the stresses and strains in the steel skeletons which are so largely used in modern building. As an interesting example consider the bridge truss. Assume that the load is concentrated at the center. The two central rods must support the load. Only the vertical components are effective — this is the problem of the hammock, and for the angles given $f_1 = \frac{1}{2} \times 10/\sin 60° = 5.8$ tons. (The horizontal rods have no upward component — their only function is to keep the base of the truss from spreading.) Force f_1' equals f_1. f_1' in turn must be the anti-resultant of thrusts f_2 and f_3, which for the angles given each equals 5.8 tons. f_2' equals f_2. f_3' equals f_3 (we are neglecting the weight of the girders); the vertical component of f_3' (5 tons) is balanced by the upward thrust of the earth (f_5), and the horizontal component (2.8 tons) is balanced by the pull of the rod f_4'. So, link by link, the engineer carries the analysis through a truss work.

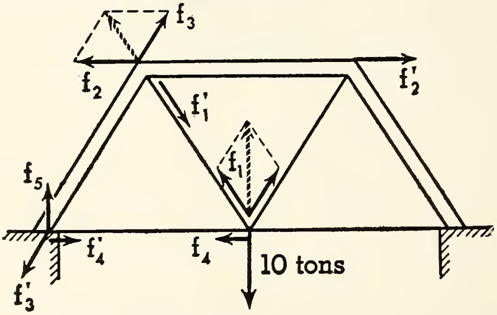

FIG. 12. Bridge truss.

76. Vectors in Three Dimensions. In general, vectors have components in three directions such as north-south, east-west, up-down. If in our examples we have discussed only two components it is because these are enough in many simple cases. The velocity of the train evidently has no upward component; obviously all the forces in the hammock problem lie in a single vertical plane. But the three components must be given for the velocity of an airplane. Evidently the resultant of a force up and one east and one north cannot be found by merely plotting on a piece of paper, but resultants of such vectors can still be found by representing the vectors as arrows in space and placing them tail to head. The components of a vector in three dimensions can be found by making the vector a diagonal in a parallelepiped; the edges of the parallelepiped give the components.

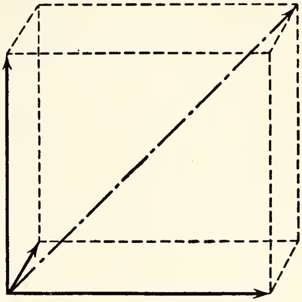

Fig. 13. Components in three dimensions.

77. Difference of Two Vectors. Obviously the difference between two vectors, each of magnitude 10 and extending in opposite directions, is 20; extending in the same direction there is no difference. What is the difference between them if their directions differ by 10°? What must be added to the one to give the other?

Figure 14 illustrates what is meant by the difference between two vectors. When we ask for the difference (Δ) between *B* and *A* we mean: What must be added (vectorially) to *A* to give *B*? Hence we can speak of the vector Δ in the figure as being the difference between *B* and *A*. As this example shows, the method of finding the difference is just opposite to that of finding the resultant: To find *B* − *A*, put arrows representing *A* and *B* with *tails together*; the vector from point of *A* to point of *B* is the difference.

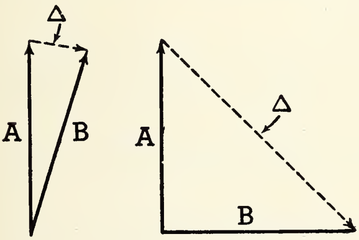

Fig. 14. Difference of vectors.

QUESTIONS

1. Can three forces, of 60 lb. each, produce equilibrium? of 10, 20, and 40 lb.?

2. Two forces, of 10 lb. each, pull out on a clock dial. What is the approximate sum when one pulls toward 12 and one toward 6? one to 12, and one to 3? one to 4, and one to 5? What is their difference when both pull toward 12? one to 12, one to 6? one to 11, one to 1?

3. What direction must a force have to balance 10 lb. N and 10 lb. E? 10 lb. N and 7 lb. S?

4. A 50-lb. boy is in a swing. What force does each rope exert? Will the answer be different if the ropes are not parallel?

5. Is it possible that the weight of a 50-lb. child may break a hammock rope which can support 100 lb. direct pull?

6. In what direction is the resultant of a speed 10 miles NE and 10 miles SE? In what direction is their difference?

7. In what direction is the resultant of equal forces up, north, and east?

8. What is the resultant of 10 lb. up; 2 lb. east; 7 lb. down; 3 lb. east, and 3 lb. down?

9. On a kite, if we combine the pull of the string and the wind thrust, the resultant of these two forces is: (*a*) downward; (*b*) upward; (*c*) perpendicular to the kite face.

10. To move 5 miles an hour straight east in a stream which flows 5 miles an hour due north a boat should be directed: (*a*) southeast, (*b*) northeast; and should move approximately: (*a*) 3, (*b*) 5, (*c*) 7 miles per hour.

Vocabulary: Statics, vector, scalar, displacement, resultant, anti-resultant, parallelogram of forces, crane, resolution, component, effective (non-effective) component, inclined plane, bridge truss.

PROBLEMS

1. A river is 1 mile wide and has a current of 3 miles per hour. A man rows 5 miles per hour. In what direction must he row to reach a point directly across stream? How long will it take?

2. A boat travels toward Europe with a speed of 20 miles an hour. A 30-mile wind blows from the northwest. How does it sweep across the deck (direction and velocity)?

3. A man pulls a weight along the ground exerting a force of 40 lb. Find the effective (horizontal) component if the rope makes an angle of 45° with the ground.

4. Continue problem 3, finding the horizontal component of force for angles of 15°, 30°, 45°, 60°, 75°. (Use the sine-cosine table, App. 1.)

5. A tent weighs 30 lb.; it is supported by a post at the center and is tied to the ground with four guy ropes, each making an angle of 45° and stretched with a tension of 20 lb. What is the vertical component of force of each rope? What thrust is exerted by the tent pole?

6. The angle between two wires supporting a picture is 120°. The tension in each is 2 lb. What is the effective (upward) component of each, and what is the weight of the picture?

7. A 150-lb. man hangs from a horizontal bar with arms at 45° from the vertical. Compute the tension in his arms.

8. A 50-lb. rug hangs from a clothesline. The line is 30 ft. long and sags 3 ft. What is the tension in the rope?

9. An archer's bowstring is 6 ft. long. He pulls it back 6 in., exerting a force of 10 lb. What is the tension in the string?

10. Neglecting friction, what force is required to propel a 3000-lb. automobile up a 10 per cent grade?

11. Neglecting friction, what force is required to pull a 100-lb. cart up a 10-ft. inclined plank, to a platform 4 ft. high?

12. A 20-lb. weight hangs by a 10-ft. rope. Pulling horizontally on another rope attached to the weight a boy pulls it 30° from the vertical. (*a*) What force does he apply? (*b*) What force is required to pull the suspension 80° from the vertical?

13. If the weight in problem 12 is suspended and pulled by steel wires 0.001 sq. in. in cross-section, through what angle can it be pulled before the wire breaks? Which wire will break first?

14. A board weighs 50 lb. It is pulled by the following forces: 30 lb. horizontally to the right; 50 lb. 60° below the horizontal; 40 lb. 60° above the horizontal; 20 lb. 120° around (counterclockwise) from the horizontal. (All in the same vertical plane.) (*a*) Find the vertical and horizontal component of each force. (*b*) Find the resultant force. (*c*) What force must be added to hold the board suspended?

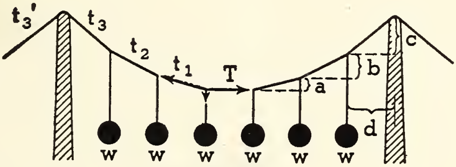

FIG. 15.

15. The Suspension Bridge. The balls in Fig. 15 (each with weight w) represent the roadway of a bridge. Neglecting the weight of the cable, show:

(*a*) The tension t_1 has a horizontal component equal to T (the tension at the midpoint) and a vertical component equal to w.

(*b*) The horizontal component of tension in t_1, t_2, t_3 remains constant; the vertical component equals w, $2w$, $3w$.

(*c*) The cable rise between junctions (a, b, c) increases as 1, 2, 3. (The resulting curve approximates a parabola.)

(*d*) At the tower the slope (c/d) of the cable is the ratio of the weight of half the bridge to the midpoint tension T.

(*e*) Each tower supports a vertical thrust equal to the weight of the bridge and no lateral thrust (assuming $t'_3 = t_3$, with equal slope).

Forces on a Clock Dial

16. Two forces act outward on a clock dial: a force of 70 grams toward 2 o'clock and 40 grams toward 5 o'clock. Find the direction and magnitude of the resultant.

17. Find the resultant of 30 grams toward 10 o'clock, 20 grams toward 12 o'clock, and 10 grams toward 1 o'clock.

18. (*a*) Which of the following forces have the same upward component: 87 grams toward 12 o'clock, 100 grams toward 1 o'clock, 174 grams toward 10 o'clock, 50 grams toward 3 o'clock? (*b*) Which have the same horizontal component?

19. Using the table of sines and cosines, find the horizontal components of the following forces: 30 grams toward 1 o'clock, 30 grams toward 4 o'clock, 41 grams toward 8 o'clock, 11 grams toward 11 o'clock. Show that the sum of these components equals zero.

20. Find the sum of the vertical components and of the horizontal components in problem 19. Combine these results to get the resultant of the four forces.

CHAPTER 8

STATICS: BALANCED TORQUES

If one pushes up on one end of a meter stick and down on the other, the forces may be balanced (in the sense that they are equal and opposite); nevertheless, the stick will not be in equilibrium because a couple is acting. The second law of equilibrium is: *In order that there shall be equilibrium for rotation the sum of the torques must be zero.*

78. Torque (or Moment of Force). It is not always convenient to pair forces into couples (§11). Speaking loosely, the *moment* of a force (or its *torque*) means its importance as a rotating agent. Observation shows us that the effectiveness of a force in producing rotation increases as its distance from the axis of rotation increases. A light touch applied at the outer edge moves a door; the force is only half as effective if applied halfway out; it is very difficult to move the door by pushing on it an inch from the hinge; it is impossible to open or close it by pushing upon the hinge itself.

The effectiveness of each of the forces F_1, F_2, F_3 (Fig. 1) in producing rotation depends jointly upon the magnitude of the force and upon the perpendicular distance (l_1, l_2, l_3) from the axis to the "line of the force." This perpendicular distance is called the **lever arm.** We define the **moment of force** (or **torque**) as the product of the force by the lever arm:

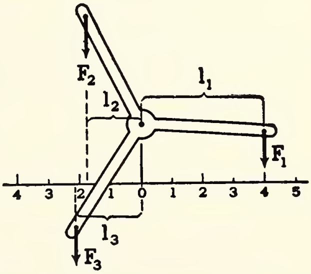

FIG. 1. Clockwise and counterclockwise torques.

$$\mathcal{T} = F \times l. \qquad \text{(Def.)}$$

If F_1 is 5 pounds and F_2 and F_3 each 4 pounds find the torque of each about the center. Which way will the system rotate?

It is customary to consider torques tending to produce counterclockwise rotation about the reference point as positive. Five forces are shown acting in Fig. 2. The moments about the point *c* are +27 (9 × 3), +9, −18, and −2 and −16. The clockwise moments equal the counterclockwise.

79. Equilibrium. This body is in equilibrium. (1) Because the forces balance it will not be translated (i.e., move as whole) — note that the two 9-pound forces balance, and the 6-pound downward force balances the 2-pound and 4-pound upward forces. (2) Because the sum of the moments equals zero, the body will not rotate. (What would happen if the 6-pound force were moved one division to the left?)

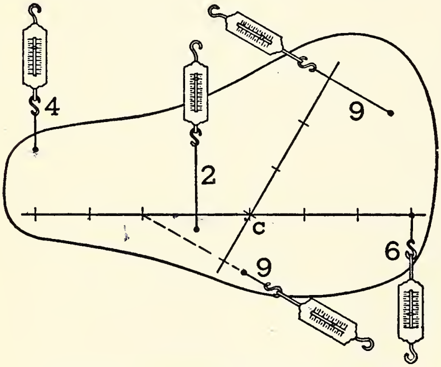

FIG. 2. Equilibrium of torques.

Any point can be chosen as reference point (*center of moments*) for computing equilibrating moments. The value of any single torque will depend upon the point chosen, but for any chosen center of moments the sum of torques will still be zero.

Summarizing, the two conditions necessary for equilibrium of a rigid body are:

(1) The resultant of the forces acting on the body equals zero. (Equilibrium for translation, Chapter 7.)

(2) The sum of the torques computed with respect to any chosen point (center of moments) equals zero. (Equilibrium for rotation.)

80. Lever Problems. (1) The lever furnishes the simplest illustration of the second condition of equilibrium. Everyone has used the lever for prying; we know how a comparatively small force on the longer arm of the lever results in a large force on the shorter arm. Taking the fulcrum as the center of moments we have from the second condition of equilibrium

$$F_1l_1 = F_2l_2,$$

or the forces vary inversely as the lengths of the lever arms. A device by which an increase in force is obtained is said to have a large *mechanical advantage.* In the lever the mechanical advantage depends on the ratio of the two lever arms. Archimedes, who discovered this law of the lever, boasted: "Give me a fulcrum and (with long enough lever arm) I can move the world." (The first condition of equilibrium shows that the force exerted by the fulcrum upward on the lever equals $F_1 + F_2$.)

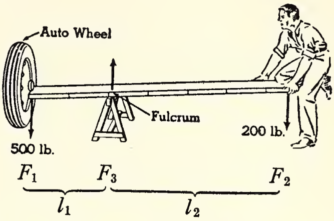

FIG. 3. Force varies inversely as lever arm.

(2) *The balanced beam.* A meter stick is balanced at the center on a straightedge (the fulcrum). The weight of the stick is 200 grams — this is considered as acting at the center (§81). Three weights are placed on the stick as in the figure. For equilibrium the fulcrum must exert an upward force of 450 grams (first condition). Choosing the fulcrum as center of moments we find one counterclockwise moment of 4000 (i.e., 100 × 40) gram-centimeters; the clockwise moments are 1000 (i.e., 50 × 20) and 3000 gram-centimeters. The moments balance, and there is no rotation.

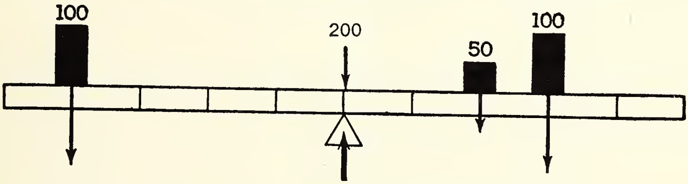

FIG. 4. Equilibrium of meter stick.

This same balance is obtained using another point as center of moments. For example, choose the left end of the stick. Of the five forces only the up-thrust of the fulcrum tends to produce counterclockwise rotation about this center. (Imagine the stick as hinged here, and consider the effect of each force separately.) This counterclockwise torque is $450 \times 50 = 22{,}500$ gram-cm. The clockwise moments are $100 \times 10 + 200 \times 50 + 50 \times 70 + 100 \times 80 = 22{,}500$. Notice that a slight simplification resulted when the midpoint was taken as center of moments, since about this point two of the forces (stick weight and fulcrum thrust) had zero lever arm and zero torque.

Find the clockwise and counterclockwise moments about the 70-cm mark.

(3) *Streetcar problem.* A 2000-pound streetcar is 4 rods long; wheels are 2 rods apart. The motorman weighs 200 pounds; three men on the back platform weigh 200 pounds each. Compute the forces on the wheels. Consider the weight of the car as acting at the center. Compensating the three downward forces (600, 200, and 2000 pounds), the ground pushes up on the wheels. Let these upward forces be x and y.

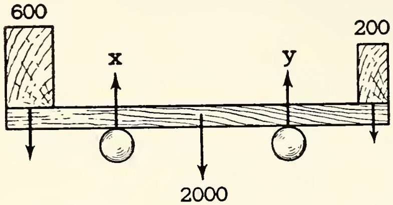

FIG. 5. Streetcar problem.

First condition of equilibrium: $x + y = 2800$ lb. (*A*)

Second condition of equilibrium: Take rear (left) wheel as center of moments.

Counterclockwise moments = Clockwise moments.

$$600 \times 1 + y \times 2 = 2000 \times 1 + 200 \times 3 \qquad (B)$$

$$2\,y = 2000 \text{ and } y = 1000 \text{ lb.}$$

From (*A*) $x = 1800$ lb.

81. Center of Gravity. In the above problems the weight of the body was considered as a concentrated force acting at the center. Actually the earth-pull (weight) acts on all parts of a body (Fig. 6). But if we choose a point in the body such that the combined moments of all the earth-pulls on the one side equal the sum of the moments of the earth-pulls on the other (and such that the equality persists for any orientation), the whole weight of the body can, for present purposes, be considered as concentrated here. This point is called the *center of gravity*. In a symmetrical body, like the meter stick or the streetcar, the center of gravity is evidently located at mid-length. In a hammer, it is close to the head. Let us compute the position of the center of gravity for the mallet shown in Fig. 6. The weight of the head is 10 pounds, of the handle 2 pounds. These weights are considered as acting at the center of gravity of

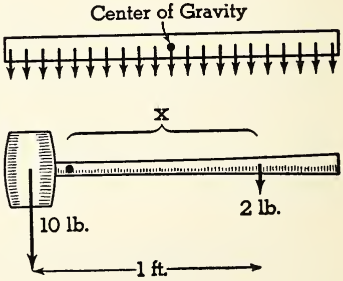

FIG. 6

each of these parts, 12 inches apart. The position of the center of gravity of the whole mallet must be such that the moments balance. Hence from the figure

$$2x = 10\ (12 - x);\ \text{therefore } x = 10 \text{ in.}$$

Of course the center of gravity of this mallet or of a baseball bat or golf club can be determined experimentally by balancing the body. For a more complicated shape (chair, table) the center of gravity can be found by suspending the body first from one center of support, then from another, dropping plumb lines from the supports and observing the point of intersection of these two lines. For in equilibrium the center of gravity is directly below the center of support.

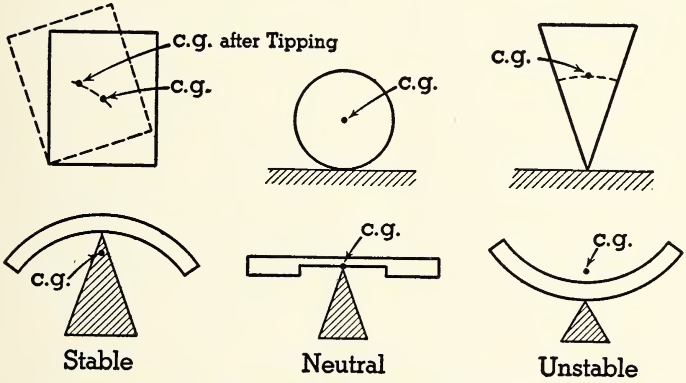

FIG. 7. Stable and unstable equilibrium.

The center of gravity seeks the lowest possible position. An object may be balanced on a knife edge with the center of gravity directly *above* the support, but the equilibrium is unstable and the body falls to one side or the other. For stable equilibrium the center of gravity must be below the supporting edge. . . When the body rests on a base of support the center of gravity must be directly over the base.[1] The lower the center of gravity and the larger the base the more stable is the equilibrium. Compare the stability of a brick, standing on its end and lying on its side. In either situation the center of gravity must rise before falling, but the rise is slight when the brick is standing on end.

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[1] This " base of support " is the figure of least perimeter drawn around the elements of the base: for a tripod, a triangle; for a table, a rectangle.

Of course these general conditions of stable equilibrium are unexpressed matters of common knowledge. When, as young infants, we learned first to roll, then crawl, then walk, these earliest mechanical experiments dealt with the problems of balance and we gradually learned to raise our center of gravity higher and higher over a smaller and smaller base. Anatomists, studying posture, have given considerable attention to the location of the center of gravity. In man it is about at the hips; in the apes, swinging and crawling, and with less developed lower limbs, it is somewhat higher.

82. The Chemical Balance. In the chemical balance the weight pans *swing freely* from agate knife edges (*a* and *b*) which are in the same plane as the fulcrum (*c*). The result is to locate their (effective) center of gravity at the fulcrum; this alone would produce neutral equilibrium. The center of gravity of the cross beam can be raised and lowered by screwing up or down the small weight (*d*). It is essential that the center of gravity be lower than the fulcrum, but not too low, since, for great sensitivity, the stability must not be very great.

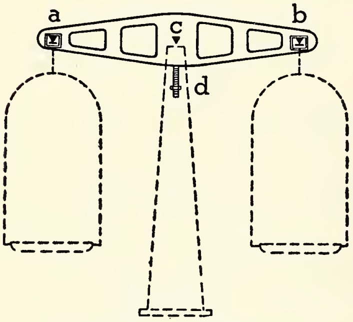

Fig 8. Chemical balance.

83. Examples of Torques and Levers. Two boys struggle to twist a baseball bat; one grips it in the small of the handle; the other at the larger end. Who will win? Why are large-handled screwdrivers more powerful than small, what advantage do we get with a monkey wrench, why use door knobs, why is wind more likely to topple a tall tree than a short one, why anchor the end of a springboard with a rock? Many other examples of moments of force will occur to the reader. The crowbar is the first example of a lever of which one thinks. But there are many other examples about us: scissors and pincers and nutcrackers, the baggage truck, the shoehorn, the crank of the ice-cream freezer, the trigger on the gun, and so on. Each joint in our human body is the fulcrum of a lever which is operated by muscular contraction.

Is the fulcrum of a lever always between the load and applied force? Compare a wheelbarrow and a seesaw. Compare the arm, which is bent at the elbow by the biceps pulling upon the forearm, and the foot, which is extended by an upward pull of the tendon which is attached to the heel.

84. Rotation and Translation. If the sum of the torques about the center of gravity of a body is zero but at the same time there is a resultant force acting on the body, then the body moves as a whole (translation without rotation). (Example: Imagine the fulcrum in Fig. 4 to be pushed upward with a force of more than 450 grams; or think of this supporting force as being removed; what will happen?) On the other hand, when the forces are themselves balanced (first condition) but not the torques, then the center of mass of the body remains at rest and the body is set into rotation. (Example: Suppose in Fig. 4 a 40-gram downward force is applied to the right end and 40-gram upward to the left.)

A body is set in rotation (without translation) by the application of a *couple*, the pair of equal and opposite forces which has been referred to in Chapter 2. (The two 9-pound forces in Fig. 2 represent a couple.) The strength (or moment) of the couple is the product of one of the forces and the distance between their lines of action. This is the same as the torque which the two forces will give about any center.

85. Examples of Second Condition of Equilibrium.

1. Given on a meter stick downward forces of 1 kg at 10, 2 kg at 30, and 4 kg at 60. What fourth force is needed for equilibrium? (Neglect the weight of the stick.) Let the force be x, and the point of application measured from the zero end of the stick be y. *Draw the figure.*

First condition: $1 + 2 + 4 + x = 0$; $x = -7$ kg. (Obviously.)

What does the negative sign mean?

Second condition (taking 0 as center of moments):

$$1 \times 10 + 2 \times 30 + 4 \times 60 = 7\,y$$

$$y = 44\tfrac{2}{7} \text{ cm.}$$

The required force is 7 kilograms upward applied at a point $44\frac{2}{7}$ cm from the zero end of the stick. If a straightedge is placed under the stick at this point it will exert this upward thrust and the stick will be in equilibrium.

2. Our examples have hitherto involved only parallel forces. We now give an example illustrating the balancing of moments among non-parallel forces. A 10-ft. ladder rests against a wall. Its foot, 6 ft. from the wall, is prevented

from slipping by a peg. The ladder weighs 40 lb. Compute the force against the wall and at the peg.

The forces on the ladder are:

(1) The downward force of gravity (*Wt*).
(2) The outward force at wall (F_1).
(3) The force obliquely upward on foot (F_2). F_2 can be resolved into an upward force F_2' and an inward force F_2''.

Take moments about the foot.

$$\begin{aligned} \text{Counterclockwise} &= \text{Clockwise} \\ F_1 \times 8 &= 40 \times 3 \\ F_1 &= 15 \text{ lb.} \end{aligned}$$

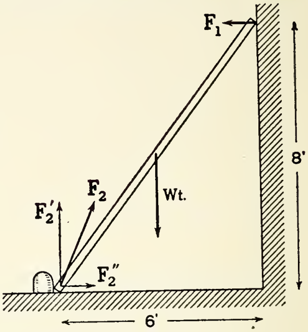

FIG. 9. Forces on ladder.

Now consider *first condition of equilibrium.* There are two vertical forces on ladder: the weight, 40 lb. down, and F_2' up. From first condition

$$F_2' = 40.$$

There are two lateral forces: F_1(15 lb.) and F_2'';

$$\therefore F_2'' = 15.$$

The total force here at the foot is $\sqrt{40^2 + 15^2} = 43$ lb. in direction shown.

QUESTIONS

1. Why should guy ropes of a mast be fastened as high as possible and as far out on the ground as possible?

2. Why is it easier for a dog to learn to walk than a child?

3. Why does a football player crouch? Why spread the legs in bracing against a charge?

4. Why not walk a tightrope with arms tied?

5. The tent peg is less likely to break out of the ground when driven as in *A* rather than as in *B*. Why?

6. When a wheel is spun on its axle by pushing on its rim, is a couple applied to the wheel?

7. Is a claw hammer a lever?

8. Give an example of a lever with fulcrum at the end; with applied force between the load and the fulcrum. Does the law of the lever (§80) apply to these cases?

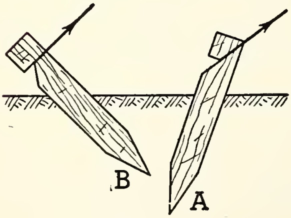

FIG. 10

9. Where is the center of gravity of the four weights in Fig. 16–3?

10. Is the meter stick in Fig. 4 in stable or unstable equilibrium?

11. Why is the peg necessary at the foot of the ladder (Fig. 9)? There is less tendency for a ladder to slip when a man steps on a lower rung; more when he is near the top of the ladder. Why?

12. When a body is in equilibrium the sum of the applied torques, clockwise and counterclockwise, equals zero when reckoned about: (*a*) the fulcrum; (*b*) the center of gravity; (*c*) any arbitrarily chosen point.

13. A ladder is suspended by the central rung. Is it in: (*a*) stable; (*b*) unstable; (*c*) neutral equilibrium?

Vocabulary: Torque, moment of force, couple, lever, lever arm, fulcrum, first (second) condition of equilibrium, center of gravity, stable (unstable, neutral) equilibrium.

PROBLEMS

✓**1.** A boy carries a 3-lb. fish on the end of a 4-ft. pole (horizontal over his shoulder). What are the force of his hand and the thrust on his shoulder if he grips the other end of the stick 1 ft. in front of his shoulder?

✓**2.** A 10-gram pencil 6 in. long rests on the table edge, with $2\frac{1}{2}$ in. extending over. If a 4-gram insect walks out on the pencil, how far does it go before the pencil topples?

3. *A* and *B* carry a 10-ft., 40-lb. ladder. A 50-lb. pail hangs 3 ft. from *A*. How much weight does each carry?

✓**4.** How many more 200-lb. men can stand on the rear platform in the streetcar problem (Fig. 5) before the car tips?

5. John weighs 100 lb.; Bill, 60. How must they sit to balance on a seesaw?

✓**6.** Where should the fulcrum of a 10-ft., 50-lb. seesaw be placed if John and Bill (problem 5) are to sit on the ends?

7. Two 80-lb. boys are on one side of a seesaw, one on the end and the other halfway out. Can a 150-lb. man balance the two?

8. A pail and a 6-ft. pole each have the same weight. A man and a boy carry the pail on the pole between them. Where can the pail be placed so that the man carries twice as much weight as the boy?

9. Is a meter stick with the following forces applied to it in equilibrium; if not, how does it move? (*a*) 1 lb. down at 30; 2 lb. up at 50; 2 lb. down at 60. (*b*) 1 lb. down at 30; 3 lb. up at 60; 2 lb. down at 70. (*c*) 1 lb. up at 20; 3 lb. down at 60; 2 lb. up at 80. Use middle of stick as center of moments. (Neglect the weight of the stick.)

✓**10.** From a 200-gram meter stick supported at the center are suspended the following weights: 300 grams at 0; 400 grams at 40; 100 grams at 60. What weight must be applied at the end of the stick (100 cm) to produce equilibrium? What force is then exerted by the support?

11. Work problem 10 with the support at 40 cm (instead of 50 cm). Where must the support be placed in problem 10 to produce equilibrium without the addition of a further weight?

✓**12.** Find the force against the wall and against the ground if a 100-lb. boy stands on the top rung of the ladder in Fig. 9.

✓**13.** A 10-ft. ladder weighing 30 lb. leans against a wall, its foot 5 ft. out. Find (*a*) the thrust against the wall and (*b*) the thrust against the ground.

✓ **14.** The beam of a crane is 20 ft. long and weighs 200 lb. It is inclined 30° from the vertical, supported by a horizontal rope at the top. Find (*a*) the torque of its weight about the base and (*b*) the tension in the rope.

✓ **15.** The beam of a crane is 20 ft. long and weighs 200 lb. It is held by a horizontal tie rope 12 ft. long and supports a 400-lb. weight. Find the tension in the rope and the force at the base of the beam.

✓ **16.** Find the forces in problem 15 if the tie rope is 15 ft. long and extends perpendicular to the beam.

17. A table is 3 ft. high and 4 ft. square. The top weighs 20 lb., and each leg weighs 4 lb. Find its center of gravity.

18. The pans and cross-beam of a chemical balance weigh 300 grams and have their center of gravity at the fulcrum. How much will the center of gravity be lowered if the adjusting nut (weight 10 grams) is lowered 1 cm?

19. A carpenter's square has arms of 10 in. and 15 in. It is hung over a nail at the vertex. What angle will the longer arm make with the vertical?

CHAPTER 9

UNIFORMLY ACCELERATED MOTION

If in 5 seconds an automobile increases its speed from 20 miles an hour to 30 miles an hour, it has gained 2 miles per hour speed in each second. Its average speed is 25 miles per hour. . . Although what we have to say in this chapter is no more recondite than this, not until about three centuries ago were these simple laws of motion understood. Ancient knowledge of mechanics was confined to statics.

86. Galileo. Galileo Galilei[1] worked in northern Italy about the year 1600 — about the time of Shakespeare in England, about the time of the first English colonies in America. Galileo studied the motions of bodies, analyzing them as we shall do. He observed that heavy and light bodies fall to the earth in the same time. (This experiment is always associated with the famous Leaning Tower at Pisa.) He observed that the falling body has constant acceleration. He studied the oscillating motion of the pendulum. This is the beginning of dynamics, and Galileo is often considered the founder of modern physical science. When Galileo dropped bodies of different weights from the Leaning Tower of Pisa and so appealed to an experiment instead of consulting the ancient lore, he challenged the authority of Aristotle, and, in matters of science, Aristotle's authority was then held almost sacred. But, more serious, he later challenged the old conception of the universe as centered about this earth of ours. He invented the telescope and discovered the moons about Jupiter. This supported the theory of Copernicus and so brought him into conflict with the church at Rome.

This is Galileo's chapter. Much of what we say here can be found in his famous (and very readable) book: "Dialogues on the New Sciences."

87. Motion at Constant Velocity. If an object moves at constant velocity, the distance traversed (s) is equal to the velocity times the time.

$$s = vt. \qquad \text{(Constant velocity.)} \qquad (2)$$

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[1] That is, Galileo Galileo-son (like John Johnson).

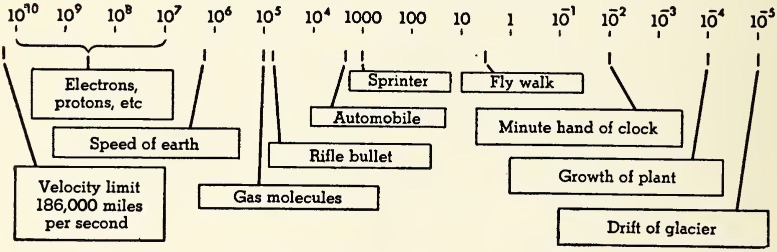
An automobile traveling 30 miles per hour travels how many miles in 2 hours? A ball rolling 10 feet per second travels how far in a minute?

This relation holds only for constant velocity. But if the speed varies it is still true that the distance equals *the average velocity* times the time. Walking, sometimes fast, sometimes slowly, if a man averages 2 miles an hour he will walk 20 miles in 10 hours. Representing the average velocity by $\bar{v}$

$$s = \bar{v}t. \tag{2'}$$

TABLE 8

Characteristic Velocities (cm per sec.)



88. Exact Significance of Velocity. When the velocity is changing what do we mean by the velocity at any instant? What does it mean when we speak of an automobile as having at a given moment a velocity of 30 miles per hour? Certainly we do not mean that the car is necessarily to go 30 miles in the next hour. More nearly we mean that the car goes $\frac{1}{2}$ mile in a minute or still better $\frac{1}{120}$ mile in a second or $\frac{1}{12{,}000}$ mile in $\frac{1}{100}$ second. We define velocity as the ratio of the change in distance (or the displacement $s_2 - s_1$) to the elapsed time $(t_2 - t_1)$ where, except in the cases of constant velocity, it is necessarily understood that the time interval is to be taken very small. The strict definition is

$$v = \left\{\begin{matrix}\text{Limit}\\ \text{approached}\\ \text{by}\end{matrix}\right\} \frac{s_2 - s_1}{t_2 - t_1}\left(\text{or by } \frac{\Delta s}{t}\right)$$

as $t_2 - t_1$ (or t) is made very small. This is what is meant when we define velocity as the distance per unit of time.

Velocity is a vector; *speed* refers to the magnitude of the velocity without regard to its direction.

89. Acceleration. If in 5 seconds a car speeds up from 20 miles per hour to 30 miles per hour, its speed changes by 2 miles per hour in 1 second. We say that it has an acceleration of 2 miles per hour per second. **Acceleration** is the rate of change of velocity; that is, it is the change of velocity per unit of time.

$$a = \frac{\Delta v}{t}. \qquad \text{(Def.)}$$

Instances of large acceleration are familiar. In Fig. 1 the ball is given a velocity of 64 feet per second in 1 second (average acceleration 64 feet per second per second). A rifle bullet is given a speed of some thousands of feet per second in less than a hundredth part of a second. Even greater is the negative acceleration when the bullet is stopped in perhaps a ten-thousandth part of a second by a plank or in a hundred-thousandth part of a second by a steel plate. The acceleration of the baseball striking the bat is not quite so large; but it is slowed up, stopped, and started back in the reverse direction, all within the thousandth part of a second (see Fig. **10**–4).

Observe: (1) Velocity is a vector, and the difference between two velocities (Δv) is a vector; hence acceleration is a vector. Example: The acceleration due to gravity is downward; the acceleration of the pitcher's baseball is forward.

(2) Acceleration is not the actual velocity change *in* a unit of time (unless the acceleration remains constant) but is the change *per* unit of time; it is a rate of change in a small time interval. (Compare §88.)

(3) The unit of acceleration involves the time unit twice. In the illustrations above, two different time units are used, hours and seconds. A mile an hour is 5280 feet in 3600 seconds or approximately $1\frac{1}{2}$ feet per second.[1] The automobile changed its speed from 30 feet per second to 45 feet per second in 5 seconds and had an acceleration of 3 feet per second per second.[2] (A little reflection will show that to speak of an acceleration of 2 miles per second or 3 feet per second is entirely meaningless.)

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[1] More exactly, 1.47 ft. per sec. The approximate relation is worth remembering and should be used in problems.

[2] This is often written "3 ft. per sec.^{2}" but read "3 feet per second per second."

(4) Acceleration is nearly the same as the "pick-up" which we speak of for the automobile. Pick-up, however, is not a vector. There is a change in velocity (hence acceleration) when an automobile rounds a corner at constant speed.

(5) If the initial velocity is u and the velocity t seconds later is v and if the acceleration is constant during the interval, we can compute the acceleration by dividing the velocity change $(v - u)$ by the elapsed time: $a = \frac{v - u}{t}$.

(6) If the final velocity (v) is smaller than the initial velocity (u) the acceleration is negative. Illustration: an automobile coming to rest.

90. Types of Motion. When the automobile (or elevator, Fig. 1) is started there is acceleration. The acceleration gradually decreases as the car gets up to speed. Soon the car proceeds at constant velocity. This constant-velocity motion, already discussed, is the simplest type of motion. The acceleration is zero.

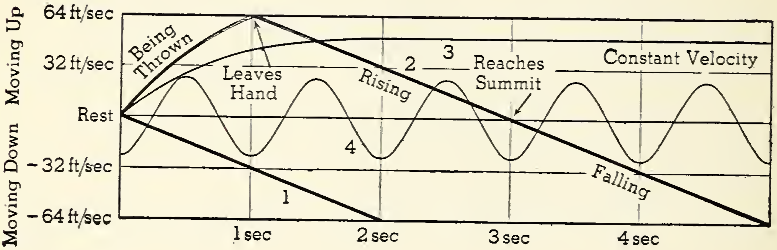

FIG. 1. Velocities: (1) Body falling from rest; (2) Ball thrown upward; (3) Elevator starting upward; (4) Bob oscillating on spring.

Next simplest, discussed in this chapter, is motion in which the acceleration is constant. In later chapters we shall discuss motion where the velocity vector is changing its direction but not its magnitude (central acceleration, §121); also we shall discuss the case where, as in the oscillating spring, the acceleration varies directly with the displacement.

91. Motion with Constant Acceleration: Velocity Attained. Suppose that an automobile picks up speed at the rate of 3 feet per second each second. In 2 seconds with this acceleration (a) its speed has increased by 6 feet per second; in 3 seconds by

9 feet per second, and so on. The change in speed is given by

$$\Delta v = at.$$

If its original speed was u, after t seconds its speed will be

$$v = u + at. \tag{3a}$$

Its average speed during the time will be

$$\bar{v} = u + \tfrac{1}{2} at.$$

If the automobile were originally going 30 feet per second, after 2 seconds of acceleration its speed will be 36 feet per second and its average speed in the interval is 33 feet per second.

92. Motion with Constant Acceleration: Distance Traversed. Going at this average speed our automobile would in the two seconds cover a distance of 66 feet. In general for a body with constant acceleration we have (from 2′)

$$s = \text{average speed} \times \text{time}$$

or

$$s = (u + \tfrac{1}{2} at)t$$

or

$$s = ut + \tfrac{1}{2} at^2. \tag{3b}$$

Observe: If the body starts from rest ($u = 0$) these relations (3*a*) and (3*b*) become

$$v = at$$
$$s = \tfrac{1}{2} at^2.$$

Or, starting from rest, the velocity is proportional to the time and the distance is proportional to the square of the time. A train starts from rest. During the first second it travels 1 foot; in the first 2 seconds, traveling longer and on the average faster, it travels 4 feet.

93. Falling Bodies. The best example of motion with constant acceleration is furnished by a body falling under the force of gravity. When Galileo dropped a heavy body and a light body from the Tower of Pisa, and watched them fall side by side, he showed that, except for the force due to the resistance of the air, all bodies fall to the earth with the same acceleration. Today, in a vessel from which the air has been removed, we show that a penny and a feather fall in equal times.

As a body falls it increases its downward speed by 32 feet (or 980 centimeters) per second in each second. If it starts from rest, after 1 second its speed is 32 feet per second, in 2 seconds 64 feet per second, in 3 seconds 96 feet per second, etc. This constant acceleration of falling bodies is usually represented by g. Falling from rest

$$v = gt. \qquad \text{(eq. } 3a \text{ with } u = 0.)$$

TABLE 9

VELOCITY AND DISPLACEMENT OF FALLING BODY

Time seconds	Speed ft. per sec.	Average Speed ft. per sec.	Distance ft.
0	0	0	0
1	32	16	16
2	64	32	64
3	96	48	144

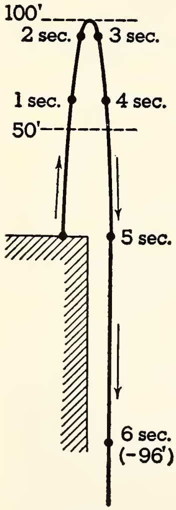

FIG. 2

We can find the distance traversed from eq. (3*b*). But let us rather find it once more from first principles. Let us find for instance the distance a body falls in 1 second. Initial speed (u) is 0; final speed v is 32; average 16—falling for 1 second at this average rate the displacement is 16 feet. During the first 2 seconds: the average speed is 32 feet per second, the distance 64 feet. Similarly in 3 seconds the fall is 144 feet. Evidently the distance varies as the square of the time (compare the first and last columns of Table 9). The relation is (from [3*b*], putting $u = 0$):

$$s = \tfrac{1}{2} gt^2.$$

(What will be the fall in 4 seconds? in 10 seconds? in $\frac{1}{2}$ second?)

The acceleration is 32 feet per second downward whatever the initial velocity. If a ball is thrown downward, leaving the hand with a speed of 80 feet per second, its speed 2 seconds later will be 144 feet per second. (The average between 80 and 144 can be found and the distance obtained.)

If a ball is thrown upward, it decreases its speed by 32 feet per second per second. Both positive and negative velocities are involved — in such a case it is customary to take upward as positive. Suppose the initial speed is +80 (upward); the acceleration is −32 feet per second per second (downward); after 1 second the speed will be +48; in 2 seconds, +16; in 3 seconds, −16; then −48, and so on. How many seconds will it be before the ball reaches its highest point and stops? What is its average speed during this time, and how high does it go? (Fig. 2.)

In metric units the value of the acceleration due to gravity (g) is approximately 980 centimeters per second per second.

94. Variation of *g* with Locality. The value which has been given (980 centimeters per second per second or 32 feet per second per second) for the gravitational acceleration is only approximate because this acceleration of fall depends upon the locality. We shall find that the effect of gravity becomes less at points distant from the earth. It is reduced slightly on high mountains. On account of the flattening of the earth the gravitational attraction is greatest at the poles. Partly owing to the greater radius of the earth and partly to the tendency of objects to fly from a rotating object, the acceleration of fall is least at the equator. The value of g varies from 978 centimeters per second per second at the equator to 983 centimeters per second per second at the poles. (See Table 15.)

95. Problems by Formula. The student has probably learned that most problems in physics can be thought through from the beginning or, on the other hand, can be worked with much less mental effort by inserting values in formulas and solving. The important formulas for motion with constant acceleration are:

$$\begin{aligned} v &= u + at & (a) \\ s &= ut + \tfrac{1}{2} at^2 & (b) \\ 2\,as &= v^2 - u^2 & (c) \end{aligned} \qquad (3)$$

(Eq. c will be derived in the next section.) Of the five quantities s, t, u, v, a, four occur in each formula. Three values will be given and one asked for in each problem. The proper formula can be chosen and the equation solved.

Examples. 1. An automobile moving 20 miles per hour is brought to rest in 5 sec. Find the acceleration.

u = 20 miles per hour = approx. 30 ft. per sec.;
$v = 0$; $t = 5$ sec.; $a = ?$

Use (*a*); $a = \frac{0 - 30}{5} = -6$ ft. per sec.^{2}

This could be solved very easily without a formula — a speed of 30 ft. per sec. is lost in 5 sec.

2. A ball is thrown upward with a speed of 80 ft. per sec. How fast will it be going after 3 sec.?

$a = -32$ ft. per sec.^{2} (It is a freely falling body.)
$u = 80$ ft. per sec. (Positive because upward.)
$t = 3$. $v = ?$

Use (*a*); gives $v = -16$ ft. per sec. (See Fig. 2.)

3. A ball is thrown upward with a speed of 30 meters per sec.; where will it be after 4 sec.?

$u = 30$ meters per sec.; $a = -9.8$ meters per sec.^{2};
$t = 4$ sec.; $s = ?$

Use (*b*); $s = 30 \times 4 - 4.9 \times 16 = 41.6$ meters.
Is the ball on its way up or down?

96. Motion with Constant Acceleration: Distance and Velocity. Let us now take one final example: An automobile starts off with an acceleration of 4 feet per second per second. How far must it travel to attain a speed of 20 feet per second?

$u = 0$; $a = 4$ ft. per sec.^{2};
$v = 20$ ft. per sec. $s = ?$

This problem cannot be worked *directly* by either (*a*) or (*b*). From (*a*), however, we can find the time (*t*) required (5 seconds) and then find the distance (*s*) from (*b*). (What is the answer?)

But we must now derive eq. (*c*) by which such problems as this last (involving *u*, *v*, *a*, and *s* — not *t*) can be worked in a single step. Let us eliminate *t* from (*a*) and (*b*). From (*a*) we have

$$t = \frac{v - u}{a}.$$

Substituting this for *t* in eq. (*b*) we have

$$s = u\frac{v - u}{a} + \tfrac{1}{2}a\frac{(v - u)^2}{a^2},$$

which the student can simplify to give

$$s = \frac{v^2 - u^2}{2a}$$

or

$$2as = v^2 - u^2. \qquad (3c)$$

Formulas (*a*), (*b*), and (*c*) should be memorized; they will suffice for solving any problem in uniformly accelerated motion.

Inserting the values of our problem in this equation we have

$$2 \times 4 \times s = 400 - 0$$

$$s = 50 \text{ ft.}$$

97. Motion on Inclined Plane. An automobile rolls down hill with constant acceleration (assuming constant slope). This acceleration is due to gravity, but it is, of course, much less than g; only one component of the gravitational acceleration can be effective, the component along the plane. We may refer again to Fig. 7–10, giving the vectors now a different significance. Let us relabel the arrows, calling the vertical vector g (instead of W), the downward acceleration of gravity, and calling the component along the plane a, the effective acceleration. Then, because of two similar triangles,

$$\frac{a}{g} = \frac{H}{L} \quad \text{(compare eq. 1)}$$

or

$$a = \frac{H}{L} g.$$

Thus the acceleration is reduced from the free fall value in the ratio of $H : L$; on a 5 per cent grade ($H/L = 1/20$) the acceleration is 1.6 ft. per sec. per sec.

We can show that, starting from rest and neglecting friction, the speed at the bottom of the incline is exactly the same as for free fall from the same height. By eq. (3*c*) the velocity for free fall is obtained from

$$v^2 = 2\,gH.$$

On the other hand along the incline eq. (3*c*) becomes

$$v^2 = 2\,aL.$$

Or using the relation just obtained

$$v^2 = 2\,\frac{H}{L}\,g \cdot L = 2\,gH,$$

as for free fall.

98. The Trajectory. If a stone is thrown horizontally from a cliff with a velocity of, say, 80 feet per second it keeps this horizontal velocity unchanged. Its displacement in this horizontal direction is proportional to the time ($s = vt$). At the same time it falls; its downward component of velocity increases by 32 feet per second every second. Its rate of fall is exactly the same as for a stone which is dropped; it falls 16 feet in the first second, 64 feet in 2 seconds, etc. (Fig. 3). The resulting path, with horizontal displacement proportional to the time and vertical displacement proportional to the square of the time, is the parabola (§720).[1]

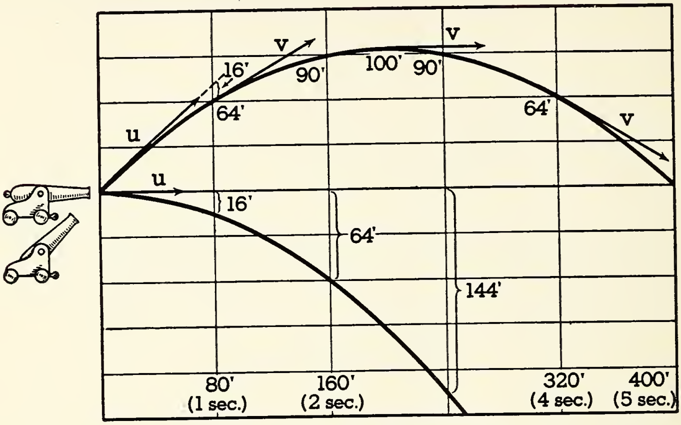

Fig. 3. Trajectories: initial velocity (*a*) horizontal and (*b*) 45° upward. Horizontal component 80 feet per second in each case.

Thus the problem of the projectile breaks up into two problems — horizontally the velocity is constant and the displacement is given by (2). Vertically the acceleration is constant and the displacement is given by (3*b*). Consider the ball thrown at an angle of 45° with initial speed of 113 ft. per sec. The velocity has equal upward and horizontal components, each 80 ft. per sec. ($113 \times \sin 45° = 113 \times \cos 45° = 80$.) The trajectory is shown in Fig. 3. Horizontally the motion is constant. The vertical motion is the same as that graphed in Fig. 2. In 1 sec. the ball is 64 ft. above the ground, having fallen 16 ft. off of the initial 45° line of flight. In 2 sec. it is 90 ft. high, in $2\frac{1}{2}$ sec. it has reached its summit, and so on. The trajectory is the same parabola as before, somewhat displaced.

The velocities along this trajectory are also interesting. The initial direction of the velocity vector (represented by the arrow) is 45°. The vertical component is initially 80 ft. per sec., decreases by 32 ft. per sec. each second, becomes zero in $2\frac{1}{2}$ sec., and in 5 sec. is 80 ft. per sec. downward. Follow these changes of the velocity vector in the figure.

99.* Relativity of Motion; The Velocity Limit. Perhaps it has occurred to the reader that motion is (like many other things) purely relative. We speak of a tree at rest, a train as moving, because there is rest or motion relative to ourselves or relative to the ground. But a person in the train regards himself and other objects in the train as at rest, the ground as in motion. When we speak of motion, we mean motion with reference to something, and this something which we have in the background of our minds is called the *frame of reference.* In this chapter we have tacitly chosen the ground for our frame of reference; we might as well have chosen a *uniformly* moving train. Everything happens exactly the same in the train as on the ground — indeed there is no way to tell whether it is the train or the earth which is " really moving."

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[1] The path of a projectile is called its trajectory (from the Latin *trajicere*, to throw across). If we allow for the non-parallelism of gravitational force — it points toward the earth center — we get for the trajectory an ellipse rather than a parabola (Chapter 17). The two curves are not distinguishable over a small arc. Of more importance is the effect of air resistance which very considerably alters the shape of the trajectory.

There is no *logical* reason why we should not use any frame of reference whatsoever. But from a *practical* point of view it is usually inconvenient to use a reference system which we recognize as being itself accelerated. Many things behave differently in a train which is rounding a curve or suddenly stopping. Our ordinary laws of physics (particularly our concept of force) are not patterned to apply to such frames of reference.

It is remarkable fact that there is in nature a greatest value for velocities which can be approached but never exceeded. In 1905 Einstein showed that no material body can travel faster than the speed of light (186,000 miles a second or 3×10^{10} cm per sec.). It appears that the inertia of a body increases, its acceleration (under any force whatever) decreases, as this limiting speed is approached. And this is true for all observers; *it is true for any frame of reference with respect to which the motion may be described!* This was a consequence of Einstein's *Theory of Relativity.* Looking about us at ordinary things moving, some faster, some slower, but all very slow in comparison with the limiting speed, we should never suspect such a limit. At very high velocities mechanics must be modified.

100.* Dimensions. On first acquaintance the equations of physics look strange. So many inches on one side of the equation seem to equal so many seconds, or grams, or what not, on the other. Were the equations really so, they would be strange indeed. As a matter of fact the equations always balance. Grams equal grams, lengths equal lengths. For example, consider our eq. (*3b*). If it is remembered that velocity is expressed (for example) as feet per second (ft. per sec.) and acceleration as feet per second per second (ft. per sec.^2) it will be seen that this equation is

$$s = ut + \tfrac{1}{2}at^2.$$

$$\text{ft.} = \text{ft. per sec.} \times \text{sec.} + \tfrac{1}{2}\,\text{ft. per sec.}^2 \times \text{sec.}^2$$

The time cancels out, and we have a distance equal to a sum of two distances. So it is for every physical equation.

An apparent exception is our equation for the pressure of water (§32) (using grams per square centimeter and centimeters):

$$P = h.$$

Here the pressure (force per unit area) is set equal to a length. But this is not the complete equation. As we find for any other liquid the complete equation is 1″ (or 1)

$$P = d'h,$$

$$\text{Pressure}\left(\frac{F}{A}\right) = \frac{\text{Weight}}{\text{per cm}^3} \times \text{Height (cm)},$$

and the equation balances; $\frac{\text{Force}}{\text{Length}^2} = \frac{\text{Weight (which is a force)}}{\text{Length}^2}$. For water the weight per cubic centimeter happened to be unity and for simplicity was suppressed.

Such *dimensional analysis* is often used, not to derive the exact equation (for it gives no information as to the values of numerical constants), but to detect erroneous or incomplete equations which do not balance.

101. Units. The time has arrived to select a definite system of fundamental units, or rather two such systems — a fundamental metric system and a fundamental British (or engineering) system. In the past we have used grams or kilograms, meters or centimeters, seconds or hours indiscriminately. This introduces no confusion as long as we use proportion. Pressure is proportional to depth: $P_1/P_2 = h_1/h_2$, and this is true whether the depths (h) are expressed in inches or in feet or in miles. Proportion will be useful throughout physics, and *when using proportion any convenient units may be chosen.*

TABLE 10

BASIC UNITS

	Metric	British
Time	Second	Second
Length	Centimeter	Foot
Mass	Gram	———
Force	———	Pound
Area	Square centimeters	Square feet
Volume	Cubic centimeters	Cubic feet
Velocity	Centimeters per second	Feet per second
Acceleration	Centimeters per second per second	Feet per second per second
Pressure	———	Pounds per square foot
Density	Grams per cubic centimeter	———
Torque	———	Pound-feet

Not so when using the algebraic method, where the equations must balance. Hence we shall choose two alternative systems of units; all constants will henceforth be given in these units, and these units must be employed in all equations (except when using proportion). The fundamental units are given in Table 10.

All physical units can be derived from the three fundamental units of length, mass, and time. In the metric system the centimeter, gram, and second are chosen as these fundamental units and the system is called the c.g.s. system.[1]

The unit of force (and hence of pressure and torque) has been omitted from the c.g.s. units. In the next chapter it will develop that it is desirable to use as the unit, not the weight of a gram mass (which would give that mass an acceleration of 980 centimeters per second per second) but that smaller force which will give it unit acceleration. The unit of mass in the British system will also be defined in the following chapter.

QUESTIONS

1. A baseball and a rubber ball with the same speed strike the floor. Which has the greater acceleration as it stops?

2. A hunter fires point blank at a monkey on a distant tree. At the moment it sees the explosion the monkey drops from the tree. Is it hit?

3. A train traveling 20 miles an hour comes to rest in a minute. What is its acceleration?

4. What is the average speed of a stone during its first second of fall, and how far does it fall?

5. How do the acceleration and speed and displacement of a body falling from rest vary with the time?

6. How far does a body fall in 1 sec., 2 sec., 3 sec.? How far in $\frac{1}{2}$ sec.?

7. A ball is thrown upward with a speed of 96 ft. per sec. How long does it rise; what is its average speed; how high does it go?

8. At what time does the oscillating bob in Fig. 1 have a positive acceleration; zero acceleration? When does the thrown ball reach its summit?

9. A boy throws a ball backward from a moving train; does he increase or decrease its velocity?

10. Inspect problems 2, 8, and 9 below. What three quantities are given and what asked for in each problem? Indicate opposite each which formula will be used for solution.

11. For a body starting from rest with constant acceleration is the speed proportional to: (*a*) the distance directly; (*b*) the square root of the distance; (*c*) the distance squared?

12. For a projectile is the horizontal distance covered proportional to: (*a*) the time; (*b*) time squared; (*c*) square root of the time?

Vocabulary: Velocity, initial (average, final) velocity, acceleration, uniformly accelerated motion, trajectory, velocity limit, Theory of Relativity, dimensional analysis, c.g.s. units, British (engineering) units.

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[1] Another system of units, based on the meter, kilogram, and second (m.k.s. system) is described in Appendix 3. These units are of convenient size for many practical problems. However in this text the only system of metric units which will be used consistently is the c.g.s. system.

PROBLEMS

1. A baseball diamond is 90 ft. between bases. A fast thrown baseball travels 100 ft. per sec.; a fast runner goes 10 yd. per sec. Assuming that it takes $\frac{1}{2}$ sec. for the catcher to return the throw, how much lead off from first base must a runner have to steal second?

✓**2.** A train starting from rest acquires a speed of 40 miles an hour in 2 min.; what is its acceleration?

3. A ball rolls down an inclined plane with an acceleration of 3 ft. per sec.^{2} How far does it travel in 5 sec.? How fast will it be traveling 10 ft. from the start?

✓**4.** A train starts from the station with an acceleration of 1000 ft. per min.^{2} How long will it take it to reach a speed of a mile a minute? How far will it go in 120 sec.; in 150 sec.?

✓**5.** I find that coasting in neutral my automobile slows down from 50 miles per hour to 30 miles per hour in 30 sec. What is the acceleration? How many yards does it travel during this time?

6. An automobile traveling 30 miles an hour can be brought to rest in 40 ft. (Fig. 12–3). Compare the acceleration with that of gravity.

✓**7.** Airplanes are launched from ships by means of a catapult. A 2-ton plane was pushed along a 60-ft. track and launched with a speed of 70 miles per hour. What was the acceleration?

✓**8.** As the result of an experimental investigation it was found that 20-gram arrows leave the bow with a speed of about 50 meters per sec. The arrow was pulled back 50 cm. Find its average acceleration.

9. If a ball is thrown upward with a speed of 100 ft. per sec., how long will it rise, and how high?

10. The acceleration of gravity on the moon is $\frac{1}{6}$ as much as on the earth. If a ball were thrown upward from the moon with a speed of 100 ft. per sec. how high would it rise?

11. The muzzle velocity of the U. S. Army rifle is 800 meters per sec.; how high will a bullet rise, fired directly upward (neglecting air resistance)?

12. Find the time of flight in problem 11.

13.[1] A ball is thrown upward from the edge of a cliff with a velocity of 40 ft. per sec. (*a*) When will it be 24 ft. above its starting point? (*b*) 24 ft. below its starting point? (*c*) 40 ft. above its starting point? Use equation and interpret the two, one, and zero possible answers in these respective problems. (*d*) Find the velocities in each case.

14. Find the acceleration of a block sliding down a frictionless plane which makes an angle of 30° with the horizontal.

15. Neglecting friction, how long does it take a sled to descend a toboggan slide 16 ft. tall and 48 ft. long? What is the final velocity?

16. Starting from rest, how long will it take an automobile to roll 160 ft. down a 10 per cent grade? What is the final velocity?

---

[1] Involves quadratic equation.

17. A cannon is located 16 ft. above water level. If a shell is fired horizontally with a muzzle velocity of 2000 ft. per sec. where will it strike the water?

18. Assuming a speed of 100 ft. per sec., how far does a baseball drop from the straight line in going from the pitcher to the plate (approximately 60 ft.)? How much when the catcher throws to second?

19. A cannon is fired at an elevation angle of 30° with muzzle velocity of 2000 ft. per sec. What is the upward component of velocity? How long will the shell be in the air? What is the horizontal component of velocity? Where will the shell strike the ground?

20. The large German cannon (" Big Bertha ") in the first World War had a muzzle velocity of a mile a second. What was the range, and how high did the shell rise (neglecting air resistance)? Assume an elevation angle of 45°.

21.[1] A roof is 8 ft. long and has a 30° slope. The eaves are 12 ft. from the ground. A toy cart rolls down the roof. When and where will it strike the ground?

22. What speed must be given to a golf ball for it to travel 200 yd., assuming that it is projected at an angle of 30° above the horizontal?

23. In the lower trajectory of Fig. 3 u is assumed to be 80 ft. per sec. On the figure represent this velocity vectorially and the velocities after 1, 2, 3 seconds. Use a scale: 1 mm = 5 ft. per sec.

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[1] Involves quadratic equation.

lb = slug $\frac{ft}{sec.^2}$

momentum is
not lost

CHAPTER 10

FORCES AND MOTION

1. When the total force on a body is zero: *A body continues in its state of rest or of uniform motion in a straight line unless acted upon by a force.*

2. When the total force on a body is not zero: *The rate of change of momentum is proportional to the applied force and is in the direction of that force.*

3. Forces on interacting bodies: *The force of action on one body is equal in magnitude to the force of reaction on the other and is opposite in direction.*

102. Newton. These are Newton's laws of motion, and they are the fundamental laws of mechanics. Newton was born the same year that Galileo died (1642). He went to Cambridge University. While still an undergraduate he invented a method of treating rates of change (such as velocities and accelerations) mathematically. We know this today as Calculus. He was still quite young when he made many of his great discoveries — in astronomy, in optics, and in mechanics. He introduced the concept of mass into Galileo's laws of motion. His mechanics he presented in his famous book, " The Principia "; his three laws summarize his system and are the basis of mechanics today. Newton is generally recognized as one of the great men of all time.

103. The First Law. A body at rest remains at rest unless a force acts on it. Upon the chair, upon the table, gravity pulls downward, but, equally strongly, the floor pushes upward. The total force is zero and there is equilibrium.

It is perhaps less obvious that a body once in motion continues in motion unless acted upon by a force. Most things about us soon come to rest; this is due to a retarding force: friction. But imagine a meteor out in space, far away from the earth and so far removed from all other bodies that the force of gravity can be neglected; it will stay at rest if at rest or if moving it will continue to move in a straight line until some force acts to speed

F = mA.

it up, to slow it down, or to deflect it. However, the law means more than this. Any object about us which is moving at a constant rate is an example of this law. The horse pulling the wagon at constant speed pulls with a force exactly equal (but opposite) to the frictional force of the wheels; the total force on the wagon is zero. In an automobile the drive wheels push forward just as much as the front wheels and wind resistance push backward. The elevator rope pulls up with a force equal to the downward weight of the elevator. All this for constant velocity (or rest). A little more force (by the second law) is required to start the wagon or automobile or elevator, a little less to stop it. But upon any body moving with constant speed in constant direction the total force is zero. Near the ground raindrops fall at constant speed — how does the retarding force of wind resistance on a drop compare with its weight? I carry a satchel in an elevator — does the satchel feel heavier because the elevator is moving upward? If I wish to raise a weight with my hand is it necessary (not to start it but to keep it moving) to push a little harder than its weight? According to Newton's first law the answer is: No.

This is the law of equilibrium. In Chapter 7 we defined equilibrium too loosely. When forces balance, a body need not necessarily be at rest; if moving the motion is unchanged. Would not equilibrium occur in a moving train? (As a matter of fact, can it be said that all the " stationary " things about us — stones, trees, houses — are *really* at rest?)

104. Momentum. Momentum (or " quantity of motion," as Newton called it) is the product of mass and velocity. It is a quality possessed by a heavy train coming down the track 50 miles an hour. A fly crawling slowly on the wall has comparatively little momentum. Which has the more momentum, a 10-gram bullet going 300 meters a second or a kilogram weight thrown with a speed of 10 meters a second?

$$\mathfrak{M} = mv. \qquad \text{(Def.)}$$

Momentum, like velocity, is a vector. The total momentum of a body or of several bodies is made up of the sum of the momenta of the parts. For instance, the total momentum of two like balls moving with opposite velocities is zero. What is the momentum of a wheel turning on its axis? In a large body at rest, the molecules are all in motion. Each one has momentum. But as many must go in one direction as in the opposite, and the total momentum is zero. (See Fig. 6.)

According to the second law of motion a large force is required to change momentum quickly. A large force is needed to hurl the cannon ball suddenly through space; when it stops with extreme suddenness at a sheet of armor plate a very great backward force is exerted on it. It is harder to give speed to a 16-pound shot than to a baseball. The nail exerts a large force on the hammer head as it suddenly brings it to rest. The heavy and fast halfback is hard to stop. These are examples of Newton's second law. But we must discuss the law quantitatively.

105. The Second Law of Motion. Since the mass of a body does not change, a momentum change implies a change in velocity. If u is the initial velocity and v the final, the change in momentum is $mv - mu$; dividing by the elapsed time we have the rate of change of momentum. Thus Newton's second law may be written

$$\mathbf{F} = k\frac{mv - mu}{t},$$

where k is the constant of proportionality. This may be written as

$$\mathbf{F} = km\frac{v - u}{t}$$

or

$$\mathbf{F} = kma. \tag{a}$$

This last is the form in which the law is most frequently used. Second law of motion: *The force is jointly proportional to the mass and the acceleration.*

We are familiar with joint proportion. As a rather crude illustration consider an automobile. To compensate for friction we can suppose that it is on a slight incline, just such that it will retain its velocity unchanged. If we push on the car we shall speed it up or slow it down. The acceleration will be proportional to the force applied:

$\mathbf{F} \propto a.$ (This is for constant mass.)

Let us on the other hand have two automobiles, differing in mass; it requires a stronger push to give the heavier car the same pick-up as the lighter:

$\mathbf{F} \propto m.$ (This is for constant acceleration.)

Or given the same force, the accelerations of the two cars will vary inversely as their masses:

$$ma = c. \quad \text{(This is for constant force.)}$$

These relations can be shown in a laboratory experiment. The car shown in Fig. 1 can be accelerated by the falling weight. A timing device (not shown) measures the travel in successive small time intervals and hence allows the accelerations to be determined. When masses are removed from the car its acceleration becomes greater — (inversely proportional to mass). Similarly, keeping the mass unchanged the hanging weight can be varied, showing the proportional relation between **F** and a.

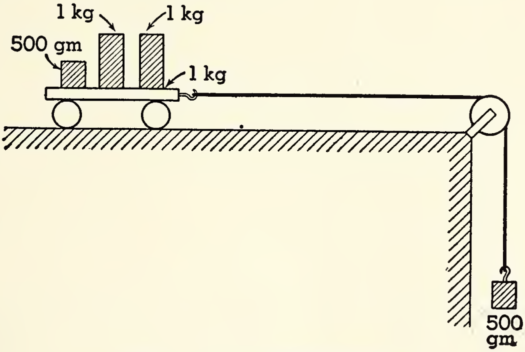

FIG. 1. $\mathbf{F} = 490{,}000$ dynes.
$m = 4000$ gm.

106. Weight. The force making a body fall is the earth-pull (weight). All falling bodies have the same acceleration. It follows (eq. a) that

$$\mathbf{F} \propto m.$$

The force of gravity acts on all bodies in proportion to their masses. The boulder and the small stone thrown from a cliff pick up speed at the same rate because the boulder, though it may be a thousand times harder to start than the stone, has then acting upon it exactly a thousand times as much force.

107. Acceleration by Proportion. Since we know the value of this acceleration given by this weight-force we can work all other acceleration problems by proportion.

Examples. What acceleration will a 100-lb. force give to a 1-ton automobile? (We know that a 1-ton force, i.e., its own weight, will accelerate it [falling] 32 ft. per sec.^{2})

What acceleration will the 500-gram weight give to the 4000-gram (total) mass in Fig. 1? (We know that a 4000-gram force will accelerate it 980 cm per sec.^{2})

108. A New Unit of Force. Any acceleration problem can be worked in this way. However, it is more usual to use eq. (*a*) directly. Before giving the value of the constant of proportionality in this equation we must decide upon what units we shall use in future. We could continue to use the ordinary weights (earth-pulls) as units of force. (This would make $k = \frac{1}{980}$, approximately, in the metric system.) But this earth-pull on a gram mass is not absolutely constant even on the surface of the earth. (It gives an acceleration of 978 at the equator and 983 centimeters per second per second at the poles.) Instead then of this unsatisfactory unit based on weight, the unit of force in the c.g.s. system is taken as *that force which will give a 1-gram mass an acceleration of 1 centimeter per second per second.* It is approximately $\frac{1}{980}$ of a gram weight (depending on the locality). It is called a **dyne.** Using this unit the proportionality constant is unity. Equation (*a*) becomes

$$\mathbf{F} = ma. \tag{4'}$$

This may be regarded as the fundamental equation in dynamics (mechanics of motion). It and the many other equations which will be derived from it are valid only if (in the c.g.s. system) the force is expressed in dynes — and of course mass in grams and acceleration in centimeters per second per second.

Example. Find the acceleration of the car in Fig. 1.

$$\mathbf{F} = 500 \times 980 = 490{,}000 \text{ dynes}$$

$$m = 4000 \text{ grams}$$

$$a = \frac{\mathbf{F}}{m} = \frac{490{,}000}{4000} = 122.5 \text{ cm per sec.}^2$$

109. Fundamental British Units.[1] In the c.g.s. system we have started with the unit of mass, and for the sake of simplicity in the fundamental formula of dynamics have introduced a new unit for force. Engineers are primarily interested in forces, and they have reversed this procedure. Engineers use the pound as a unit of force. If he is to still use eq. (4′) the engineer must then choose a proper unit of mass. One pound force will accelerate a pound mass 32 feet per second per second or will accelerate a 32-pound mass 1 foot per second per second. Hence 32 pounds is taken as the unit of mass — it is called a slug. **The slug** is the mass which is given an acceleration of 1 foot per second per second by a 1-pound force.

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[1] For other systems of fundamental units see Appendix 3.

Example. What force is required to give a 3200-lb. automobile an acceleration of 3 ft. per sec.^{2}?

$$m = 100 \text{ slugs}, \quad a = 3 \text{ ft. per sec.}^2$$
$$\mathbf{F} = ma = 100 \times 3 = 300 \text{ lb.}$$

110. The table of units on p. 96 should now be completed by the student. It should be noted that many of the units which we have used previously are not fundamental units. For instance, the c.g.s. unit for pressure is not the atmosphere or the gram-weight per square centimeter, but it is the dyne per square centimeter. This unit is called the *barye.*[1] An " atmosphere " is a little greater than a megabarye ($980 \times 1033 = 1{,}013{,}000$ baryes [§44]). In the engineering system the fundamental pressure unit is the pound per square foot (not per square inch). . . Density is mass per unit volume. *The unit of density in the British system is the slug per cubic foot* — the density of water is almost 2 slugs per cubic foot. In the following chapters these units must always be used in problems of forces and motion (unless the problems are worked by proportion). For the problems of hydrostatics, of static stresses and strains, of equilibrium no such special unit system was necessary.

111. Examples of Newton's Second Law. Just as any body which is at rest or in uniform motion in a straight line furnishes an illustration of Newton's first law, so any body whose velocity is changing furnishes an example of the second law. It must be remembered that the force here referred to means the *total force* acting on the body. To distinguish this total force from the partial forces we use the bold-face letter (**F**). I drag a sled. As long as the velocity is constant, my forward pull ($F_1 \rightarrow$) just equals the backward force of friction ($F_2 \leftarrow$); $\mathbf{F} = 0$. . . But while I am starting the sled into motion $F_1 \rightarrow$ must be increased. $\mathbf{F} = F_1 + F_2$ and $\mathbf{F} = ma$, where m is the mass of the sled (in slugs or grams).

When I drag the sled, my forward pull must exceed the backward force of friction by *ma*; both forces are equal as long as the velocity is constant. . . When a hammer is stopped the acceleration is very great and the force on it is very great; it equals *ma* (upward). . . When, at the report of the gun, the sprinter gets under way, his leg pushes his body forward with a force equal to *ma*.

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[1] Pronounced " barrie." The unit has in the past quite commonly been called a *bar.* Today this latter term is used to represent a million baryes (megabarye).

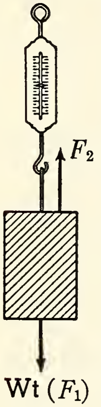

FIG. 2

I hold a 20-gram weight on a spring balance in an elevator. Suppose that the elevator starts upward with an acceleration of 98 centimeters per second per second. What does the balance read? In equilibrium the balance (Fig. 2) would read 20 grams (upward force) and the total force would be zero. But here

$$\mathbf{F} = ma = 20 \times 98 = 1960 \text{ dynes (or 2 gram weights).}$$

This *total force* is 2 grams; upward, because a is upward. Gravity pulls down with a force of 20 grams; the balance must pull upward with a force of 22 grams. For the net force (1960 dynes) on the weight equals *ma*.

Suppose that our elevator drops with an acceleration of $\frac{1}{2}$ g. Then the total force is $\mathbf{F} = ma = 20 \times -490 = -9800$ dynes (down). The weight force (20 grams) is $-19{,}600$ dynes (down); the spring balance must pull upward with a force of 9800 dynes to give a total force of -9800 dynes (down).

$$\mathbf{F} = F_1 + F_2 = -19{,}600 + F_2 = -9800 \text{ dynes,}$$

and F_2 (the pull of the balance) equals $+9800$ dynes (upward).

These problems can be solved readily by "common sense." If the elevator is at rest the upward force on the weight is 20 grams; if the elevator and things in it fall freely ($a = -980$ cm per sec.^{2}) the weight is not supported by the balance at all. If the elevator falls with half this acceleration the supporting force is half the weight. What would be the pull if the elevator rose with an acceleration equal to g?

112. Newton's Third Law. *Action is always equal and opposite to reaction.* In order to avoid confusion with the partial forces (F_1 and F_2) acting on a body, which are also *sometimes* equal and opposite (when?), remember that action and reaction always act on different bodies. I push the table, it pushes back on my hand; the earth pulls down on the weight, the weight pulls up on the earth. These are action and reaction: *always* equal.[1]

In our elevator problem the scale pulled up on the weight with a force of 22 grams; what was the reaction to this?

In effect, the third law says that a body cannot act on itself. A stone cannot set itself in motion; a man cannot pull himself up by his bootstraps. For every action there is an agent. If a force is exerted on some body toward the right, an equal force is exerted on some other body to the left; if on one body there is an upward force, on the other there is a downward force. The significance of this will presently appear.

113. Motion of Large Bodies. These laws of Newton apply directly to forces acting on small particles. They say nothing about rotation. However, their importance would not be great if they could not be applied to the large bodies about us. We can, indeed, use them as applying to the translation of any large body if we consider the position of the body as represented by its center of mass. (For all practical purposes the center of mass is the same as the center of gravity.) Without force, this center of mass of a body remains either at rest or in motion with constant velocity. When force acts, the center of mass of the body is accelerated in the direction of the force. A stick may be hit on one end or in the middle; wherever the point of impact, its center of mass moves in the same path for the same force. The motion is as though all the mass were concentrated here, a single particle acted upon by the given force . . . A football may spin or tumble, if kicked upon the side or end, but in any case its center moves exactly in the direction of the applied force! . . . When an unsymmetrical object (like a

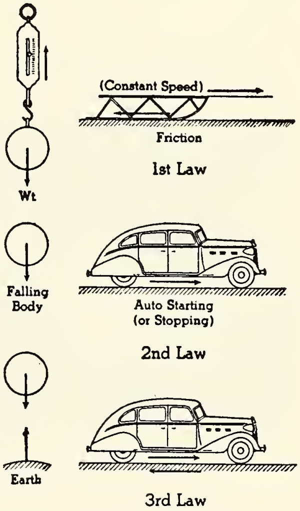

FIG. 3. Examples of Newton's laws of motion.

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[1] I push down on the table — the floor pushes up on the table. These are F_1 and F_2 which enter into the total force of the second law. . . The earth pulls down on the weight, perhaps I push up on it — these forces may or may not be equal, depending on whether we have equilibrium or not.

hammer) is thrown through the air perhaps it seems to wobble in uncertain manner in its flight; actually its center of mass moves, under the force of gravity, smoothly in a parabola.

114. Impulse. Let us now state the second law in a slightly different way. As implied in Newton's statement (§105)

$$\mathbf{F} = k\frac{mv - mu}{t},$$

or using fundamental units

$$\mathbf{F} = \frac{mv - mu}{t}. \qquad (4)$$

This is equivalent to

$$\mathbf{F}t \text{ (or ``impulse'')} = \text{Change in momentum} = \Delta\mathfrak{M} \qquad (4')$$

The product of force and time is called **impulse.** Newton's second law can be stated as: Change in momentum is (in fundamental units) equal to the impulse. This form of the law is particularly useful in the case of sudden blows, where the exact rate of change of momentum is difficult to estimate.

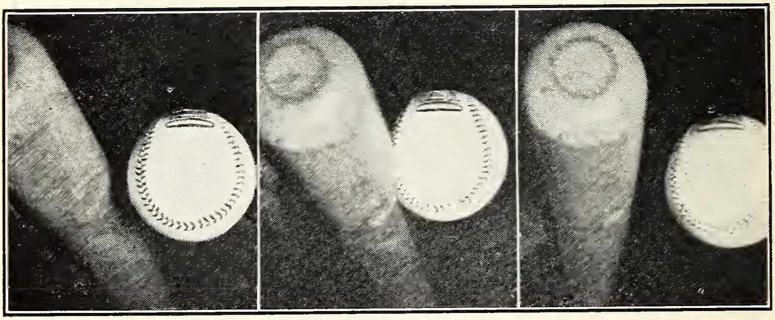

FIG. 4. Impulse produces change of momentum. Instantaneous (spark) photographs.

Example. What is the impulse when a 16-lb. sledge hammer, moving 6 ft. per sec., strikes a stake?

$m = \frac{1}{2}$ slug; $u = 6$ ft. per sec.; $mu = 3$; final momentum zero.
Impulse $= \Delta\mathfrak{M} = -3$ lb-sec.

Possibly the force is 300 lb. for 0.01 sec. or perhaps 30 lb. for 0.1 sec.

115. Combination of Second and Third Laws — Conservation of Momentum. If there is a force to the right on one body there is a force to the left on another (third law). The one force

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acts as long as the other. Hence impulses are equal. Therefore, whenever one body changes its momentum to the right, some other body changes its momentum to the left (second law). Therefore *the total momentum of the two interacting bodies cannot change.*

A simple experiment will illustrate this. Suppose that two small cars, connected by a spring, are held several feet apart. The momentum now, at rest, is zero. If each is released they come together, the lighter car coming the faster so as to make the two momenta just equal and opposite. The one car has a → momentum, the other an equal ← momentum. The sum of the two momenta is still zero.

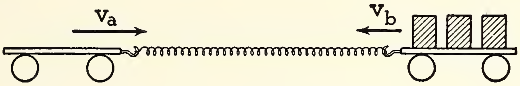

FIG. 5. Action and reaction.

This is the *Law of Conservation of Momentum.* If the whole interacting system is considered, the momentum of this complete system cannot change.[1] Consider the cannon and the cannon ball. Each gets the same impulse from the explosion. The light ball goes to the left rapidly; the heavy cannon goes to the right slowly — momenta are equal and opposite.

$$m_c v_c = -m_b v_b.$$

A man jumps from a rowboat — it is the same story. The man momentum and boat momentum are just opposite; their sum equals zero (because it was zero at the beginning). . . Consider a more complicated case: A shell traveling along its trajectory suddenly bursts in mid-air — one piece is projected forward, another to the rear, perhaps several to right and left — but the sum total of all their momenta still equals that of the original shell. Indeed (neglecting the momentum given to the air), the center of gravity of all these scattering pieces proceeds forward in exactly the original trajectory and hits the target! (But of course a center of gravity does no damage.)

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[1] A **complete** or **isolated** system includes all elements which have an essential bearing on a problem. All particles in the universe interact more or less with one another and hence the universe as a whole is the only strictly complete system. But physical analysis has been possible because problems can be simplified into interactions between a few, practically isolated, bodies. In the collision between two balls, the balls are considered as a complete system; in gravitational problems the earth and weight are a complete system.

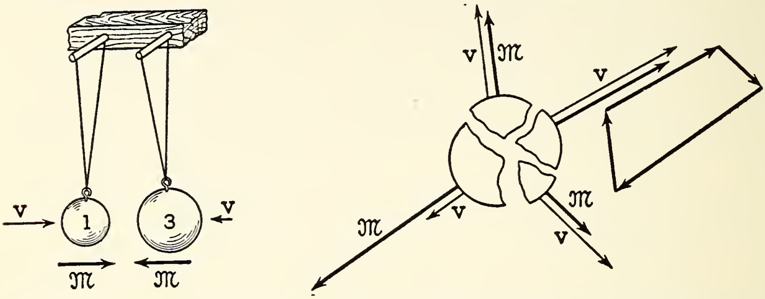

FIG. 6. Conservation of momentum. In bursting shell, momenta are shown relative to center of gravity.

There is grandeur in this law. There are no exceptions. But there are certainly many cases in which momentum *seems* to change. What if I, at rest, suddenly spring upward? *My* momentum certainly changes. But the student will realize that I am only a part of the complete system. I am pushed up from the earth, the earth is pushed downward — and the downward momentum given the earth is exactly equal to my upward momentum. It is just like jumping from the boat — but here it is a very large " boat," and the velocity given the earth is very small indeed! But we must not attempt to apply the law here without considering this momentum of the earth.

Consider again the firing of the cannon. Perhaps it is on a boat. The cannon is fired — shell moves to right, cannon more slowly to left, momenta equal. But the boat stops the cannon — and the boat is pushed very slowly to the left. But water stops the boat, and the earth stops the water, and now it is the earth which has a leftward momentum, and still this is equal to that of the ball and the total is still zero. Finally the ball strikes the earth — both stop — momentum still zero.

The total momentum stays constant. This is one of the fundamental principles of this world of ours. It may not always be the most convenient principle to use, when, as in the examples just considered, the reacting body is the earth itself. Such motions can be treated in other ways. But for small isolated systems — like the man and the boat, the cannon and the ball — the principle is very useful in determining the motion.

116. Examples in Conservation of Momentum.

1. A 10-gram bullet is fired from a kilogram gun, suspended to move freely. If the speed of the bullet is 500 meters per sec. what is the speed of recoil of the gun?

Total momentum must be zero after collision.

$$\text{Hence: } m_g v_g = -m_b v_b$$
$$1000 \times x = 10 \times 500$$
$$x = 5 \text{ meters per sec.}$$

2. This bullet now enters a block of wood, mass 990 grams. Find the velocity given to the block. Before collision: momentum = 10 × 500 gram-meters per sec. After collision: the total mass is 1000 grams. Hence $1000\,v = 5000$; $v = 5$ meters per sec. (Why is it not necessary to reduce meters to centimeters in these problems?)

117. Résumé. 1. As summary of Newton's laws let us consider the horse pulling the sledge. With its feet the horse pushes backward on the ground (f_1'); the ground pushes forward on the feet of the horse (f_1). The horse pulls forward on the sledge (f_2); the sledge pulls back on the horse (f_2'). The ground retards the sledge (f_3'); the sledge pushes forward on the ground (f_3). Which of these forces are to be paired as action and reaction?

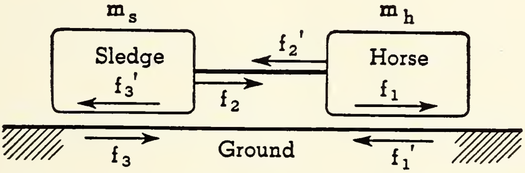

FIG. 10. Vectors are shown for constant velocity toward the right. When velocity is increasing f_2 (and f_2') are greater than f_3' (and f_3); f_1 (and f_1') are still greater.

If the horse and sledge move forward at constant rate, $f_1 = f_2'$ (opposite directions). Similarly for the sledge, $f_2 = f_3'$. This is Newton's first law.

Suppose, however, that the horse starts up; then $f_1 - f_2' = m_h a$; $f_2 - f_3' = m_s a$. This is Newton's second law. The total force on a body = ma.

Newton's third law says that under all conditions $f_1 = f_1'$; $f_2 = f_2'$; $f_3 = f_3'$ (opposite directions).

2. Momentum is mass times velocity. The change in momentum is equal to the impulse (Ft); its change per second (ma) is equal to the force. Since for every force acting on one body there is an opposite reacting force on another body, in a "complete" system the total momentum remains unchanged (Conservation of Momentum).

QUESTIONS

1. State the three laws of motion with examples of each.

2. When is the sum of the forces acting on a body equal to zero? When is action equal to reaction? Illustrate.

3. What is the reaction to (*a*) the force of friction which stops the automobile; (*b*) your weight; (*c*) the air-push of an electric fan; (*d*) the blow of the hammer on a nail.

4. How is it that a hammer can exert a force many times its weight?

5. What happens to the center of mass when a couple is applied to a body?

6. An old lady carried a basket of eggs into an elevator. It was all she could carry. What happened when the elevator started up, and why?

7. In an elevator a man stands on a platform balance (lever and weight type). Will his apparent weight change when the elevator starts to rise?

8. Name the fundamental units (in the c.g.s. and British system) of area, force, mass, pressure, density, velocity.

9. In which of the following formulas is it necessary to use fundamental units:

$v = u + at$; $ft = mv - mu$; f (action) $= -f$ (reaction); $f_1l_1 = f_2l_2$ (levers); $f_1 : f_2 = A_1 : A_2$ (hydraulic press); $fs = \frac{1}{2}mv^2$ (Chapter 12).

10. What happens to a kilogram ball when a kilogram force is applied to it? when a dyne is applied? when 1000 dynes are applied for 2 sec.?

11. I throw a ball upwards; it slows down, stops, returns, and strikes the earth. Is there conservation of momentum?

12. What force is required to give a kilogram an acceleration of 980 cm per sec.^{2}; to give a slug mass an acceleration of 5 ft. per sec.^{2}?

13. The frictionless plane problem: A man is on a frictionless plane. Is there any way he can get off?

14. A shell moving 1000 ft. per sec. bursts into two parts of equal weight. One goes forward with a speed of 1300 ft. per sec. What happens to the other?

15. A 1000-lb. cannon fires a 10-lb. ball with a speed of a 1000 ft. per sec. What is its speed of recoil?

16. Compare the momentum of a pound shot going 1000 ft. per sec. and a ton automobile going 1 ft. per sec. Suppose the two should meet; which way would the car move?

17. An elastic rubber ball and an equally heavy stone are thrown against a wall. Which will give the greater impulse?

18. You (with any equipment desired) are in a closed box. (*a*) Can you tell whether or not the box has a constant velocity? (*b*) Perhaps the box is going faster and faster (constant acceleration); could you detect this? Question (*a*) led Einstein to his Theory of Special Relativity; (*b*) led him to the General Theory of Relativity.

19. Is a house (*a*) at rest or (*b*) moving with constant velocity or (*c*) accelerated?

20. When two bodies collide there is conservation of momentum because: (*a*) a body remains in equilibrium if the resultant of all forces acting on it is zero; (*b*) when there is no rotation the sum of the moments of force equals zero; (*c*) action is equal and opposite to reaction.

21. Action is equal to reaction: (*a*) always; (*b*) when there is no friction; (*c*) when the system is in equilibrium.

Vocabulary: Newton's Laws of Motion, momentum, dyne, slug, barye, center of mass, impulse, Conservation of Momentum, complete (isolated) system.

PROBLEMS

1. What is the c.g.s. unit of force? What is your weight in c.g.s. units? What is your mass in the engineering system of units? What is the standard atmospheric pressure in dynes per square centimeter? How many dynes of force are required to break an iron rod 1 sq. cm in cross-section?

2. What acceleration is given to a 10-lb. mass by a force of 5 lb.? To a kilogram mass by a 100-gram force?

3. Neglecting friction, how fast will a 1-ton car move if we push it with a 100-lb. force for 10 sec.?

4. How much force must a bowler apply to give a 16-lb. ball a velocity of 30 ft. per sec. in 2 sec.?

5. The maximum pressure developed in a 12-in. naval gun is about 20 tons per sq. in. The shell weighs 400 lb.; what is its maximum acceleration? Assuming an average acceleration in the barrel (50 ft. long) of half this value, compute the muzzle velocity.

6. Find the force applied to (*a*) the airplane and (*b*) the arrow in problems **9**–7, 8.

7. The automobile in problem **9**–5 weighed 3000 lb. Find the retarding force (friction and air resistance).

8. I hold a 100-gram weight in my hand. Compute the acceleration if I push upward with (*a*) a force of 102 grams; (*b*) 90 grams.

9. A 1-ton (1000-kilogram) elevator moves upward with an acceleration of 1 meter per sec.^{2} What is the tension in the cable?

10. What is the tension in the cable of a 3200-lb. elevator (*a*) at rest? (*b*) when it rises with an acceleration of 10 ft. per sec.^{2}? (*c*) when it descends with an acceleration of 30 ft. per sec.^{2}?

11. Two weights of 90 and 110 grams, respectively, are suspended by a string passing over a pulley. What is the effective force tending to move the system? What is the total mass to be moved? What is the acceleration, and where will the weights be after 1 sec.?

12. Compute the tension in the string in problem 11.

13. Two 75-lb. boys are balanced on a seesaw. A 10-lb. weight is handed to one of the boys. Compute his downward acceleration.

14. What impulse is applied in driving a 1.6-oz. golf ball with a speed of 50 yd. per sec.?

15. A pitcher throws a 5-oz. ball 100 ft. per sec. The batter knocks it back with a speed of 120 ft. per sec. What impulse does he apply?

16. A rifle weighs 2 kilograms. It fires a 20-gram bullet with a speed of 1 kilometer per sec. What is the speed of recoil? What is the momentum of the bullet and of the gun? What impulse was applied by the powder?

17. Bullet velocities are determined by firing them into a block and measuring the momentum. A 10-gram bullet fired into a 990-gram wooden block gave the block a velocity of 2 meters per sec. What was the velocity of the bullet?

18. A 100-lb. boy stands in a 100-lb. rowboat. A 5-oz. baseball is thrown to him with a speed of 60 ft. per sec. He catches the ball and throws it back with the same speed. What speed of drift does this give to the boat?

19. A 400-lb. cart is rolling on a smooth roadway with a speed of 5 ft. per sec. How will it move if a 100-lb. boy running 20 ft. per sec. jumps onto the rear end?

Saturday

1 3 5 8 9 11 12 15 16

on 125

CHAPTER 11

EXAMPLES OF FORCES

118. Friction. Perhaps of all forces the most familiar are the force of gravity (weight) and the force of friction. We operate an automobile at constant speed on a level road or drag a sled or push a lawn mower or drive a nail — in each instance forces are exerted to " overcome friction."

You push upon a block resting on a board; yet if your force is less than 30 or 40 per cent of its weight the block does not move. Why? Owing to the interlocking roughnesses in the surfaces of block and board or to actual adhesion between the surfaces there is a force opposing the motion — the force of friction. When a force is applied tending to move the block in any direction along the board, this frictional force is ready to oppose it, up to a certain limit. You must exert a certain force to pull the block uniformly along the board. The frictional force is resisting. In steady motion your force and the frictional force together equal zero (Newton's first law of motion).

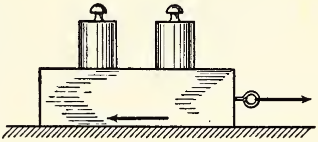

FIG. 1. Coefficient of friction is the ratio of the frictional force to the force (weight) pushing the surfaces together.

The force required to pull the block along the board is proportional to its weight

$$F_{\text{frict.}} = k \cdot \text{Weight}. \qquad (5')$$

The constant k is called the *coefficient of friction;* it depends on the nature of the surface. For wood sliding on wood its value is usually about 30 or 40 per cent; for metal on wood, about 25 per cent; and for polished metal on metal, 15 or 20 per cent. What force is required to push a 100-pound box on a smooth floor?

In eq. (5′) the force pressing the surfaces together is due to the weight. In general,

$$F_{\text{frict.}} = kF_{\perp}, \qquad (5)$$

where $F_{\perp}$ represents the force, whatever its cause, perpendicular to the two surfaces pressing them together. For example, for a body on an inclined plane (Fig. 2) this perpendicular force is one component of the weight.

The force of friction does not depend on the area in contact or upon the speed of motion.[1] However, when an object is at rest the frictional force which must be overcome is somewhat greater than the force required to move it while in motion. "Starting friction is greater than moving friction." When the two surfaces are at rest the molecules appear to get settled down into a better interlocking and adhesive forces are stronger. In ordinary language, there is a tendency for things to stick.

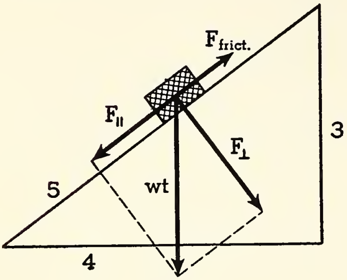

FIG. 2

TABLE 11

COEFFICIENTS OF FRICTION

Wood on wood	0.3 –0.5	Leather on metal	0.55
Metal on metal	0.15–0.2	Brick on brick	0.6–0.7

AVERAGE VALUES FOR BEARINGS

Dry surfaces	0.2	Perfectly lubricated surfaces	0.006
Oiled surfaces	0.03	Ball or roller bearings	0.003

Examples. 1. A force of 30 lb. acts on a 96-lb. weight. Coefficient of friction is 25 per cent. Find the acceleration.

Force of friction (eq. 5) is 24 lb. Hence net forward force is 6 lb. Mass = 3 slugs.

$$a = \frac{F}{m} = \frac{6}{3} = 2 \text{ ft. per sec.}^2$$

2. An inclined plane is 5 ft. long and 3 ft. high, with a 4-ft. base (Fig. 2). Find the acceleration of a block down the plane if its coefficient of friction is $\frac{1}{2}$.

By similar triangles the gravitational component along the plane ($F_{\parallel}$) is $\frac{3}{5}$ of its weight. The component perpendicular to the plane ($F_{\perp}$) is $\frac{4}{5}$ of its weight; hence the force of friction is $\frac{2}{5}$ of its weight. Thus the total force down the plane is $\frac{1}{5}$ of its weight, and hence the acceleration is $\frac{1}{5}\,g$.

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[1] What has been said applies to solids in sliding contact, without lubrication. The laws of fluid friction, involving viscosity (§154), are quite different.

119. Rolling Friction. The most effective way of reducing friction is to substitute rolling for sliding — by using wheels instead of runners on our vehicles, by using ball bearings instead of sliding bearings. If an ideal rigid wheel rolled on an ideal rigid track there would be no friction. But when a steel wheel rolls on a steel rail the wheel is slightly flattened and the rail indented; there is a retarding force because in effect the wheel is always climbing slightly uphill, always crushing down a ridge which lies before it. This effect is greater for small wheels (where the "hill" is steeper) than for large ones — small wheels (or small ball bearings) have more rolling friction than large ones. But, except in sand or mud or snow, rolling friction is usually very much less than sliding friction.

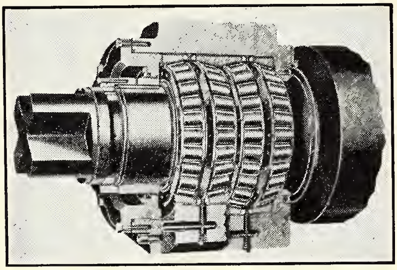

FIG. 3. Four-race heavy-duty roller bearing.

In the wagon wheel there still remains sliding friction at the bearing (reduced by lubrication). This is not very serious, because the axle has a small radius, and even a very considerable frictional force exerts only a small torque on the wheel.

CENTRIPETAL FORCE

120. Centripetal Acceleration. Among the illustrations of Newton's second law we have considered only forces which change the *magnitude* of the velocity. We must now consider changes in direction. Consider a boy on a bicycle. At one moment his velocity (say 20 feet per second) is represented by the vector CD (Fig. 4). Suppose that 2 seconds later his direction is changed by 30° (CE). The difference in these two velocities (Δv in figure) is about 10 feet per second, and the acceleration ($\Delta v/t$) is 5 feet per second per second. A force is required (inward) to produce this acceleration. All know how the rider must lean inward on rounding a curve — he thus makes the road thrust on the tire furnish the inward force (Fig. 5). We must now consider this motion more carefully.

121. Uniform Circular Motion. We must find the value of the acceleration of a body which moves around a circle with constant speed. (" Speed " is the magnitude of velocity, without reference to direction.) Suppose that the radius of the circle is r and the speed v. It is understood that the *velocity*, which is a vector, is changing. In t seconds the body has gone a distance vt and its velocity changes from $v \rightarrow$ to $v \searrow$. The difference between these velocities (Δv) must be found by the vectorial method, and this is done at the top of the figure.

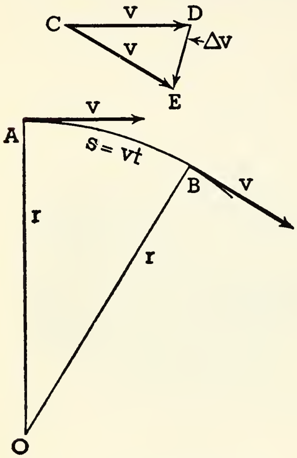

FIG. 4. Centripetal acceleration.

The path AB can be considered as approximately a straight line. The triangle ABO is similar to DEC (the angle between the velocity vectors equals the angle between the radii). Hence

$$\frac{\Delta v}{v} = \text{(approx.)}\ \frac{vt}{r}.$$

Here this is only approximately true because the path s is not a straight line. But acceleration is the instantaneous rate of change of velocity. That is

$$a = \left\{\begin{matrix}\text{The limiting value}\\ \text{as } t \text{ is made very}\\ \text{small of}\end{matrix}\right\}\frac{\Delta v}{t}.$$

And as t is made smaller, the angle decreases, the curved path AB approaches nearer and nearer a straight line, and in the limit when the angle (in both figures) is made indefinitely small the equation becomes exact. Transposing we have

$$a = \frac{\Delta v}{t} = \frac{v^2}{r}. \tag{6}$$

This is the value of the rate of change in velocity as a body goes about the circle at constant speed. The direction of Δv gives the direction of the acceleration — it is directed to the center of the circle. This acceleration toward the center is called *centripetal acceleration.*

122. Centripetal Force. From Newton's second law $\mathbf{F} = ma$. Therefore if a body is to move uniformly in a circular path it must be pulled toward the center with a force

$$\mathbf{F} = \frac{mv^2}{r}. \tag{6'}$$

The force varies as the square of the speed, inversely as the radius.

This is *centripetal force,* acting on the moving body to deflect it inward from its natural (force-free) motion in a straight line. The student will recall many examples of such force from his experience. If a mass is rotated on the end of a string the force required may be many times the weight of the mass. The string may not be able to furnish this force; it will then break and the mass fly off tangentially, in a straight line. Flywheels may break. Rotary clothes driers throw the water out from the clothes. (Or, more accurately, we should say that the rotating basket pulls the clothes in toward the center, away from the water, which simply continues moving at a tangent from the basket.) Always a force — centripetal force — is required to keep the body in the circle; if the force fails, the body moves in the straight line, by Newton's first law.

Corresponding to this inward force upon the moving body, there is the opposite, outward force of reaction acting upon the agency which produces the force. The mass swung around on a string pulls outward on my hand; the spokes of the flywheel pull outward on the hub; the car rounding a curve pushes outward on the road. This outward force of reaction (*not acting on the moving body*) is called *centrifugal force.* (For a different use of this term see §124.)

Examples. 1. Find the centripetal force acting on a 128-lb. boy riding a bicycle 10 miles an hour (15 ft. per sec.) about a circle of 20-ft. radius.

$$m = 4 \text{ slugs}; \; v = 15 \text{ ft. per sec.}; \; r = 20 \text{ ft.}$$

$$\mathbf{F} = m\frac{v^2}{r} = \frac{4 \times 225}{20} = 45 \text{ lb.}$$

The centripetal acceleration is 225/20 = 11.25 ft. per sec.^{2}, about one-third the value of g. (See Fig. 5.)

2. A 5-gram weight on a string 10 cm long is swung about in a circle, 2 revolutions per second. What is the tension in the string?

$$v = 2\pi r \times 2 = 125.6 \text{ cm per sec.}; \quad m = 5 \text{ grams}; \quad r = 10 \text{ cm}.$$

$$\mathbf{F} = \frac{mv^2}{r} = \frac{5 \times 125.6^2}{10} = 7900 \text{ dynes}.$$

This (about 8 grams weight) is the tension in the string neglecting the effect of gravity. The weight of the bob (5 grams) is added to this force at the bottom of the swing, subtracted from it at the top. At the top of the swing the string still pulls downward with a 3-gram force.

123. Illustrations of Centripetal Force. A weight on a string can be whirled around in a vertical circle; a pail of water can be swung around over the head without spilling; as a circus stunt a car can "loop the loop." Here we are apparently defying gravity; why do these bodies not fall? Actually they do fall; as the weight or the water or the car passes over the summit, unsupported, it is attracted downward by gravity, but the gravitational acceleration (980 cm per sec.^{2}) acts simply as the centripetal acceleration of the circular motion. The speed must be such that the central acceleration (v^2/r) is (at least) equal to g. Then at the summit the earth-pull (mg) furnishes the centripetal force. If the speed is greater than necessary, the centripetal force is greater than the weight and the pail actually pushes downward on the water or the track pushes downward upon the car.

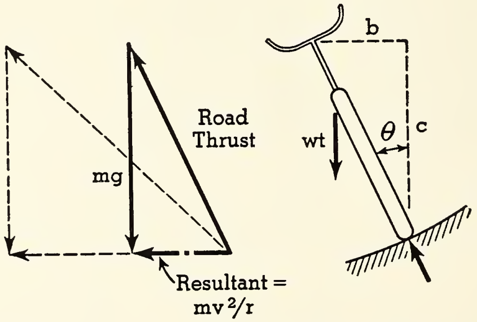

FIG. 5. Bicycle turning a curve. Vector diagrams are drawn for two different speeds, with centripetal acceleration (v^2/r) equal 15 and 32 ft. per sec.^{2}

Consider again the bicycle rider leaning inward as he turns a curve. Two forces act on the bicycle: the down pull of gravity and the thrust of the road directed obliquely upward (Fig. 5). The resultant of these forces is mv^2/r directed inward. The upward component of the road thrust is equal to the weight (mg); its inward component furnishes the centripetal force mv^2/r. Evidently the rider must lean at an angle such that $b : c = v^2/r : g$. That is, when $v^2/r = g$ the rider leans inward at 45° ($b = c$); for still higher speeds he leans inward still more ($b > c$).[1]

To avoid slippage the thrust should be nearly perpendicular to the roadway; hence the highway is banked at curves. Sometimes at county fairs motorcycle riders race around a small bowl-shaped track; v^2/r is very much greater than g; the sides of the bowl are quite vertical and the motorcycle is nearly horizontal.

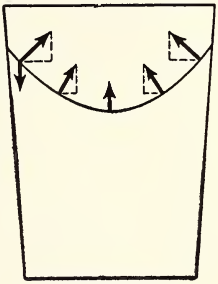

FIG. 6

A similar banking of the surface occurs in a rotating liquid. When we spin a glass, partly filled with water, the water is thrown outward, rising up the walls until its surface assumes the shape of a paraboloid (Fig. 6). A small element of water at the surface (or a floating cork) is subject, like the bicycle, to two forces: the down pull of gravity and a buoyant thrust by the supporting liquid perpendicular to the surface; the resultant is the inward centripetal force.[2] As the velocity (v) increases with the distance from the center, the angle of banking steadily increases, resulting in the parabolic shape. . . On a much larger scale the waters of the oceans are " banked up " near the equator, owing to the rotation of the earth. The " level " surface of the ocean is actually about 13½ miles farther from the center of the earth at the equator than at the poles. The Mississippi River is in this sense " higher " at the mouth than at the source — the water is thrown southward by " centrifugal force " (§124).

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[1] The ratio $b : c$, the opposite divided by the adjacent side, is called the *tangent* of the angle θ (to be compared with the sine and cosine). Here the tangent of the leaning angle ($\tan \theta$) is equal to the ratio $v^2/r : g$.

[2] The student must distinguish between the body as a whole (the glass of water) which has no translation and constant rotation and is in equilibrium (Chapter 8) and a small part of the body which has translation in a circular path and is therefore accelerated, not in equilibrium.

In the strictest sense a body at rest on the earth is not in equilibrium. The speed of the rotating earth (at the equator) is about 1000 miles per hour (or 1500 ft. per sec.). Hence the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{1500^2}{4000 \times 5280} = \frac{2{,}250{,}000}{21{,}000{,}000} = 0.1 \text{ ft. per sec.}^2 \text{ or } 3 \text{ cm per sec.}^2$$

(approx.)

The gravitational force upon a 1-gram mass at the equator is really 981 dynes, the supporting upthrust of the earth (the apparent weight) is 978 dynes, leaving a net downward (centripetal) force of 3 dynes, sufficient to produce this acceleration.

It is more convenient, however, to consider motion relative to the earth itself, to consider houses and trees as in equilibrium (apparent equilibrium), considering the force of gravity (981 dynes per gram at the equator) as reduced by an outward " centrifugal force " (3 dynes per gram) giving the apparent resultant force which we observe (978 dynes per gram).

124.* Apparent Equilibrium in a Rotating System. From the point of view of a person on a rotating turntable, objects fixed upon the table appear to be at rest. The centripetal force seems (from this rotating point of view) merely to compensate an outward-flinging " centrifugal force," which appears to act on all bodies in the rotating system.[1] This " centrifugal force " does not (in the strictest sense) exist; it is an " apparent force," introduced to account for the apparent equilibrium in the rotating frame of reference (§99). We may, for example, suppose the different points in the glass of water (Fig. 6) to be in equilibrium by imagining another, outward-flinging, " force " which just compensates the resultant inward force shown in the figure. We think then of the liquid as piled up around the outside not because of its tendency to move off along a tangent (because we are treating it as being at rest) but because of the action of this radial " force." The student should understand that this rotating system is not in equilibrium and that this outward-flinging force is fictitious. However, it is sometimes convenient to adopt this point of view and to use this fictitious outward-flinging " force "; for example, we shall now discuss the centrifuge, treating it as if it were in equilibrium.

125.* The Centrifuge. Under the influence of gravity finely divided particles suspended in a liquid gradually sink as a sediment to the bottom (as in the clearing of muddy water) or, if lighter than the liquid, rise to the surface (as in the separation of cream globules from milk). The centrifuge is a rapidly rotating cylinder in which such a suspension can be more quickly separated, by " centrifugal " instead of gravitational force. For high velocities of rotation each particle tends to " fall outward " because of a " centrifugal force " (mv^2/r) many times greater than the force of gravity (mg). Each particle is " buoyed " inward by a force equal to the centrifugal force (instead of weight) of the liquid which it displaces. As a result denser particles " sink " to the outside, particles less dense than the liquid " float " to the center; because of the great magnitude of the forces the separation is much more rapid than separation under gravity. In this way in the ordinary centrifuge cream is separated from milk, sugar crystals from syrup, blood corpuscles from the blood fluid. The small, very high-speed *ultra-centrifuge*, turning thousands of revolutions per second, gives a " centrifugal force " some hundreds of thousands of times greater than the force of gravity. With this enormous force even the molecules themselves may be " unmixed "; for example, the heavier oxygen may be separated from the lighter nitrogen in the air.

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[1] The term *centrifugal force* is used in two distinct senses: (1) as the real force of reaction (not acting on the rotating body) (§122), and (2) as this fictitious " force " on the rotating body introduced when we wish to treat it as if in equilibrium (rotating frame of reference). For clearness we use quotation marks when using the term in this latter sense. Because of this ambiguity in meaning the beginning student is advised not to use the term; the only force on a rotating body is centripetal. It is introduced here at the risk of confusion because many students will certainly meet this concept of apparent equilibrium and " centrifugal force," probably in connection with the centrifuge. In advanced mechanics our " fictitious force " is classed as a *generalized force.*

FORCES DUE TO IMPACT*

126. The Water Jet. An object thrown against a wall produces an impulse (Ft) equal to the change in momentum ($\Delta\mathfrak{M}$). Two cases should be distinguished: (1) If there is no rebound (inelastic collision) the momentum simply changes from $\mathfrak{M}$ to 0 and the impulse equals $\mathfrak{M}$. (2) For a perfectly elastic collision (an ideal rubber ball) the momentum is reversed, changes from $\mathfrak{M}$ to $-\mathfrak{M}$, and the impulse is twice as great. (There is an impulse as the ball is brought to rest and an additional impulse as it springs back.) But in any event the impulse equals the change in momentum. Imagine a surface subjected to an incessant bombardment of particles; imagine, for example, rain pounding against a roof or sand poured from a height upon a board or bullets from a machine gun striking a target. The repeated impulses combine to produce a nearly steady force, and (by Newton's second law) this force is

$$F = \frac{\Delta\mathfrak{M}}{t},$$

where $\Delta\mathfrak{M}/t$ is the momentum change of the particles striking the surface per second.

Consider now the force due to a water jet splashing against a wall. Let A be the cross-sectional area of the jet and v its velocity; a volume of liquid Av reaches the surface per second; the corresponding mass is dAv (where d is the density). The momentum is thus $dAv \cdot v$; this is the momentum lost at the wall per second. Thus

$$F = dAv^2 \quad \text{(no directed velocity after impact).}$$

This result can be easily checked in the laboratory by measuring the force when a jet splashes against a board.

This assumes no velocity after impact. In the water turbine the blades are hollowed out; the jet instead of splashing sideways is redirected, effectively rebounding from the surface; corresponding to the greater change in momentum a greater force is obtained, $2dAv^2$ from a blade at rest, assuming no losses.

127. Force of the Wind; Turbulent Resistance. This same general argument applies to wind striking a sail or to the force of a river current against a stationary object. The moving air or water is deflected and loses only a part of its momentum — only a small part if the obstacle is " streamlined " in shape. Nevertheless we may expect the force to be proportional to the density of the medium, to its velocity squared, and to the area of the obstacle:

$$F \propto dAv^2.$$

This assumes that the lines of flow have the same general pattern about the obstacle, with the same proportional change in momentum, for different speeds and different cross-sections. As a matter of fact, this relation usually holds fairly well. The value of the constant of proportionality and the actual magnitude of the force depends upon the shape of the obstacle, becoming quite small for a well-streamlined shape.

The problem is exactly the same when a body is in motion in still air. The air resistance upon a ball or a parachute or an automobile, or the water resistance upon a boat, varies as the square of the speed. This resisting force is equal to the momentum given to the air or water per second. Motion given to the medium breaks up into eddies and soon through viscosity (§154) degenerates into heat. The general phenomenon is called turbulence; for ordinary moving bodies the major part of the air friction is usually due to turbulence (§156).

QUESTIONS

1. Is the force of friction of a weight on a board reduced when the board is tilted?

2. On a roller coaster does the seat always support exactly the weight of the passenger?

3. A weight hanging on a string is made to revolve in a horizontal circle (the string describing a cone). Does the vertical component of the string tension still equal the weight of the body?

4. If you whirl a weight on a string around once a second, the centripetal force is doubled when the string length is doubled. Why?

5. Compare the air thrust upon an automobile traveling 30 and 60 miles per hour.

6. Supposing that the crew of a boat, hoping to decrease its displacement, proceeded to relieve it of their weight by continually jumping from the deck. Explain why they would not succeed.

7. When an automobile turns a corner the net force acting on it is: (*a*) outward; (*b*) inward; (*c*) zero.

8. If a rubber ball and a lead ball are thrown (with equal momenta) against a stone wall: (*a*) the impulse will be greater for the rubber ball; (*b*) the impulse will be greater for the lead ball; (*c*) the impulses will be equal.

Vocabulary: Coefficient of friction, starting (moving) friction, centripetal acceleration (force), "centrifugal force," centrifuge, turbulence.

PROBLEMS

1. If the coefficient of friction is 30 per cent, what force is required to move a kilogram weight along a table with constant velocity? What force to give it an acceleration of 98 cm per sec.^{2}?

2. A weight started sliding over ice with a speed of 16 ft. per sec. comes to rest in 4 sec. What is the coefficient of friction?

3. A weight is resting on a piece of paper. Assuming a coefficient of friction of 30 per cent what is the greatest acceleration which can be given to the weight by pulling the paper?

4. Assuming a coefficient of friction of 2 per cent between rubber and ice, how long will it take for an automobile on ice to attain a speed of 10 miles per hour?

5. A kilogram weight rests on the table top. It is tied to a similar weight dangling from a pulley over the table edge. Assume a coefficient of friction of 25 per cent. Find the acceleration of the weights.

6. Starting at 20 miles per hour my automobile will coast for 200 yd. on a level roadway. Compute the effective coefficient of friction.

7. Assuming an effective coefficient of rolling friction of 2 per cent, what speed will a 3000-lb. automobile acquire if I push it on a level roadway with a force of 80 lb. for 20 sec.?

8. A typical modern passenger locomotive weighs 135 tons (110 tons on drive wheels), can exert a tractive effort (pull) of 20 tons, and can develop 2250 horsepower. How great must the coefficient of sliding friction between wheel and rail be to avoid slippage when the engine is exerting its greatest tractive effort?

9. At 20 miles per hour the effective coefficient of friction of a freight car is about 0.3 per cent. What tractive effort is required to haul thirty 96-ton cars (*a*) on the level, (*b*) up a 1 per cent grade? (*c*) The largest freight locomotive can exert a tractive effort of 70 tons; what acceleration can it give this train on the level?

10. A toboggan slide has a 30° slope. A sled weighs 100 lb. What is the effective component of the weight along the incline tending to accelerate the sled? What is the component perpendicular to the slide? If the coefficient of friction is 3 per cent, what is the force of friction? What is the acceleration of the sled?

11. What is the acceleration of an automobile moving 10 ft. per sec. around a circular track of 100-ft. radius?

12. What is the acceleration of a 100-gram ball swung about once a second on the end of a meter string? What is the tension in the string?

13. A kilogram ball swings about on a steel wire 20 cm long and 1 sq. mm in cross-section. How many revolutions per second can it make without breaking the wire?

14. An automobile goes over the crest of a small hill which can be considered circular with a radius of curvature of 30 ft. How fast must the automobile travel to leave the ground?

15. What speed must a car have to loop the loop, if the loop is 8 meters in diameter?

16. A ball, suspended from the ceiling by a 5-ft. string, is thrown so as to describe a horizontal circle 6 ft. in diameter. What is its speed?

17. Show that a bicycle rider, riding around a circular track, 100 meters in circumference, in 8 sec., must lean at an angle of approximately 45°.

18. A " cyclodrome " is 36 ft. in diameter; motorcycles race around it at 40 miles per hour. Compare the centripetal acceleration with gravity, and estimate the angle of lean.

19. A centrifuge 6 cm in diameter makes 3000 revolutions per second. Compare the centrifugal force with the force of gravity.

20. A block of wood is on a merry-go-round, 10 ft. from the center. How fast must the platform rotate to throw the block outward? Once started will the block continue to slip toward the edge of the platform? (Coefficient of friction, 0.3.)

21. According to the text the centripetal acceleration at the equator is about 3 cm per sec. Using proportion, find how many times faster the earth would have to rotate to throw bodies off into space.

22. The nozzle of a fire hose has a 1 sq. in. cross-section; the velocity of the jet is 50 ft. per sec. Find the force of impact against a wall.

23. An electric fan 20 cm in diameter gives the air a velocity of 3 meters per sec.; find the force of reaction against the fan.

Tuesday

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CHAPTER 12

WORK AND ENERGY

128. Work. The last two chapters have dealt with force and impulse (*Ft*) and momentum; this chapter deals with force and work (*Fs*) and energy. When, through the application of a force, something is moved, work is done. Work is the product of force and distance.

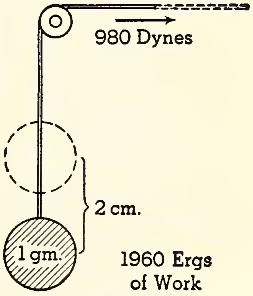

FIG. 1. Lifting work (ergs) equals weight (dynes) × height (cm).

I work when I throw a ball or raise a weight or drag a sled. If, applying a horizontal force of 20 pounds, I drag a sled 5 feet, I do 100 foot-pounds of work. If I lift a gram weight a centimeter (force 980 dynes) I do 980 dyne-centimeters of work; if I lift it 10 centimeters the work is 9800 dyne-centimeters, and so on. Work is force times distance.

Does this agree with our everyday meaning of physical labor? In a general way, yes. Consider spading the garden. In merely pushing on the spade we exert a force but do no work — we work when we drive the spade and when we lift up the earth. Work is force times distance. True we may say that it is " hard work " merely to hold a heavy weight, but perhaps this is not exactly what we mean (§295). Certainly nothing is accomplished; the weight could be supported as well by a post.

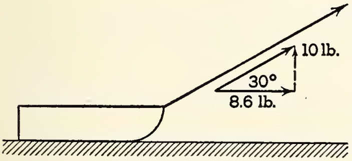

FIG. 2. Component in direction of motion is effective.

If the force vector is not parallel to the displacement, it is only the component of force in the direction of motion which is effective. The exact definition is: **Work** is the product of the displacement and the component of force in the direction of the displacement.

$$W = Fs \quad (F \text{ in the direction of } s). \qquad \text{(Def.)} \qquad (7)$$

Example. How much work is required to drag the sled in Fig. 2 100 ft.? The effective component of force is 8.6 lb., and the work done is 860 ft-lb.

129. Work Units. When 1 pound force is exerted through 1 foot distance, 1 **foot-pound** of work is done. This is the engineering unit. Similarly the dyne-centimeter is the fundamental unit in the c.g.s. system — the work done by a dyne acting through a centimeter. This unit has been given a name, **erg.**

The erg is a very small unit. A **joule** is 10 million (10^7) ergs. This is the *practical unit* of work in the metric system. Since approximately 980 dynes make a gram weight and 454 grams equal 1 pound and 30.5 centimeters equal 1 foot, there are approximately 13,500,000 ergs in a foot-pound, or a joule is about $\frac{3}{4}$ foot-pound.

Examples. 1. How much work is done (by gravity) upon a 1-lb. stone during the first second of fall?

$$F = 1 \text{ lb.}; \; s = 16 \text{ ft.}; \; W = Fs = 16 \text{ ft-lb.}$$

2. How much work is required to lift a kilogram a meter? $F = 980{,}000$ dynes; $s = 100$ cm; $W = Fs = 98{,}000{,}000$ ergs $= 9.8$ joules.

130. Energy. Work is always done by some agent. To be able to do work a body must have *energy*. There is chemical energy in food and in gasoline, and hence a man can lift a weight and an automobile can climb a hill. There is energy in the heated steam which moves the piston (heat), in the spring which closes the door (potential energy), in the moving hammer which drives the nail (kinetic energy). **Energy** is the ability to do work. And as work is done the energy of the working agent decreases.

Work is always done *upon* some body, and, as a result, the energy of that body increases. (1) The pitcher throws the ball; his energy decreases, and the ball receives a corresponding ability to work — to exert a force through distance; it drives back the catcher's glove. The energy which a body has in virtue of its motion is called **kinetic energy.** (2) We lift a weight and it has increased energy; it can force its way back to the ground and do work. Such energy due to position is called **potential energy.** (3) We drag a weight along the floor, and perhaps it is not obvious that we have made it more energetic but actually it is a little hotter and *heat* also has the ability to do work. . . What energy the working agent loses, the body worked upon gains. The total energy remains unchanged. This is the principle of *conservation of energy.*

The energy of a body is measured by the work it can do; hence energy is measured in the units of work: foot-pounds or ergs or joules.

131. Kinetic Energy. The velocity of a body changes when an unbalanced force acts on it. From eq. (3*c*) (Chapter 9) we can readily show how the velocity depends on the work done. This equation may be written as

$$as = \tfrac{1}{2}v^2 - \tfrac{1}{2}u^2.$$

Multiply both sides by m and we have (remembering that $ma = \mathbf{F}$)

$$\mathbf{F}s = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

or

$$W = \Delta \text{ Kinetic Energy} \qquad (8)$$

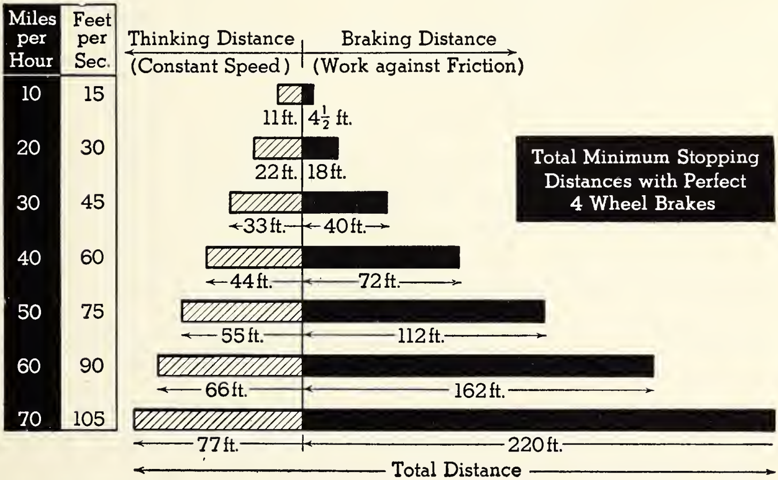

FIG. 3. Stopping distances of automobile. (Dry cement road.)

This quantity $\frac{1}{2}mv^2$, which is increased by the work done by the unbalanced forces, is the **measure of kinetic energy.** Its increase equals the work done upon a body in speeding it up;

the decrease in kinetic energy is equal to the work done by the moving body as it comes to rest.

Example. Find the kinetic energy of a 1-lb. ball after a free fall of 1 sec. Mass = $\frac{1}{32}$ slug; $v = 32$ ft. per sec.; k.e. = $\frac{1}{2}\,mv^2 = \frac{1}{2} \cdot \frac{1}{32} \cdot 32^2 = 16$ ft-lb. of kinetic energy. (Compare with example 1 of §129.)

When a body starts from rest, that is without initial kinetic energy,

$$W = \tfrac{1}{2}\,mv^2. \tag{8'}$$

Notice that the kinetic energy depends on the square of the velocity. An automobile going 30 miles an hour has 9 times as much capacity for work as one traveling 10 miles an hour. It can go up a hill 9 times as high or will drag along 9 times as far after the brakes are set. The student can think of many instances of kinetic energy — in the wind which turns the windmill, in the water jet which turns the water wheel, in the cannon ball which rips up the embankment.

132. Potential Energy. When we raise a weight or stretch a spring we increase potential energy. The increase in energy is measured by the work we do. Hence if $\bar{F}$ is the average force which we exert in this way:

$$\text{Potential Energy} = \bar{F}s.$$

The most important case is the potential energy due to the gravitational field. Here the force is the lifting force; it is equal to the weight mg. We must take something as a starting point to measure heights. Suppose we consider the energy on the ground as zero. Then the energy at any height h is

$$\text{Potential Energy}_{\text{grav.}}\left\{\begin{matrix}\text{ergs}\\ \text{ft-lb.}\end{matrix}\right\} = \text{Weight}\left\{\begin{matrix}\text{dynes}\\ \text{lb.}\end{matrix}\right\} \times h\left\{\begin{matrix}\text{cm}\\ \text{ft.}\end{matrix}\right\}$$

or

$$\text{Potential Energy} = mgh.^{1} \tag{9}$$

We shall find the potential energy of a stretched spring in Chapter 15.

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[1] The first form of the equation (line above) is more convenient to use with British units. (To find mg in eq. 9 we should first have to change the weight in pounds to the mass (m) in slugs, and then, multiplying by g, change back to pounds weight again.)

Examples. How much capacity for work has a cubic foot of water at a height of 10 ft.? Show that a liter of water has 98 million ergs of potential energy at a height of 1 meter. (How many joules?)

133. Increasing and Decreasing Energy. If the motion is in the same direction as the force acting *upon* a body, work is done *on* the body; its energy increases. If the motion is in the same direction as the force exerted *by* the body, work is done *by* the body; its energy decreases. The rope exerts an upward force on each weight in Fig. 4*a*. If it happens that *A* is moving upward work is done on *A* (and the potential energy of *A* increases). But if *A* moves down, then work is done *by A* on the rope and hence *on B* (and the energy of *A* decreases).

Consider a locomotive pulling a train. The engine pulls forward on the cars; the cars pull backward on the engine. But the motion is forward, and the engine does work upon the cars. To prevent its energy from decreasing, coal must be fed to the boiler. What becomes of the energy given to the cars when the train runs at constant speed?

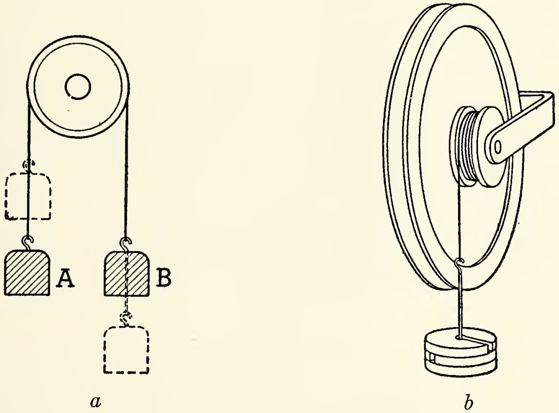

Fig. 4. Energy transformations:

a. Transfer of potential energy from *B* to *A* (*B* works on *A*).
b. Potential energy transformed to kinetic (weight works on wheel).

134. Conservation of Energy. As the weight in Fig. 4*b* falls it loses potential energy; the wheel gains kinetic energy. After unwinding, the weight is wound up again; kinetic energy is transformed back again to potential energy. The total energy remains constant. Similarly in the oscillating pendulum, in the roller coaster, in the bouncing ball, the energy is repeatedly transferred from the one form to the other. True, in each of these instances the body does not come back exactly to its original height, and after a number of oscillations the body comes to rest. This is because of friction, which gradually changes the kinetic energy into heat, another form of energy. We shall meet with energy in still other forms — electrical energy, magnetic energy, light energy, sound energy, chemical energy — for all physics is a study of energy, and in each branch of our study we shall find new methods of doing work. But since always, if one object does work, another has work done upon it, when one object loses energy, another gains, and in any complete system the total energy of all kinds remains constant. This is the law of *Conservation of Energy*. This law takes its place beside the laws of Conservation of Mass and Conservation of Momentum as one of the great basic laws of nature.

Many folk have "invented" perpetual-motion machines. These are machines designed to run forever, despite friction. Of course if there were no friction the wheel and weight in Fig. 4*b* or the roller coaster or any machine which is not doing work would run forever because energy must be conserved. But there is always a generation of frictional heat energy and other forms of energy must gradually decrease, so that unless energy is continually supplied any machine must soon come to rest. Perpetual-motion machines are delusions.

135. Power. How quickly can work be done? **Power** is the rate of doing work, the work done per second:

$$P = \frac{W}{t}. \qquad \text{(Def.)}$$

Examples. 1. If an elevator raises a ton load 100 ft. in 10 sec., the work done is 2000 × 100 = 200,000 ft-lb. and the power is 20,000 ft-lb. per sec.

2. If a 3200-lb. automobile (100 slugs) increases its speed from 20 to 30 ft. per sec. in 5 sec. the change in kinetic energy is

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2 = \tfrac{1}{2} \times 100 \times 900 - \tfrac{1}{2} \times 100 \times 400 = 25{,}000 \text{ ft-lb.}$$

This is the work done in 5 sec.; hence the power is 5000 ft-lb. per sec.

3. If a 100-gram mass (98,000-dyne weight) is lifted a meter per second the work done per second is

$$\frac{Fs}{t} = Fv = 98{,}000 \times 100 = 9{,}800{,}000 \text{ ergs per sec.}$$

Work is force times distance; as illustrated by this example, power is force times the rate of covering distance or

$$P = Fv. \ (F \text{ in direction of } v.)$$

The fundamental units of power evidently are *ergs per second* and *foot-pounds per second.* Other units are in practical use. Five hundred and fifty foot-pounds per second equals 1 **horsepower** (hp). A joule per second is called a **watt.** It is left to the student to reduce the answers in the above examples to these units. A kilowatt — 1000 joules per second — is frequently used. A horsepower is approximately $\frac{3}{4}$ kilowatt, or more exactly

$$1 \text{ hp} = 746 \text{ watts.}$$

The *power rating* of a machine is the power which it delivers when operating at full capacity. Typical values are:

Automobile 75–150 hp.
Airplane motor 50–1500 hp.
Locomotive (average) 2000 hp.

The steam turbine which spins the electric generator in the public-utility plant may deliver as much as 100,000 horsepower — or in one or two giants as much as 200,000 horsepower.

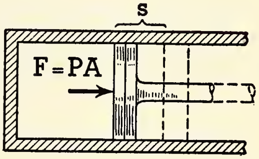

Fig. 5. Work equals Pressure × Volume increase.

136. Work of Fluids. When the steam in the cylinder of a steam engine or the water in a hydraulic press pushes upon the piston and forces it back, work is done. It is easy to show that the work here equals the pressure times the increase in volume. For the force against the piston is PA; when the piston is moved a distance s, the work is

$$W = Fs = PAs = P \cdot \Delta V \qquad (7')$$

since the cross-section area times the distance equals the increase in volume. It is understood that, to get the work in foot-pounds, the pressure must be expressed in pounds per square foot; to get the work in ergs the pressure must be in dynes per square centimeter (baryes).

137. Work in Turning. When, as in most machines, the work is furnished by a rotating wheel, it is more convenient to measure the work in terms of torques and angles than in terms of forces and distances. Of course, a torque does no work unless something is turned by it; the work is jointly proportional to the torque acting and to the angle turned through:

$$W \propto \mathcal{T} \cdot \theta.$$

The exact relation will depend upon the units in which the angle is measured (in minutes or degrees or radians or complete revolutions). It will be shown in Chapter 16 that when the angle is measured in radians the relation is

$$W = \mathcal{T} \cdot \theta. \tag{7''}$$

138. Work and Impulse; Energy and Momentum. Impulse is force times time; work is force times distance. Impulse changes momentum, and $\mathbf{F}t = \Delta\mathfrak{M}$; work changes energy, and $Fs = \Delta E$.

The difference between momentum and energy is well illustrated by consideration of the cannon and the cannon ball. A cannon is fired. The exploding gases push with equal forces, forward on the ball, backward on the cannon, and act upon both for the same brief time. Hence the impulses are equal and each gets the same momentum in opposite directions:

$$m_b v_b = -m_c v_c.$$

But the kinetic energies are by no means equal. For, if we write these energies as $\frac{1}{2}(m_b v_b)v_b$ and $\frac{1}{2}(m_c v_c)v_c$, it is evident (since the qualities in parentheses are equal in magnitude) that the energies remain proportional to the velocities. It would certainly be unfortunate (for the cannoneer) if the cannon had as much energy to rip and tear as the ball. Analyzing the action we see that, since the ball moved much farther than the cannon under the influence of the force, the work on the ball was much the greater. Given the same mass, the momentum — ability to exert impulse — is proportional to the velocity; kinetic energy — ability to do work — is proportional to the square of the velocity.

Kinetic energy is not, like momentum, a vector. Similarly, potential energy and work are scalar quantities. This makes the summation of energies simpler than the summation of momenta. No vector diagrams are necessary. The total energy of a system is obtained by simple addition of energies. A ball or molecule or automobile going north and another one going (equally fast) south can, in both stopping, do twice the work that one of them alone can do. But their total momentum is zero.

139. Résumé. Work is done when a force moves a body, and it is measured by the effective component of force times the distance. Its fundamental units are foot-pounds and dyne-centimeters (ergs). The joule is 10^7 ergs.

Power is the work done per second; the units are foot-pounds per second and ergs per second. Other units in common use are: horsepower = 550 foot-pounds per second, and watts = joules per second.

Energy is the ability to do work. Mechanical energy is either kinetic ($\frac{1}{2}mv^2$) or potential (for gravity mgh). There are also heat energy, electrical energy, etc. A body's energy decreases when it does work, increases when work is done on it, but the totality of energy is never changed. This is the Law of Conservation of Energy.

Work can also be expressed as proportional to torque times angle, the proportionality constant depending on the units of angle.

The work done by a fluid is the product of its pressure and the change in volume.

QUESTIONS

1. How much work is done in lifting a gram a centimeter; in lifting a pound an inch?

2. How much kinetic energy has a gram mass moving 1 cm per sec.; a pound ball moving 1 ft. per sec.?

3. What are the fundamental units of work, energy, power?

4. What are the practical units of energy and power?

5. A ball strikes a half circular trough and rolls around and out. Is any work done? Are the momentum and energy changed?

6. A lead ball strikes a wall and falls down; a tennis ball strikes a wall and bounces back. Is there a transformation of energy in either case? Assuming equal masses and velocities, which has the greater change of momentum?

7. Compare the kinetic energy of a 1-lb. ball moving 2 ft. per sec. and of a 2-lb. ball moving 1 ft. per sec.

8. An engine puts on the brakes to come to rest. What are the directions of the forces on engine and on train at the coupling; does engine or train do the work? (The brakes are on the engine.)

9. Which has more energy, a 1-gram ball 1 cm high or a 1-gram ball moving 2 cm per sec.?

10. What becomes of my energy when I throw a ball, raise a book, drag a chair? Where did my energy come from?

11. Do you do physical work in going upstairs; downstairs?

12. How much power is used when a 200-lb. man runs up a 180-ft. stairway in a minute?

13. Trace the changes in energy in a roller coaster.

14. Which does work: the hammer or the nail; the engine or the train; the powder or the bullet; the catcher or the baseball; the baseball or the bat; the door or the door spring in opening, in closing? Why?

15. How many times more kinetic energy has a car at 30 miles per hour than at 20 miles per hour?

16. Does a centripetal force do work?

17. If the mass of a hammer is halved and its speed doubled its momentum is (*a*) unchanged; (*b*) doubled; and its energy is (*a*) unchanged; (*b*) doubled.

18. A " watt-second " of energy is: (*a*) 1 erg, (*b*) 10^7 ergs, (*c*) 10 joules.

Vocabulary: Erg, foot-pound, joule, kinetic (potential) energy, Conservation of Energy, power, horsepower, watt, kilowatt.

PROBLEMS

1. How much work do you do in climbing 30 ft.? How many joules of work are required to lift a kilogram weight a meter?

2. Compute the work in joules required to lift a pound weight (454 grams) a foot (30.5 cm).

3. Assuming that the handle of a lawn mower makes an angle of 30° with the ground and that you exert a 25-lb. thrust upon it, how much work do you do in pushing it 10 yd.?

4. If it takes a 20-kilogram force to push a saw and it takes eighty 20-cm strokes to cut a block, how much work is required?

5. If the coefficient of friction is 0.3, how much work is required to pull a kilogram weight 1 meter along a table?

6. If the coefficient of friction is 0.3, how much work is required to pull a kilogram weight up a 30° inclined plane 1 meter long?

7. What power is required to cut the block of problem 4 in a minute? How long would it take to cut it on a 2-kilowatt mechanical saw?

8. How long should it take a horse to hoist a ton of hay into a 30-ft. barn? (Assume 1 hp as the power rating of a horse; actually horses today average better than this.)

9. How fast can a 100-watt motor hoist a 1-ton (1000-kilogram) elevator?

10. At 50 miles per hour it is estimated that wind resistance on an automobile is 100 lb., rolling friction of tires 25 lb. Assuming no transmission losses, what horsepower is required to drive a car at this speed?

11. Approximately what is the potential energy of a gallon (approx. 4 liters) of water above Niagara Falls (height 55 meters)?

12. A man hauls up a 50-kilogram anchor from a depth 5 meters below the water surface onto a deck 2 meters above the surface. By how much does he increase its potential energy?

13. A 10-lb. wooden plank is floating, 40 per cent submerged. Compare its potential energy when we (*a*) submerge it 10 ft. under water and (*b*) raise it 10 ft.

14. How much work can be done by a 10-gram bullet traveling 1 kilometer per sec.? by a 1-oz. bullet traveling 1000 ft. per sec.?

15. Compare the energy of a ball (*a*) when it has a speed of 490 cm per sec. and (*b*) when it is at rest at an elevation of 245 cm.

16. Find the percentage increase in kinetic energy of a body when its speed is increased from 100 to 101 cm per sec.

17. What work is required to speed up a 3200-lb. car from 20 to 30 miles per hour?

18. It is advertised that a certain automobile (weight approximately 3200 lb.) can pick up from a speed of 5 miles per hour to 60 miles per hour in 21 sec. Find the effective horsepower, neglecting frictional losses.

19. A 96-lb. hammer of a pile driver falling 30 ft. strikes a pile and drives it in 1 in. What is the resisting force met by the pile? (Neglect the comparatively large heat loss at point of impact.)

20. With a 2000-gram pull a sling shot is stretched 40 cm. What is the average force during stretching? What is its potential energy? What speed does it give to a 20-gram stone?

21. When a 1000-kilogram automobile traveling 30 meters per second is brought to rest, how much heat energy (in kilojoules) is generated in the brakes?

22. Neglecting friction, can an automobile traveling 60 miles an hour coast up a hill 50 ft. high? If so, what speed will it have at the summit?

23. A 320-lb. car is pulled up to the 100-ft. starting summit of a roller coaster. (*a*) How much work is done? (*b*) The track now drops 80 ft.; assuming no frictional loss, find the potential energy, kinetic energy, and speed at the bottom of the dip. (*c*) The next crest is 80 ft. from the ground; find the kinetic energy here.

24. Assuming it to work at maximum efficiency, how long should it take a 3000-lb. 80-hp automobile to attain a speed of 40 miles an hour? of 60 miles an hour? How fast can it travel on a 10 per cent upgrade?

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CHAPTER 13

SIMPLE MACHINES

140. Law of Machines. A machine is a device for the application of force for a useful purpose. A pair of pincers, a hammer, a crowbar — these are very simple kinds of machines; in modern factories we find an innumerable variety of more complicated machines — giant presses to bend or shear or stamp; intricate automatic machines, machines which almost seem to show intelligence as they select and judge and sort and weigh. But all machines have this in common: (1) A force is applied *to* the machine and pushes something, pulls something, squeezes something, or turns something — this is the *work input*; (2) a force is delivered *by* the machine, and this force lifts or squeezes or turns or pushes — this is the *work output*. And for any machine this work output equals, or, if there is friction, is less than, the work input. For the ideal machine (without friction)[1]

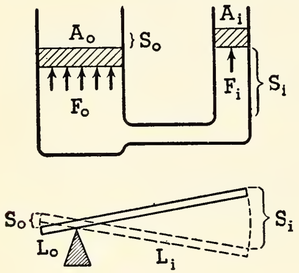

FIG. 1

$$\text{Work in} = \text{Work out}†$$

$$F_i s_i = F_o s_o.†$$

We may write this

$$\frac{F_o}{F_i} = \frac{s_i}{s_o}.† \qquad (10)$$

The output and input forces vary inversely as the displacements. For example, consider the hydraulic press. It has already been found (§34) that the forces on input and output sides are proportional to the areas of the cylinders. The distances traveled by the two pistons are inversely as their areas. Hence

$$\frac{F_o}{F_i} = \frac{A_o}{A_i} = \frac{s_i}{s_o},†$$

in accordance with the law of machines.

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[1] The symbol † is used in this and the following chapter to indicate relations which hold only in so far as friction can be neglected.

Or, again, take the lever. When the lever turns through an angle θ it is evident that the distances s_i and s_o are proportional to the lever arms L_i and L_o. Hence by the law of machines the forces are in the ratio

$$\frac{F_o}{F_i} = \frac{s_i}{s_o} = \frac{L_i}{L_o}.\dagger$$

That is: the forces vary inversely as the lever arms. This is the law which was derived (using the equality of torques) in Chapter 8.

141. Mechanical Advantage and Velocity Ratio. Often it is the purpose of a machine to deliver a large force out from a small force in. The ratio F_o/F_i is called the **mechanical advantage** (m.a.) of the machine. It must be remembered that when we gain force we lose distance. If the mechanical advantage of a lever is 10, the load is raised only 1 inch while the applied force moves 10 inches. There is a loss in speed. The ratio s_o/s_i is called the *velocity ratio.* Neglecting friction, the mechanical advantage is equal to the reciprocal of the velocity ratio.

$$\text{M.a.} = \frac{s_i}{s_o}.\dagger \tag{10'}$$

As illustration, suppose that a machine is hidden in a box — perhaps a hydraulic press, or a system of levers or gears or pulleys — its nature is unknown. Two chains come out. Suppose that every time we pull the one chain out horizontally 1 inch the other is pulled upward 4 inches. The velocity ratio is 4 : 1, the mechanical advantage is $\frac{1}{4}$, and a 100-pound force (input) on the one chain will give a 25-pound force on the other. This is the maximum possible value of the output force — if there is friction the output will be less.

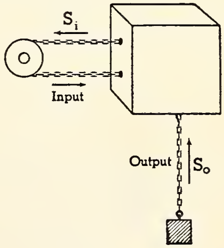


FIG. 2

Frequently one wishes to obtain a large force output, a large mechanical advantage (as in a hydraulic press and in a crowbar). Other machines are designed to give a large velocity ratio — the bicycle is a familiar example. In the automobile the gear shift allows different couplings — " low gear " giving higher mechanical advantage for starting and hill climbing, "high gear" giving a higher velocity ratio.

142. The Block and Tackle. A single pulley attached to the load (Fig. 3*b*) has a mechanical advantage of 2. Clearly for every foot the load is raised (s_o) the rope *i* must be pulled through 2 feet (s_i). The velocity ratio is $\frac{1}{2}$; hence the mechanical advantage is 2. In general for pulleys:

M.a. = Number of ropes on the moving pulley.†

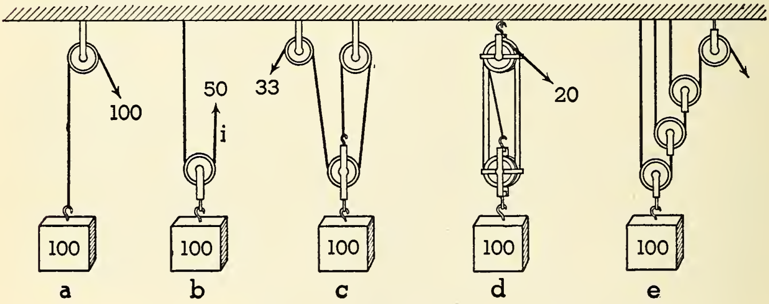


FIG. 3. Mechanical advantage of frictionless pulleys.

When the weight is raised 1 foot, there is 1 foot of slack to be taken up in each rope; hence the travel of the force is so many times the travel of the load. (Or the matter can be viewed thus: If there are N ropes, each rope supports $1/N$th of the load; this is the tension in the rope and hence is the force to be applied.)

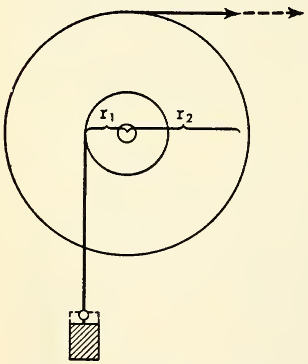


FIG. 4. Wheel and axle.

The rule which has been given does not apply to a multiple pulley system such as shown in Fig. 3*e*. The velocity ratio and the mechanical advantage in this system are left as a problem for the student.

143. The "Mechanical Powers." The Egyptians used the lever, the inclined plane, the wedge in building their pyramids. The pulley, wheel and axle, and the screw were used by the Greeks and Romans. These six devices for multiplying the force of men's muscles became known as the "mechanical powers"; these simple machines form the basic elements of many of our complicated modern mechanisms. We have treated the lever and pulley. It is left to the student to find the velocity ratios and derive the mechanical advantage for each of the other machines.

(1) *Wheel and Axle.* In Fig. 4 the load is suspended from the small cylinder, radius r_1; the force applied at the large cylinder, radius r_2. Find the velocity ratio and prove

$$\text{M.a.} = \frac{r_2}{r_1} \cdot \dagger$$

(2) *Inclined Plane.* l represents the distance through which the force is exerted, h represents the height through which the load is lifted. Show that

$$\text{M.a.} = \frac{l}{h} \cdot \dagger \qquad \text{(Compare §74.)}$$

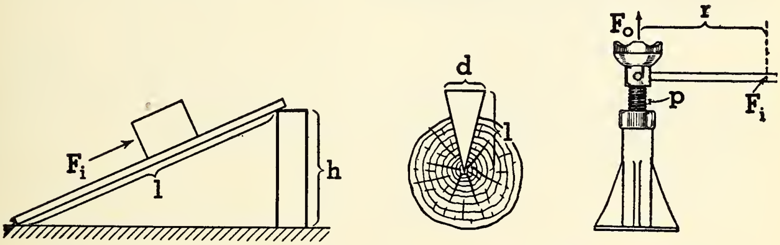


FIG. 5. Simple machines: inclined plane, wedge, screw and lever.

(3) *Wedge.* The wedge is used to pry objects apart a distance d when the wedge moves a distance l.

$$\text{M.a.} = \frac{l}{d} \cdot \dagger$$

(4) *Screw.* The figure shows a screw combined with a lever. When the lever rotates once the load is raised through the pitch (p) of the screw. Show that

$$\text{M.a.} = \frac{2\pi r}{p} \cdot \dagger$$

The student will observe that several of these simple machines are essentially alike. The wheel and axle are essentially a lever, and the mechanical advantage can be found by equating the two opposite torques. The wedge is two inclined planes; the screw is an inclined plane wrapped around on a cylinder. Other commonly used appliances are obvious combinations of these elementary machines.

144. Efficiency. The equations given above (and marked with daggers) are for the ideal mechanical advantage which would obtain in the absence of frictional losses. Actually there will always be such losses, more or less serious. The actual energy equation will be

Work in = Work out + Frictional heat developed.

The *work in* will always be somewhat larger than the *work out*, and the force developed (F_o) somewhat smaller than the ideal value $F_o^\dagger$. This ratio of output work to input work is called the **efficiency** of the machine. That is

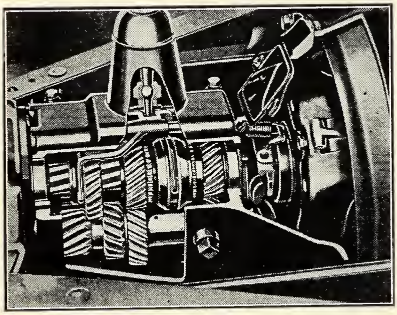


FIG. 6. Transmission gears of automobile.

$$\text{Efficiency} = \frac{F_o s_o}{F_i s_i}. \qquad (10'')$$

Since $F_i s_i = F_o^\dagger s_o$ (where $F_o^\dagger$ represents the theoretical — frictionless — value of the output force), efficiency may be alternatively defined as

$$\text{Efficiency} = \frac{F_o s_o}{F_o^\dagger s_o} \quad \text{or} \quad \frac{F_o}{F_o^\dagger};$$

that is, the efficiency is the ratio of the work actually obtained to that obtained in a frictionless machine, or it is the ratio of the force actually delivered to the force expected from the velocity ratio. If the theoretical mechanical advantage of a machine is 4 and its efficiency is 75 per cent, a 10-pound input force will produce a 40-pound output force in the ideal (frictionless) case but a 30-pound output force actually.

Usually high efficiency is desired. But not always. In a vise, in a chain hoist, in a screw jack the efficiency must be kept so low that the load cannot act to run the machine backwards as soon as the applied force is removed. The efficiency of such machines must be well under 50 per cent.[1]

145.* Rotational Mechanical Advantage. Before the machine age, men were usually interested in lifting loads or dragging them. Today there is more spinning of wheels. If a list were made today of the most-used fundamental machines probably the inclined plane and the wedge would be omitted, but we should include the belt or chain drive and the gear drive. The belt carries the power from one rotating drum to another; the chain from one sprocket to another (as on the bicycle). As the belt transfers the power from large drum to small, or *vice versa,* it steps up or steps down the *angular* velocity. At the same time it steps down or steps up to the *torque.*

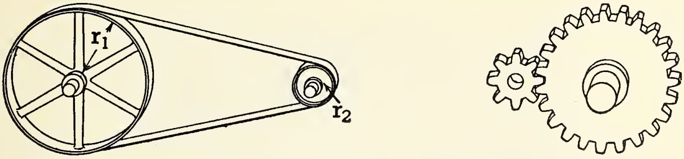
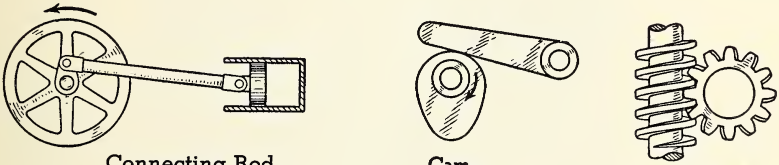
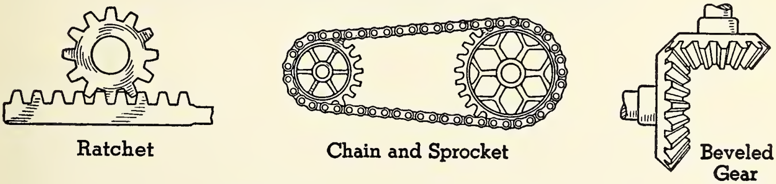


FIG. 7. Examples of rotary mechanical advantage.



Connecting Rod **Cam** **Worm Gear**



Ratchet **Chain and Sprocket** **Beveled Gear**

FIG. 8. Elements of machines.

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[1] It is easily shown that for a machine in which the friction depends only on the load, not on the applied force (for example the inclined plane or screw), the machine cannot be operated by the load if its efficiency is less than 50 per cent. This rule evidently does not apply to a pulley system.

In following the transmission of power through the wheels of a modern machine it is more convenient to consider the rotational mechanical advantage (r.m.a.) and the rotation ratio than the ordinary mechanical advantage and the velocity ratio. *The rotational mechanical advantage is the ratio of the torque delivered* ($\mathcal{T}_o$) *to the torque applied* ($\mathcal{T}_i$). We may write the law of the perfect machine as

$$\mathcal{T}_i\theta_i = \mathcal{T}_o\theta_o \dagger$$

or

$$\text{R.m.a.} = \frac{\mathcal{T}_o}{\mathcal{T}_i} = \frac{\theta_i}{\theta}, \dagger \qquad (10''')$$

where θ represents the angle turned through. The rotational mechanical advantage (neglecting friction) is the reciprocal of the rotational ratio (θ_o/θ_i).

In the belt drive the peripheral speed of both drums is the same, but the angles turned through are inversely as the radii. Hence

$$\text{R.m.a.} = \frac{\mathcal{T}_o}{\mathcal{T}_i} = \frac{\theta_i}{\theta_o} = \frac{r_o}{r_i} \cdot \dagger$$

The torques on the two wheels are directly as their radii. Similarly for the spur or beveled gear drive the rotational mechanical advantage depends on the number of teeth on the two meshing gears — the angular velocities vary inversely as the number of teeth.

Example. Compare the torques which must be applied to the hour hand, the minute hand, and the second hand of a clock, to stop the clock.

146. Transformation of Power. Power is force times velocity. The same power may be developed by the slow-moving water wheel driven by a high head of water as by a fast-spinning steam turbine blown by the comparatively gentle pressure of the moving steam. But the power must be adapted to its use, and for this purpose we use machines. For the stamping of metal in a giant press we cannot use " high-speed — small-force " power directly; the force must be stepped up (and the speed down). This same power which operated the press might have been used to spin an air fan, but then we should wish to step up the velocity (and hence step the force down). This is one of the functions of a machine. By means of its appropriately high or low mechanical advantage the machine steps the power into a form that can be effectively used. But always the power out is equal to, or (because of frictional losses) less than, the power in.

In addition to this stepping up or stepping down of force (and speed), of course the machine is designed to do a specific job of cutting or pounding or squeezing or turning or to push a needle or to write letters or to do any of the multiform tasks demanded in present-day civilization. This has introduced a great number more " mechanical powers " than the six of earlier times. The back-and-forth reciprocating motion of the piston must be linked to the wheel; one wheel operates another by means of a belt drive or by a chain drive or by a gear drive. (For a few of these elementary mechanisms see the illustrations under " Power Transmission " in the Encyclopædia Britannica.) And not only is power transmitted mechanically by belts or shafts or chains or gears; it may be transmitted hydraulically or pneumatically or, most important of all, it may be transmitted electrically. Today all the machines of a city may be driven by one central engine,

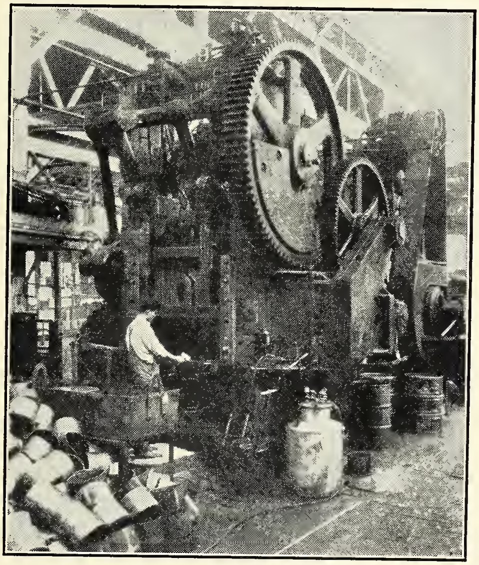

FIG. 9. Large stamping press; punching cylindrical vessels from sheet metal.

not by belts or chains, but by the invisible flow of electrons in small wires. Here, in a very real sense, the electric fan in the home, the electric lathe in the factory and the electric generating plant are a part of one single machine. The *force in* is the steam push on the turbine of the electric generator; the *force out* is applied by the fan or the lathe. And linking the input and output we have — not belts or chains — electrons flowing in wires. We may not know the meaning of this electrical mechanism which lies between; it is for us at present much like our box (Fig. 2), with only its two visible chains. Nevertheless, for this complicated giant machine, with its arms stretching from the central power plant into all our homes and factories, we can figure the mechanical advantage from the velocity ratio (the speed of the fan or the lathe as compared to that of the generator), and we have the general law of machines, valid except for the energy waste as heat:

$$\frac{F_o}{F_i} = \frac{s_i}{s_o}.\dagger$$

For this is simply the Law of Conservation of Energy.

QUESTIONS

1. If the theoretical mechanical advantage of a machine is 5 and the actual mechanical advantage is 4, what is the efficiency? What is the velocity ratio?

2. What force do you estimate a man can exert with a pair of pliers?

3. Neglecting friction, estimate the mechanical advantage of a typewriter key.

4. With two ropes on the moving pulley, it takes a 60-lb. effort to lift a 100-lb. weight. What is the efficiency?

5. Why is it impossible to ride a bicycle up a steep hill?

6. What is the rotational mechanical advantage of a wheel and axle?

7. A painter seated on a scaffold raises it by a rope passing over a single fixed pulley. He fastens the end of the rope to the building and the rope breaks. Explain.

8. In a simple pulley system with 4 ropes on the moving pulley and efficiency 50 per cent, the velocity ratio is: (*a*) less than 1 : 4; (*b*) greater than 1 : 4; (*c*) equal to 1 : 4.

9. When an engine carries a train up an incline the work is: (*a*) greater than, (*b*) less than, (*c*) equal to, the work necessary to lift the load directly.

Vocabulary: Input, output, ideal machine, Law of Machines, mechanical advantage, velocity ratio, efficiency, rotational mechanical advantage, power transformation, connecting rod, cam, rack and pinion, spur (beveled, worm) gear.

PROBLEMS

1. What work is required to raise a 100-lb. box to a 3-ft. platform (*a*) up an inclined plane 10 ft. long? (*b*) with a crank and axle (crank 1 ft. long, axle 2 in. diameter)? Assume that the efficiency in each case is 50 per cent.

2. What force is required in problem 1?

3. If it requires a force of 200 lb. to lift a 1000-lb. weight by the pulley system shown in Fig. 3*e*, what is the efficiency of the system?

4. What force is required to lift one end of a 3000-lb. automobile by means of a screw jack if the pitch is $\frac{1}{2}$ in., the lever arm 1 ft. long, and the efficiency 30 per cent? (Assume the weight evenly distributed.)

5. A crowbar is 10 ft. long and the load is 2 ft. from the fulcrum. What is the mechanical advantage if the load is at the end? If the fulcrum is at the end?

6. What is the frictionless mechanical advantage of a 30° inclined plane?

7. A steam engine can exert a 10-ton tractive force. Neglecting friction, how heavy a train can it pull up a 1 per cent grade?

8. What is the mechanical advantage of the combination inclined plane and crank and axle in Fig. 10?

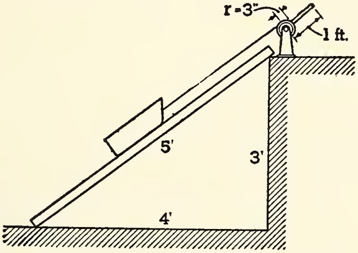

Fig. 10

9. Approximately what is the velocity ratio between a doorknob and the latch? Between the operating handle and window of an automobile? Between the rotating head and the lead of an automatic pencil?

10. The hour hand of a clock is 3 in. long, the second hand 1 in. What is their velocity ratio? Suppose that a force of 2 grams applied to the second hand stops the clock; what force must be applied to the hour hand to stop it?

11. Compute approximately the mechanical advantage of the aneroid barometer mechanism from the dimensions in Fig. 5–8.

12. What is the velocity ratio in a bicycle: wheel 28 in. diameter; pedal shaft 8 in. long; front sprocket 40 teeth; rear sprocket 12 teeth?

13. Find the rotational mechanical advantage of the bicycle in problem 12.

14. In a certain automobile the angular velocity is reduced 4.1-fold in the differential; in the transmission the angular velocity is reduced 1.7-fold, 3-fold, and 3.4-fold respectively for intermediate, low, and reverse. Find the rotational mechanical advantage for each gear and for direct drive. With

30-in. tires how many revolutions per second does the engine make at 60 miles per hour (direct drive)?

15. Five complete turns are required to wind the spring in a toy automobile; the average value of the winding couple is 5 lb.-in. The toy then travels 25 ft. on the level. What is the average value of the propulsive force?

16. Show that in rolling a barrel up an inclined plane (pushing on the top of the barrel) the mechanical advantage is $2\,l/h$.

17. Derive the expression for the rotational mechanical advantage of the worm gear.

18. Show that if more than one-half the work done in pushing a weight up an inclined plane is done against friction, the weight will not slide down the plane of itself.

CHAPTER 14*

ENERGY OF FLUIDS

147. Pressure: Potential Energy per Unit Volume. As a balloon rises, its potential energy decreases. When we blow an air bubble under water or submerge a cork or thrust a plunger into a liquid (Fig. 1) we have an increase in potential energy. The cork will push upward, the plunger will tend to be shot out, the air bubbles could be made to turn a paddle wheel and do work as they rise. The work which we do in shoving the plunger into the liquid is[1]

$$W = PAs = PV = \text{Potential energy},$$

where V is the volume of liquid displaced. (Compare eq. 7′.) This is a measure of the potential energy. From this point of view,[2] *pressure is the potential energy per unit displaced volume.*

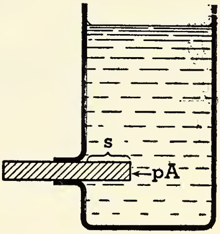

FIG. 1

$$P = \frac{\text{p.e.}}{V}.$$

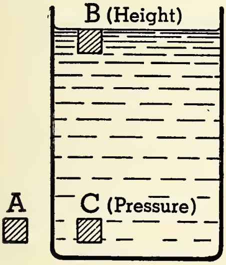

FIG. 2

We must distinguish between the direct gravitational potential energy (which increases with height) and this hydrostatic potential energy which increases with depth. A stone, in falling, loses gravitational potential energy, but, if it falls in water, this loss is, in part, compensated for by an increase in hydrostatic energy.

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[1] In all equations in this chapter fundamental units must be used: pressure in baryes (§110) or pounds per square foot; densities in grams per cubic centimeter or slugs per cubic foot, etc.

[2] This is not contradicting our earlier definition of pressure, for energy per unit volume is dimensionally equivalent to force per unit area. We may speak (loosely) of the direction of the pressure, meaning the direction of the surface used to measure it.

Compare the energy of a certain volume of water when placed at A and at B and at C (Fig. 2.). The potential energy is greater at B than at A because of the height (mgh);[1] it is greater at C than at A because of the pressure (PV). Now compare B and C. No work is required to raise or lower a cubic centimeter of water in water; it is buoyed up by the other water; it "just floats."

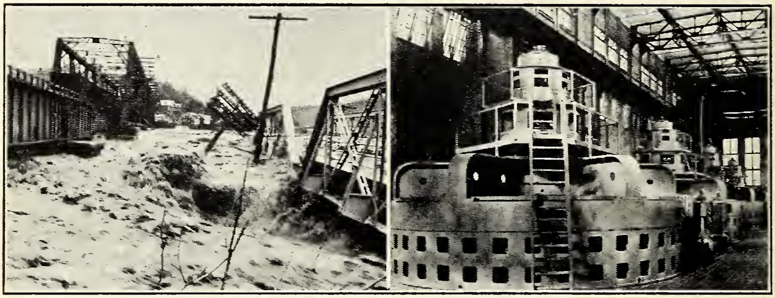

Fig. 3. Water power: flood; generators in hydroelectric plant.

Therefore the potential energies at B and C are the same. At greater depths the gravitational energy decreases but the pressure energy increases to compensate. Let us call the sum of these two energies for a *unit volume* (dgh and P) the *hydrostatic potential.* Then for a liquid in equilibrium the potential is the same at all parts.

This refers to the energy of the liquid (per unit volume) in the liquid. A stone is heavier than water; its potential energy at B is greater than at C. A cork is lighter than water, its potential energy will be greater at C than at B; similarly for a bubble — in fact, here the direct gravitational energy is negligible.

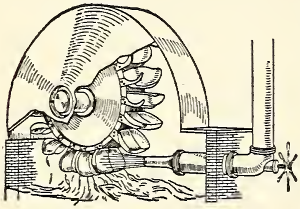

Fig. 4. Small water turbine.
Power = $P \times V/t$
Force = $\Delta\mathfrak{M}/t$

Notice that, speaking strictly, this potential energy is localized, not in the submerged cork or the balloon, but in the displaced water or air which has been lifted. In displacing the water the cork has raised the surface level slightly and increased the potential energy of this elevated water.

Example. What is the potential energy of a cubic-millimeter bubble 10 cm under water? Pressure is $10 \times 980 = 9800$ baryes. Hence the energy per cubic centimeter displacement is 9800 ergs; 9.8 ergs per cubic millimeter. Our bubble can do 9.8 ergs of work in rising to the surface.

---

[1] In this chapter h represents the height and h' or $-h$ the depth.

148. Hydrostatics. It has just been shown that the hydrostatic potential ($P + dgh$) is the same at all points in a homogeneous liquid at rest. The principles of hydrostatics given in Chapter 4 follow from this. For example:

(1) All points at the same level have the same value of dgh, hence have the same pressure (P).

(2) At the free surface of a liquid the pressure (P) is constant (atmospheric), therefore the gravitational energy (dgh) must be constant; thus the free surface of the liquid is level (h constant).

(3) Pressure changes with depth. In rising a distance h the gravitational energy per unit volume increases by dgh; hence the pressure change is $-dgh$ or dgh', where h' ($= -h$) is the change in depth. (See eq. [1], footnote p. 29.)

But each of these rules applies only to liquids at rest. Liquids tend to flow from high potential to low. When motion occurs it is necessary to consider kinetic energy and the transformation of energy into heat through viscous friction. Let us consider the kinetic energy, neglecting for the present the effects of viscosity.

149. Velocity of Efflux. As water flows out from an orifice in a tank the hydrostatic potential energy is changed to kinetic energy. Consider a unit volume. Its mass is numerically equal to the density (d). In the body of the liquid this unit volume has energy P; outside, it has kinetic energy $\frac{1}{2} dv^2$. By conservation of energy

$$\tfrac{1}{2} dv^2 = P$$

or

$$v = \sqrt{\frac{2P}{d}}\cdot\dagger \qquad (11')$$

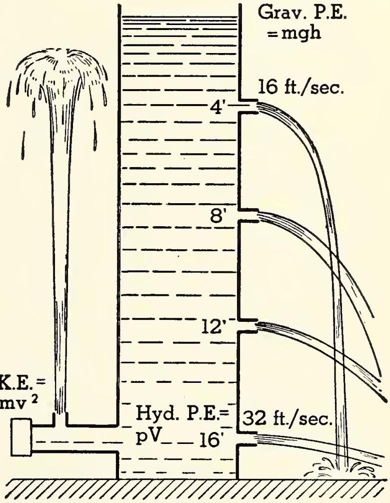

Fig. 5. Torricelli's theorem.

Example. How fast will water be injected into a vacuum? Here P = 1 atmosphere = (approx.) 1,000,000 baryes.

$$d = 1 \text{ gram per cc.}$$

$$v = \sqrt{2 \times 10^6} = 1400 \text{ cm per sec.}$$

Torricelli's Theorem. Neglecting air and liquid resistance, a jet of water issuing from an orifice in a reservoir has just sufficient speed to reach the level of the free surface in the reservoir, for the kinetic energy at efflux must equal the pressure energy at the orifice and this in turn equals the gravitational energy at the surface. The pressure at the orifice is dgh'. Using this in the above equation we have

$$\tfrac{1}{2}\, dv^2 = dgh'$$
$$v^2 = 2\, gh'.\dagger$$

This is exactly the speed that water dropped from the surface would have at the level of the opening (eq. 3*c*). This principle is due to Torricelli (§43).

150. Bernoulli Effect. Wherever the velocity of a fluid is great the pressure (measured transverse to the motion) is small. The jet issuing from the tank was one example of this. Consider a liquid flowing through a pipe of varying cross-section. The pressure at *a* and *c*, as indicated by the rise in the side tubes, is the same (neglecting viscous friction); the pressure is less at *b*. The pressure is least where the speed is greatest. The effect was observed and explained by Bernoulli[1] (1750).

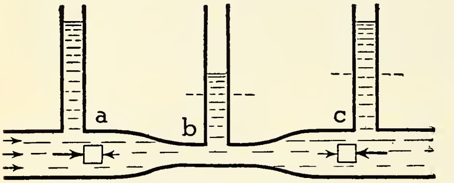

FIG. 6. Bernoulli effect.

This change in pressure seems so surprising that we wish to explain it from two different points of view:

(1) As a volume of water (*a*) approaches the constriction it is accelerated. By Newton's second law this implies an unbalanced force. The pressure on its left side must exceed that on its right. After passing through the constriction the liquid (*c*) is retarded; this demands a greater pressure on the right face. Hence to produce this necessary speeding up and slowing down the pressures in the narrower part of the tube must be smaller.

(2) The quantitative explanation follows from consideration of energy. Where the kinetic energy is greatest the potential energy is least. Let us follow a cubic centimeter of water through the tube. At *a* its potential energy is P_a and its kinetic energy is $\frac{1}{2}\, dv_a^2$. At *b* its velocity (v_b) is greater and its pressure (P_b) is less.

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[1] Daniel Bernoulli, member of a famous Swiss family of scientists and mathematicians.

For, by conservation of energy,

$$P_a + \tfrac{1}{2}\,dv_a^2 = P_b + \tfrac{1}{2}\,dv_b^2\dagger \qquad (11'')$$

or

$$P_a - P_b = \tfrac{1}{2}\,d\,(v_b^2 - v_a^2).\dagger$$

151. Illustrations of the Bernoulli Effect. If a stream of air is blown between two pingpong balls the balls will be brought together. The fast-moving air has reduced sidewise pressure, and the atmosphere forces the balls together. . . Similarly two boats anchored close together in a stream will come together; the water moving abnormally fast between them has reduced pressure. . . If air is blown at a ball while the ball is spinning on the end of a string, the ball will move to one side. Below the ball in Fig. 7 the air speed is reduced by the rotation of the ball and hence the pressure here is increased. (This is the principle of the curving baseball; in this case the air is at rest and the spinning ball moves into it.). . . In the atomizer a stream of air sucks up the liquid from the reservoir and then breaks it into spray. . . In the aspirator pump a jet of water carries the air along with it and evacuates a vessel.

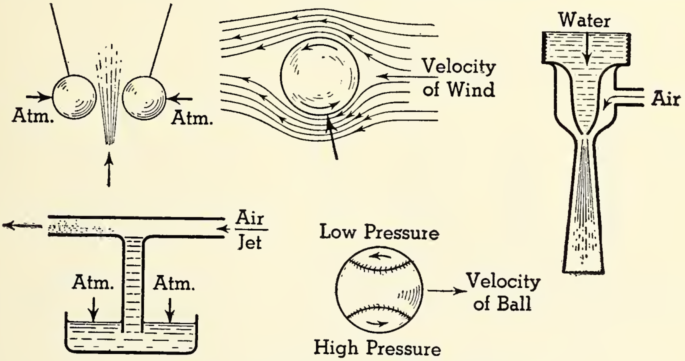

FIG. 7. Examples of Bernoulli effect.

152. General Consideration of Energy in Liquids. Since the tube in Fig. 6 was horizontal it was not necessary to consider gravitational energy. To treat more general cases we must complete eq. (11″) by including the gravitational potential energy of a unit volume: *dgh*. The equation then reads

$$\overbrace{dgh_a + P_a}^{\text{potential}} + \overbrace{\tfrac{1}{2}\,dv_a^2}^{\text{kinetic}} = \overbrace{dgh_b + P_b}^{\text{potential}} + \overbrace{\tfrac{1}{2}\,dv_b^2}^{\text{kinetic}}\;\dagger \qquad (11)$$

or the total energy (gravitational, hydrostatic, and kinetic) remains constant. This equation would give, for example, the pressure (neglecting effects of viscosity) at various heights in Fig. 4–4 when there is flow in the pipes.

This equation summarizes the properties of liquids, neglecting frictional forces:

(1) If v is zero we have hydrostatics: $P = dg\Delta h'$ (eq. 1, p. 29).

(2) If h is constant we have velocity of efflux and Bernoulli effect:

$$\Delta P = \tfrac{1}{2}\,d\,(v_1^2 - v_2^2).\dagger$$

(3) If P is constant we have velocity of free fall

$$v^2 = 2\,gh'.\dagger$$

153. Flow through Pipes. Water flowing through a pipe meets a viscous resistance which we have been neglecting; in many respects this viscous resistance resembles friction between solids. (A notable difference is that the frictional force in fluids is proportional to the speed.[1]) Because of this resistance there must be a difference in pressure between the two ends of the pipe to maintain the flow.

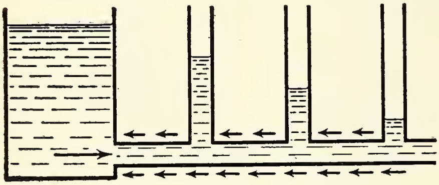

FIG. 8. Pressure drop in pipe. Arrows represent forces, viscous forces balancing pressure force, resulting in steady flow.

It is a common experience that turning on one faucet reduces the pressure at other faucets in the house. If no water were drawn from a city water system, water in every building could rise to the level of the reservoir. Actually it will not rise this high owing to the drop in pressure along the water mains. The mains are made large enough to prevent too serious a " line drop " in pressure. Petroleum and natural gas are piped for thousands of miles. Pumping stations are installed at intervals to build up the pressure which is lost along the line (Fig. 9).

In the human body the heart acts as such a two-stage pumping system. The pressure is raised by the right ventricle, drops in the flow through the lungs, is raised again in the left side of the heart and gradually drops again owing to the viscous flow through the arteries and capillaries and veins throughout the body. The capillaries though small are very numerous, have an aggregate cross-section 800 times as great as the arteries, and (as can be seen in Fig. 10) actually offer less resistance to the flow.

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[1] This is for streamline flow; at high speeds or with rough piping turbulence occurs and introduces greater resistance (§156).

Thus when there is flow it is no longer true that the pressures at all points at the same level are equal. The water flows from high pressure (high hydrostatic potential) to low. The pressure gradually decreases as we go along the pipe from one end to the other. This means that potential energy is decreasing; the energy is being transformed into heat.

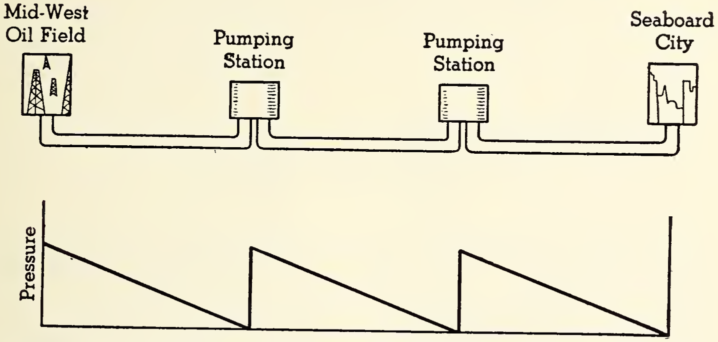


FIG. 9. Oil pipe line with viscous resistance and hydromotive forces.

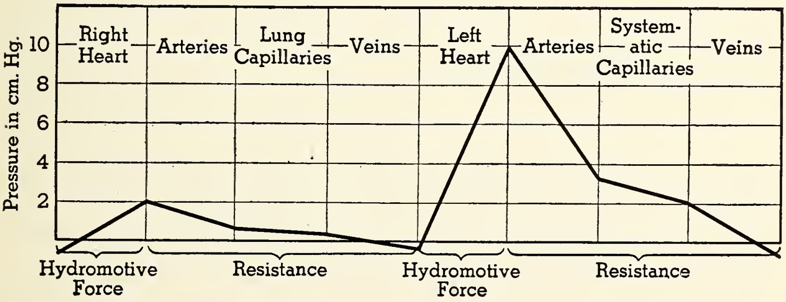


FIG. 10. Pressures in human circulatory system. (Compare Fig. 5–6.) (Mean values during heart beat.)

154. Viscosity. The viscous force depends upon the nature of the fluid (viscosity) as well as upon its speed and the size of pipe. The viscosities of several fluids relative to water are given in Table 12. The lubricants are typical winter and summer crank-case oils. Pitch is included in the list as representing a substance which, though apparently solid, in reality is a liquid a million million times more viscous than water.

TABLE 12

VISCOSITY (RELATIVE TO WATER AT 20° C)

Air	Ether	Alcohol	Glycerin	Pitch
0.02	0.23	1.2	850	1.3×10^{12}

Lubricating oils at 55° C:
S.A.E. No. 10, relative viscosity 20.
S.A.E. No. 30, relative viscosity 40.

Water passing through a pipe does not flow like a rigid cylinder, slipping at the walls. Along the center of the pipe the velocity is greatest, becomes less nearer the walls and the liquid in direct contact with the walls is at rest. There is slippage of adjacent layers, one past the other. Figure 11 represents two plane surfaces separated by a viscous medium (more concretely, say, a plank floating in shallow water). The upper surface moves to the right. The arrows show the motion of successive layers of the medium. Owing to viscosity the motion of the upper surface produces a drag to the right on the lower surface; there is a backward drag (reaction) on the upper surface. We can conceive of this viscous force as being somewhat like the frictional resistance between rigid surfaces as the successive layers of the medium are dragged past each other. The slower molecules in one layer impede the faster molecules in the layer above.

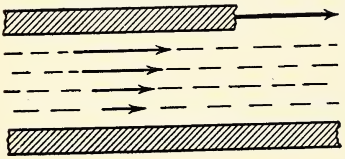


FIG. 11. Viscous drag.

The force necessary to drag the upper plate is proportional to its speed (v), to its area (A), is inversely proportional to the distance (l) between the plates: $F = \eta\, Av/l$. The proportionality constant (η) is the *coefficient of viscosity* of the medium. For water its value is 0.01 at 20° C. (If the upper surface in Fig. 11 moves 1 centimeter per second and is 1 centimeter from the lower surface, separated by water, the drag on it is 0.01 dyne for each centimeter of its area.) The value is 850 times greater for glycerin, 50 times less for air. The relative values in Table 12 must be divided by 100 to give the absolute values of the coefficients.

155.* Measurement of Viscosity. *Direct Method.* In practice viscosity cannot be measured by dragging a plank through a shallow liquid because the waves and turbulence set up by the ends of the plank may cause a greater drag than the effect which is to be measured. These end effects can be eliminated by using a rotating cylinder (Fig. 12). The interval between the two cylinders is measured and is filled with the oil or other liquid which is under test. The inner cylinder is mounted on pivot bearings; by means of a falling weight a known torque is applied to this cylinder and the velocity measured. From these data the coefficient can be obtained.

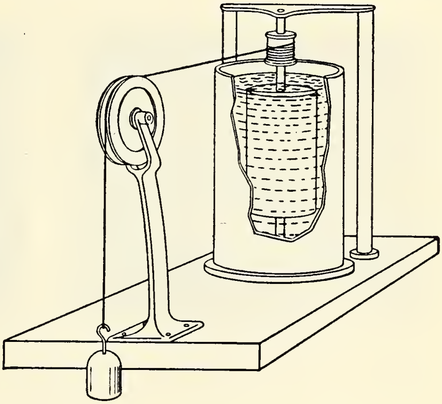


FIG. 12. Direct measurement of viscosity.

Flow through Capillaries. Often a more convenient method is to measure the rate of flow through a small capillary tube. Evidently the volume of liquid which will pass through a tube per second (V/t) under a given pressure difference (ΔP) will decrease very rapidly as the radius (r) of the tube is decreased; not only is the cross-section of the stream decreased but also its velocity is reduced by the proximity of the walls. It can be shown that $\frac{V}{t} = \frac{\pi r^4 \cdot \Delta P}{8\,\eta L}$ (Poiseuille's fourth-power law), where L is the length of the tube and η the coefficient of viscosity. The rate of flow of the liquid through a small capillary under a given pressure head can be measured and η determined.

Viscous Force on a Moving Sphere. When a small body moves, not too fast, through a viscous medium the resisting force is proportional to its radius of cross-section, its velocity, and the viscosity of the medium. For a sphere, the exact relation is: Viscous force $= 6\pi\,\eta r v$ (Stokes' Law). A body falling through a fluid is at first accelerated but soon reaches a limiting velocity at which this viscous force just equals the effective weight (allowing for buoyancy) of the body in the liquid. Then $\mathbf{F} = 0$ and the body falls at a constant speed. Evidently, from Stokes' Law this limiting speed (v) is inversely proportional to the viscosity (η). Thus viscosities can be measured by observing the rate at which a small sphere falls or a bubble rises through the medium.

Although Poiseuille's Law and Stokes' Law cannot be derived here they are given for reference. The law of capillary flow is of considerable importance in biological sciences, and Stokes' Law has many applications in advanced physics and physical chemistry.

156. Turbulence. We have been considering the liquid as flowing in parallel lines through a tube — this is "streamline" flow, and the resistance is due to viscosity. But if the tube is rough or has sharp corners, or if the velocity is high, a much more serious energy loss may occur. The stream breaks into turbulence and the energy is now lost, not gradually, by the action of molecule on molecule in neighboring streamlines, but in wholesale quantities as large volumes of the liquid are sent whirling off in eddies, churning and splashing.

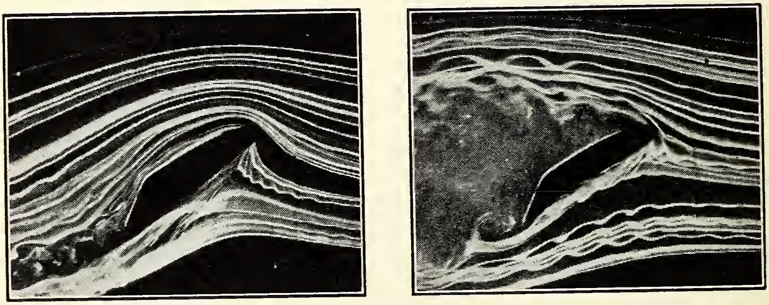


FIG. 13. Model airplane wing in wind tunnel near stalling angle; streamline flow breaks into turbulence.

When a large body — a ship or airplane — moves through the water or air, the ordinary forces of viscosity are not very serious — otherwise an airplane could not travel 300 miles an hour. The wind resistance of an airplane is almost entirely due to turbulence. It is turbulence which retards the motion of a parachute or of a rock falling through water. Turbulence can be greatly reduced by shaping the body to follow the natural lines of streamline flow. Nature has given streamline bodies to fish and to birds. And streamlining is one of the first essentials in the design of submarines, dirigibles, and airplanes.

For small objects moving slowly through a viscous medium, the resistance is primarily due to viscosity and Stokes' Law holds.[1] But in the great majority of instances — for raindrops, projectiles, automobiles, and airplanes — the resistance is due to turbulence. Since this resistance is due to the kinetic energy ($\frac{1}{2} mv^2$) fed to the fluid, the resisting force is proportional to the square of the velocity (rather than the first power as in Stokes' Law), to the density (rather than the viscosity) of the medium, and to the cross-sectional area (rather than the radius) (§127).

Because of viscosity in the whirls and eddies the turbulent motion ultimately degenerates into heat.

QUESTIONS

1. Is there an increase or decrease in energy as air bubbles rise in water?

2. One balloon is filled with hydrogen gas and another with carbon dioxide (heavier than air). Explain why the energy of one increases and of the other decreases with height.

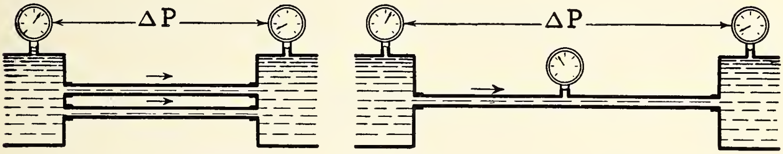


FIG. 14

3. Compare the resistance of one pipe, two pipes in series, and two pipes in parallel. Show that, for the same total pressure difference (Fig. 14), adding a second pipe in parallel increases the flow, in series decreases it.

4. Show that the total energy is constant when a liquid (*a*) leaves an orifice, (*b*) flows through a narrow neck, (*c*) rises toward the surface, (*d*) passes down tube with viscous resistance.

---

[1] For viscous resistance the retarding force is proportional to $\eta v r$ (§155)

$$F_{\text{visc.}} \propto \eta v r,$$

For turbulent resistance

$$F_{\text{turb.}} \propto d v^2 A \propto d v^2 r^2 \quad (\S 127).$$

Therefore the relative importance of turbulence to viscosity is

$$\frac{F_{\text{turb.}}}{F_{\text{visc.}}} \propto \frac{d}{\eta} v r.$$

Thus, though the exact relation (giving the constant of proportionality) depends upon the shape of the body, in general we can say that turbulence will assume greater relative importance for larger bodies at high speeds. The quantity dvr/η which serves as a criterion for the importance of turbulence is called *Reynolds' Number*.

5. A tank 10 ft. high is 20 ft. from the ground. Compare the work required to fill it with water (*a*) with a pipe to the bottom, (*b*) with a siphon over the top, (*c*) with a pipe with outlet at the top.

6. Suppose the small bottle in the Cartesian diver (Fig. 4–9) is filled with air, corked, and tied to the bottom by a thread which is just on the point of breaking. Will a sudden downward jerk break the thread?

7. Try this experiment: Place a small card against the lower end of a thread spool; blow through the spool and observe that the card does not fall off. Explain this by the Bernoulli principle. (Place a pin through the card to center it and keep it from slipping.)

8. Water flows from a small pipe into a reservoir, entering 10 ft. below the surface. The water pressure against the walls of the pipe, just before the orifice, will be: (*a*) greater than, (*b*) less than, (*c*) equal to, 4.33 lb. per sq. in. plus atmospheric.

9. For water flowing rather slowly through a glass tube the resistance to flow is: (*a*) independent of the velocity; (*b*) proportional to the velocity; (*c*) proportional to velocity squared.

Vocabulary: Hydrostatic potential energy, hydrostatic potential, velocity of efflux, Torricelli's Theorem, Bernoulli effect, viscous resistance, line drop, hydromotive force, coefficient of viscosity, Poiseuille's Law, Stokes' Law, turbulence, streamline.

PROBLEMS

1. How much work is done in filling a reservoir, capacity 1000 cu. ft., at an average height of 50 ft.?

2. How much work is done per cubic foot of water in the turbines at Boulder Dam? (Fig. 4–1.)

3. According to Fig. 10 the mean pressure differences through the two sides of the heart are 2 cm and 8 cm, respectively. About 60 cc of blood pass through the heart at each beat, and the normal pulse rate is about 75 per minute. Compute the work done by the heart per day and the power in watts.

4. An open tank 32 ft. tall is filled with water. There is an orifice halfway up the side. Where will the jet strike the ground?

5. Find the velocity of efflux at the spillway of Boulder Dam (Fig. 4–1). Where will the jet strike the river?

6. Find the velocity of efflux from the siphon, Fig. 5–7, if the outlet *c* is 1 ft. below the free surface.

7. Find the pressure at the crook in the siphon, Fig. 5–7, if this is 2 ft. above the water level and the water flows through the siphon with a speed of 8 ft. per sec.

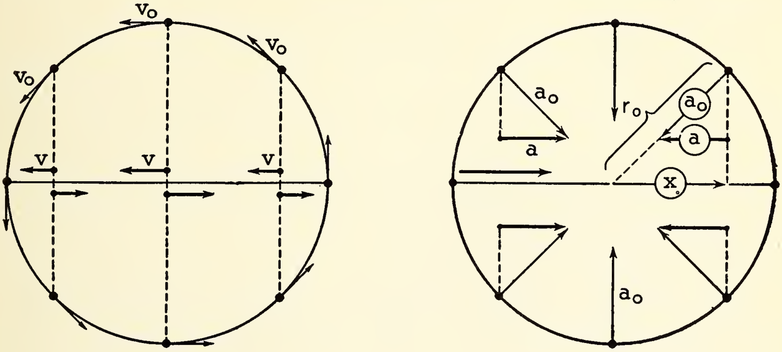
8. What is the pressure difference between points *a* and *b* of Fig. 6 if the speeds at *a* and *b* are 50 and 150 cm per sec., respectively?

9. Compare the Reynolds' Number for a body moving with the same speed in water and in air. Contrast a submarine and a dirigible balloon; imagine each to have the same shape but the submarine to be 10 times smaller (in each dimension) and to move 10 times slower; will the proportion of energy lost because of turbulence be greater or smaller for the submarine?

CHAPTER 15

SIMPLE HARMONIC MOTION

157. The Projection of Uniform Circular Motion. Let us suppose that a bob is traveling around uniformly in a vertical circle. The sun shines overhead, casting a shadow of the bob on the ground; this shadow moves back and forth in a straight line. The motion of the shadow is the projection of the circular motion. The student can picture to himself this motion. Although the bob is moving at a steady rate the speed of the shadow is constantly changing. It moves rapidly past the center; as it approaches the end of its path it moves more slowly, stops, reverses its motion, and returns. The velocity vector of the shadow (v) can be found as the projection of the velocity vector (v_0) of the circular motion (Fig. 1*a*). (For simplicity the projection of the circular motion can be represented on the diameter.)



(*a*) (*b*)

FIG. 1. Projection of uniform circular motion.

a. Projected velocity (v) greatest at center, zero at extremities.
b. Acceleration (a) proportional to displacement (x).

The acceleration is greatest at the extremities; at the center the velocity is momentarily constant, without acceleration. If we draw a vector representing the centripetal acceleration of the bob, then take the projection of this vector in the horizontal direction, we shall have the acceleration of the shadow. Suppose that a_0 represents the acceleration of the bob toward the center; then a represents the acceleration of the projected motion. It will be observed that this acceleration is directed toward the left whenever the displacement is to the right. And the acceleration is greater when the displacement is greater. Indeed we now shall show that the two are in direct proportion.

Let the displacement of the shadow from the center at some moment be represented by x, and the radius of the circle by r_0 (Fig. 1*b*). Then, since the a–a_0 triangle and the x–r_0 triangle are similar, we have

$$\frac{a}{a_0} = \frac{x}{r_0}$$

or

$$\frac{a}{x} = \frac{a_0}{r_0} \text{ (i.e., a constant).} \qquad (a)$$

That is, the ratio a/x is a constant; *the acceleration varies as the displacement.* (But observe that it is negative [←] when the displacement is positive [→].)

158. Simple Harmonic Motion. We have studied several types of motion. In the simplest type there was no acceleration (no force, constant speed). Next simplest was the motion with constant acceleration (constant force). Next we studied motion in a circle, where the acceleration was constant in magnitude (v^2/r) but varied in direction, being always directed toward the center. We now study motion in which the acceleration varies in magnitude. In **simple harmonic motion** (s.h.m.) the acceleration is proportional to the displacement and opposite to it in sign. This is the motion of a bob vibrating on a spring, of a pendulum as it oscillates to and fro; it is the projection of uniform circular motion just considered.

By Newton's second law acceleration is always proportional to force. And when an elastic body is distorted the restoring force is usually directly proportional to the displacement (Hooke's Law). So when a spring oscillates, when a tuning fork vibrates, when a hammer strikes a sudden blow on a plate, whenever one object strikes another and sets it vibrating, we have either simple harmonic motion or a vibration which is a combination of a number of simple harmonic motions of different frequencies.

Vibration is the commonest form of motion in the world around us, and simple harmonic motion is the simplest form of vibration.

Visualize the motion of the bob attached to the spring. The more the bob is displaced the harder the spring pulls back. We pull it down and let go. At first the restoring force is very great and the acceleration correspondingly large. As the bob nears its equilibrium position the force becomes less, and the acceleration is small. At just the moment it passes the equilibrium point there is no force and for the moment it is neither gaining nor losing speed. Then, as the bob passes beyond this point, there is an opposite force which slows it down; the force becomes greater and the retardation becomes greater with increasing displacement. The acceleration is proportional to the displacement, opposite in direction, and the motion is simple harmonic. In Fig. 2 the black disc represents the equilibrium position, the circles represent the limits of the oscillation.

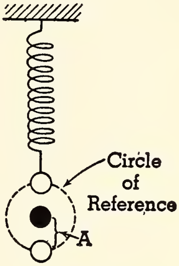


Fig. 2. Bob on a spring.

For purposes of computation it is often convenient to consider any simple harmonic motion as associated with uniform circular motion around an imaginary circle — this is called the *circle of reference.* In Fig. 2 the simple harmonic motion is projected from the circle of reference upon a vertical diameter.

159. Period, Frequency, Amplitude, Phase. The oscillating bob goes from one extremity to the other and back again. This makes a complete vibration. The time required for this we call the **period** (T); the number of complete vibrations per second is called the **frequency** (n). A spring oscillating 10 times per second has a frequency of 10 and a period of $\frac{1}{10}$ second. Evidently

$$T = \frac{1}{n}.$$

The greatest displacement from the center is called the **amplitude** of the motion. *Phase* is a term used to designate in what part of its vibration the body is. During a single period, the vibrator, starting at some given phase, passes through every other phase;

twice does its displacement become equal to the amplitude, and after one period it returns to the original phase. . . A man naturally swings his arms as he walks. What do you estimate as the period, frequency, and amplitude of this motion? Are the two arms in the same phase or in opposite phase?

The amplitude is evidently the radius of the circle of reference. The phase is usually represented by the angle in this circle. Thus two particles at opposite extremities of their motions would be said to differ in phase by 180°; there is a phase difference of 90° between a particle at an extremity and one passing through the equilibrium position.

160. General Expression for the Period. One noteworthy fact about simple harmonic motion is that the period does not depend upon the amplitude. (The bob in Fig. 2 has the same period whether it vibrates through 2 inches or 6 inches; the greater amplitude is accompanied by greater average speed.) We now show that the period depends simply upon the (constant) ratio a/x, that is, upon acceleration per unit displacement:

We refer again to Fig. 1*b* where associated with the simple harmonic motion we had motion in a circle. We found that

$$\frac{a}{x} = \frac{a_0}{r_0}$$

or, since we know that $a_0 = v_0^2/r_0$ (eq. 6), this is

$$\frac{a}{x} = \left(\frac{v_0}{r_0}\right)^2. \qquad (b)$$

Now, the period is simply the time required for the reference point to go about the circumference, and this is

$$T = \frac{2\pi r_0}{v_0}.$$

But from eq. (*b*) $(r_0/v_0)^2 = x/a$; using this value, we have

$$T = 2\pi\sqrt{\frac{x}{a}}. \qquad (12)$$

This is the fundamental equation in simple harmonic motion.

Example. When the prong of a tuning fork is deflected by 1 mm it has an acceleration of 1000 cm per sec.^{2}; find its period.

a/x = 10,000 cm per sec.^{2} per cm. Hence

$$T = 2\pi\sqrt{\frac{1}{10{,}000}} = 0.0628 \text{ sec.}$$

The frequency is 16 vibrations per second.

161. Special Cases of Simple Harmonic Motion. 1. Bob and Spring. The stiffness of a spring is usually given in terms of the **spring constant** k, which equals the force required to stretch the spring 1 cm. If a bob of mass m is attached to the spring the period of oscillation will be

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{12'}$$

Proof: Since the force in the spring equals kx we have $kx = ma$ or

$$\frac{x}{a} = \frac{m}{k}; \tag{c}$$

substituting this in (12) gives (12′).

Example. If it requires 100 dynes to stretch a spring 1 cm find x/a and the period when a gram weight is suspended from it.

162. Special Cases of Simple Harmonic Motion. 2. The Pendulum. A small bob attached to a string forms a *simple pendulum.* As it swings back and forth in a small arc under the influence of gravity it has very nearly simple harmonic motion. Its period is

$$T = 2\pi\sqrt{\frac{l}{g}}, \tag{12''}$$

where l is its length and g the acceleration of gravity.

Proof: With the pendulum as shown in Fig. 3 the force of gravity can be divided into two components: one non-effective, tending only to stretch the cord; one effective in tending to accelerate the bob. In the figure, g represents the full acceleration of gravity and a the effective component acting in the bob. In the two similar right triangles we see that

$$\frac{x'}{a} = \frac{l}{g}.$$

The actual displacement of the bob is x, measured along the curved path; but if the pendulum is displaced only a few degrees x will very nearly equal x'. We may then write

$$\frac{x}{a} = \frac{l}{g} \text{ (approx.).} \qquad (d)$$

Since l/g is a constant this proves the motion to be simple harmonic motion. Now substituting l/g for x/a in the fundamental equation gives eq. (12″). The relation evidently holds strictly only for small oscillations; however, for an amplitude of 10° the error is only about a fifth of one per cent.

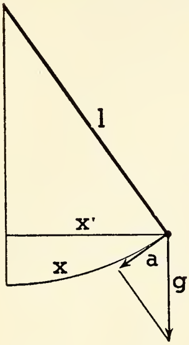


FIG. 3. Simple pendulum. $\frac{x'}{a} = \frac{l}{g}$.

Galileo was the first to investigate the motion of the pendulum; we are told that his first observations were made as he watched the swinging chandeliers in the cathedral, timing the oscillations by his pulse-beat. (This is the first wrist watch!) The pendulum later came to be used in clocks; today the balance wheel has largely displaced it.

Examples. Eq. (12″) shows that the period of a pendulum varies as the square root of the length: $T \propto \sqrt{l}$. A pendulum a meter long has a period of about 2 sec. What will be the period of a pendulum 4 meters long? How long is a pendulum with a period of 1 sec.? of $\frac{1}{2}$ sec.?

In this connection the student should recall the rule for finding the square root of numbers near unity (App. 2 [6]). $\sqrt{1.01} = 1.005$; thus if the length is increased by 1 per cent the period will be increased by $\frac{1}{2}$ per cent. $T \propto 1/\sqrt{g}$; therefore, if the value of g is increased from 980 to 981 (approximately 1 part in 1000), in what ratio will T change?

163. The Physical Pendulum. The ideal simple pendulum which we have been discussing has all the weight concentrated at a point and the suspension is weightless. No such pendulum exists, but the ordinary bob on a string can usually be treated as a simple pendulum. In the *physical pendulum* the weight is distributed. Suppose a meter stick to be suspended from one end; its period, of course, will not be that of a simple pendulum a meter long. The motion is not simple linear motion of a point but rather an angular oscillation of the whole stick about the center of support (angular harmonic motion). As a matter of fact, it can be shown that the period of a physical pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{mgh}}, \tag{12'''}$$

where m is its mass, h the distance of its center of gravity from the center of support, and I its *moment of inertia* (see Chapter 16). By comparison with eq. (12″) it appears that the physical pendulum is equivalent to a simple pendulum with length $l = I/(mh)$.

The most precise measurements of the gravitational constant (g) have been made by means of the physical pendulum. Several special types of pendulums have been devised for this purpose.

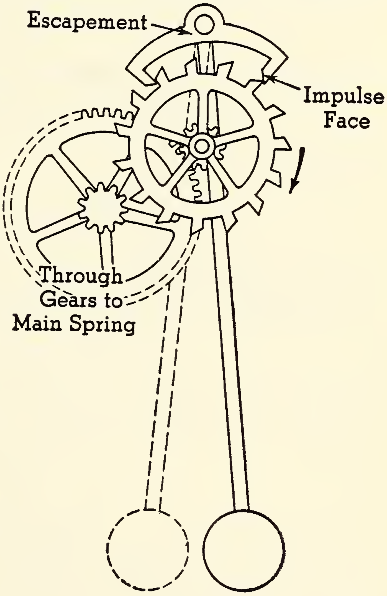


Fig. 4

164. Clocks. In ancient and medieval times, time was told by sundials, by sand trickling through hourglasses, by water dripping into a tank, by the burning away of a graduated candle. After A.D. 1000 crude mechanical clocks were occasionally devised, controlled by friction or by a spinning fanwheel. The clock in its modern form dates from the middle of the seventeenth century, when the Dutch physicist-astronomer-mathematician Huygens (§618) introduced the pendulum clock.

The clock is a machine, driven by "springpower." The mainspring, acting through the clockwork, furnishes the power to turn the hands; the pendulum, operating the escapement, releases this power at regular intervals. In Fig. 4 the mainspring is driving the toothed scapewheel in a clockwise direction. At each oscillation of the pendulum, the escapement bar, rocking back and forth, unlocks the scapewheel and allows it to advance by one tooth. But the escapement must serve one other purpose. Of itself the pendulum motion would gradually die out because of friction. Through the escapement, at each oscillation a little energy must be fed from the mainspring to the pendulum. As the tooth is released it slides against the inclined face (" impulse face ") of the escapement prong and so gives the pendulum a slight impulse.

Thus the escapement acts as a valve periodically releasing the flow of energy, the pendulum acts as the governor opening this valve at regular intervals, and the mainspring is the source of power, operating the clockwork and also at each swing feeding a little energy to the pendulum. In small clocks, in watches and chronometers, the balance wheel and hairspring take the place of the gravity pendulum. The balance wheel oscillates back and forth with angular harmonic motion, and this operates the escapement.

For good timekeeping the design of the escapement is very important. The impulse from the mainspring slightly disturbs the simple harmonic motion and more or less alters the period. The simple escapement which we have described was invented by Hooke.[1] There have been many modifications. Various compensations have been introduced — for variations in temperature, in orientation, in spring tension. Jeweled bearings have been introduced, gears improved, until the ordinary high-grade watch of today, properly regulated, may have a rate variation of only a few seconds a day.

165.* Maximum Displacement, Velocity, and Acceleration. We now wish to describe more explicitly the characteristics of simple harmonic motion. For definiteness we take a bob on a spring, for which

$$\frac{x}{a} = \frac{m}{k} \quad \text{and} \quad T = 2\pi\sqrt{\frac{m}{k}}.$$

The maximum displacement is the amplitude A. It is the same as the radius of the circle of reference:

$$x_{\text{max.}} = A \qquad (= r_o).$$

The greatest acceleration occurs at the extremity of the motion, when $x = A$ and by eq. (c)

$$a_{\text{max.}} = \frac{k}{m} A \qquad (= a_o).$$

It is the same as a_o. The greatest velocity occurs at the mid-point ($x = 0$); the velocity here ($v_{\text{max.}}$) equals the velocity of the reference point as it moves about the circle of reference (v_o). Substitute k/m for a/x and A for r_o in eq. (b) and we have

$$v_{\text{max.}} \ (= v_o) = \sqrt{\frac{k}{m}} A.$$

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[1] Robert Hooke made many discoveries, though it has been somewhat the fashion to deprecate his work. He invented the hairspring clock balance (incidentally discovering the law of elasticity). He proposed the use of the pendulum for measuring gravity and of the barometer in weather forecasting; he advocated the wave theory of light and attacked the problem of planetary motion — which Newton later solved. Also he made extensive studies with the microscope and was first to recognize the living cell.

166.* Vibrational Energy. When a spring is stretched a distance x, the force is kx, where k is the spring constant, and the average force applied in stretching it this distance is $\bar{F} = \frac{1}{2}\,kx$. The work done is the average force times the distance. Hence

$$\text{Potential energy} = \bar{F}s = \tfrac{1}{2}\,kx^2. \qquad (9')$$

The potential energy of a stretched spring is proportional to the square of the stretch.

In vibratory motion the energy is repeatedly changing from potential to kinetic and back again. At the extremity of motion, the energy is all potential, and (by 9′) equals $\frac{1}{2}\,kA^2$, where A is the amplitude. As the bob passes through the equilibrium position ($x = 0$) the energy is all kinetic ($\frac{1}{2}\,mv^2$).

Substituting here the value just found for $v_{\text{max.}}$ gives $\frac{1}{2}\,kA^2$ for the kinetic energy at the midpoint. This equals the potential energy at the extremity and is an example of conservation of energy.

167.* Displacement, Velocity, and Acceleration in Simple Harmonic Motion. In uniform motion $x = vt$, $v =$ constant, $a = 0$. In uniformly accelerated motion, starting from rest, $s = \frac{1}{2}\,at^2$, $v = at$, $a =$ constant. How shall we describe exactly the variation in displacement and velocity and acceleration in simple harmonic motion?

We have just found the maximum value of each of these quantities

$$r_0 = x_{\text{max.}} = A \text{ (amplitude)},$$

$$v_0 = v_{\text{max.}} = A\sqrt{\frac{k}{m}},$$

$$a_0 = a_{\text{max.}} = A \cdot \frac{k}{m}.$$

For the exact description of the motion it is necessary to refer again to the circle of reference. In Fig. 5 (left) the simple harmonic displacement, velocity, and acceleration have been represented as the projection of r_0, v_0, and a_0, this time for convenience upon a *vertical* axis. Examining the figure the student can show that for any "phase angle" θ

$$x = r_0 \cos\theta,$$

$$v = -\,v_0 \sin\theta,$$

$$a = -\,a_0 \cos\theta.$$

As this phase angle increases in the course of time the displacement and acceleration vary as its cosine, the velocity as its sine. These values are plotted in Fig. 5.

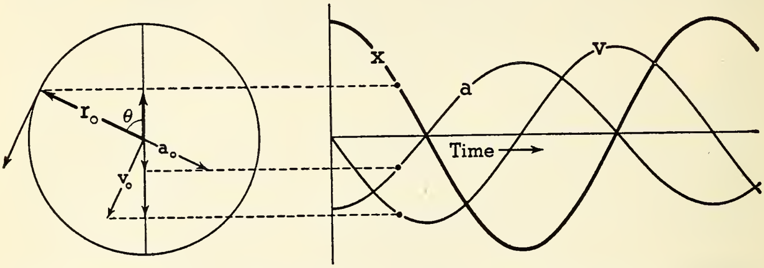

Fig. 5. Variation of displacement, velocity, and acceleration in simple harmonic motion. (Projection on vertical axis; compare Fig. 1.)

We now introduce the time into these equations instead of θ. θ is proportional to t. Since the phase angle changes 360° in T seconds we can write

$$\theta : t = 360^\circ : T,$$

or

$$\theta = \left(\frac{360^\circ}{T}\right) t.$$

Hence

$$x = r_o \cos \left(\frac{360^\circ}{T}\right) t,$$

$$v = -v_o \sin \left(\frac{360^\circ}{T}\right) t,$$

$$a = -a_o \cos \left(\frac{360^\circ}{T}\right) t.$$

These equations specify a motion not simply increasing as a power of the time but varying " sinusoidally," oscillating as the sine or cosine of a steadily increasing angle.

168. Résumé. In simple harmonic motion (s.h.m.) the acceleration is proportional to the displacement (x/a = constant). The motion can be considered as the projection of uniform circular motion (" circle of reference "), and this is useful in developing the important relations. The expression for the period is $T = 2\pi\sqrt{x/a}$.

The vibrating spring ($x/a = m/k$) and the simple pendulum ($x/a = l/g$) are important examples of simple harmonic motion.

QUESTIONS

1. What are the amplitude and period of motion in a grandfather's clock?

2. When do x, v, a equal r_0 (radius of the circle of reference), v_0, a_0, respectively?

3. Suppose that circular motion is projected upon a vertical instead of a horizontal diameter. Does the resultant simple harmonic motion differ in period, amplitude, or phase?

4. You have a spring balance but not a watch. How can you determine the period of a bob on a spring?

5. What happens to the frequency of a bob on a spring if the mass of the bob, the stiffness of the spring, and the amplitude of vibration are all doubled?

6. What are the period and frequency of pendulums 1, 2, 4 meters long, respectively?

7. Any vibratory motion is simple harmonic motion: (*a*) always; (*b*) provided that there is no friction; (*c*) if the restoring force is proportional to the displacement (Hooke's Law).

8. If the length of a pendulum is changed from 100 to 99 in. the period will be decreased by approximately: (*a*) $\frac{1}{2}$ per cent; (*b*) 1 per cent; (*c*) 2 per cent.

Vocabulary: Simple harmonic motion, circle of reference, period, frequency, amplitude, phase, spring constant, simple (physical) pendulum, phase angle, sinusoidal oscillation.

PROBLEMS

1. A bob oscillates with simple harmonic motion, with a period of $3\frac{1}{7}$ sec. The path (from end to end) is 20 cm. What is the circumference of the circle of reference? What is the velocity (v_0) of the point of reference? What is the maximum speed of the bob?

2. Find the acceleration of the bob in problem 1: (*a*) when 1 cm from the center; (*b*) at the extremity of its path.

3. If the bob in problem 2 has a mass of 600 grams what is the force of the spring at its extremity? What is the spring constant?

4. Imagine a man to pace back and forth across a 20-ft. room with simple harmonic motion and with a period of $12\frac{4}{7}$ sec. What is his acceleration (*a*) at the wall; (*b*) 6 ft. from the wall; (*c*) 10 ft. from the wall?

5. A kilogram weight hung on a certain spring stretches it 9.8 cm. What is the spring constant? If this bob is set into vibration what will be its frequency of vibration?

6. It takes a load of 200 grams to stretch a rubber band a centimeter. Find the period of vibration of a kilogram weight suspended from it.

7. If a spring stretches 1 ft. per lb., what will be its period with a 2-lb. weight attached?

8. It takes 25 lb. to depress a springboard 1 in. What is the period of oscillation when a 150-lb. man stands on it?

9. A weight hangs from a rubber band. We raise the weight until the band is just slack and then drop it. It oscillates through a range of 12 cm. What is its period? (*Hint:* What is its initial acceleration?)

10. Assuming the motion to be practically simple harmonic, what is the maximum acceleration of the piston of an automobile engine making 3600 revolutions per minute with a 4-in. stroke? If the piston weighs 2 lb. what is the accelerating force (**F**) at the end of the stroke?

11. What is the period of oscillation of a swing 8 ft. long?

12. Find the exact length of a simple pendulum with a period of 2 sec.

13. Suppose that a weight were suspended from the tower of the Empire State Building (1248 ft. high). It is displaced 10 ft. to one side and released. What is its period of vibration? Show that its greatest acceleration is about 3 in. per sec.^{2}

14. Compare the period of a pendulum on the surface of the earth, the surface of the moon, and the surface of the sun. (g on the moon is $\frac{1}{6}$, on the sun 28 times, the value on the earth.)

15. (*a*) Show that when the length of a pendulum is increased by 2 per cent its period increases by 1 per cent. (*b*) How will the period of a pendulum change if the acceleration of gravity is increased from 978 cm per sec.^{2} at the equator to 983 at the poles?

16. If a pendulum is adjusted to have a frequency of 43,200 vibrations per day at New Orleans, how many more will it have in St. Louis? (Table 15, Chapter 17.)

17. Find v_0 and a_0 for a 2-sec. pendulum swinging with a 3-in. amplitude.

18. The weight in problem 5 is pulled down 4 cm and released. Find its position and velocity 0.1 sec. later.

19. By means of the sagitta formula (§720) prove that the potential energy of a pendulum is proportional to the square of the displacement.

CHAPTER 16

ANGULAR MOTION

169. Angular Displacement, Angular Velocity, Angular Acceleration. Hitherto we have been speaking chiefly of linear displacement — bodies have moved as a whole and the distances have been measured in feet (or centimeters), velocities in feet (or centimeters) per second, accelerations in feet (or centimeters) per second per second. Now, when a body rotates, its motion as a whole cannot be described in these terms, for various parts of it move with different speeds — the speed of a rotating disc increases as we go farther and farther out from the axis; on the axis itself the velocity is zero. It is appropriate to describe such motion in terms of the angle turned through (angular displacement [θ]) and the angular velocity, and the angular acceleration. A wheel turning regularly, a ball rolling with constant speed, the earth spinning on its axis have constant angular velocity. When the wheel begins to turn or slows down, its angular velocity changes; it then has angular acceleration. **Angular velocity** (ω) is the angle turned through per second[1]

$$\omega = \frac{\theta}{t}. \qquad \text{(Def.)}$$

Angular acceleration (α) is the change in angular velocity per second

$$\alpha = \frac{\Delta\omega}{t}. \qquad \text{(Def.)}$$

(Compare these with the definitions of [linear] velocity and acceleration.)

The angle (θ) may be measured in degrees, the angular velocity (ω) in degrees per second, and angular acceleration (α) in degrees per second per second. (What is the angular velocity of a merry-go-round which turns around once in 10 seconds?) Or of course the angle might be measured in revolutions, or in minutes (of arc). However, the angle unit commonly used in physics is the radian.

---

[1] These Greek letters customarily used for angular quantities are theta (θ), omega (ω), and alpha (α).

170. The Radian. If on the circumference of a circle we measure off a distance equal to the radius, the angle subtended by this arc at the center is called a *radian*. The radian is about 57°. More exactly: Since the radius will go into the circumference 2π times, the radian is $360°/2\pi$. Approximately how many radians in 1°, 10°, 120°? How many degrees are 2π radians, $\pi/2$ radians?

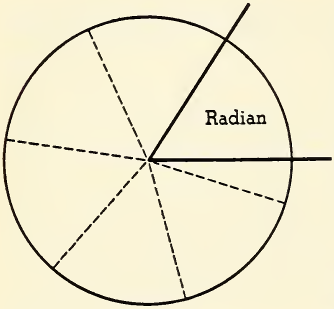

FIG. 1

171. Relation between Linear and Angular Displacement, Velocity, and Acceleration. Why use this particular unit? Because of the simplicity which it affords in changing from linear measure to angular measure. The angle (in radians) is exactly equal to the distance (s_c) (measured along the circumference) divided by the radius.

$$\theta = \frac{s_c}{r}. \tag{13a}$$

Similarly the angular velocity (in radians per second) equals the linear velocity along the circumference divided by r; the angular acceleration (in radians per second per second) equals the acceleration *along the circumference* divided by r.

$$\omega = \frac{v_c}{r}\left(\text{for } \omega = \frac{\theta}{t} = \frac{s_c}{rt} = \frac{v_c}{r}\right) \tag{13b}$$

$$\alpha = \frac{a_c}{r}\left(\text{for } \alpha = \frac{\Delta\omega}{t} = \frac{\Delta v_c}{rt} = \frac{a_c}{r}\right). \tag{13c}$$

Examples. If a horse on a merry-go-round is 20 ft. from the center and is 5 ft. long, what angle does it subtend? If it travels 10 ft. per sec. what is the angular velocity of the merry-go-round? If 5 sec. later its speed is increased to 40 ft. per sec. what is the angular acceleration?

Through what angle does a bicycle wheel turn when the bicycle goes 50 ft. (radius 1 ft.)?

Or conversely, the angular displacement being known, the linear displacement follows at once

$$s_c = \theta r.$$

Example. A barrel (radius 15 in.) rolls through an angle of 20 radians in 5 sec. How far does it move? What are its angular velocity and linear velocity? How many revolutions in 20 radians?

172. Correspondence between Linear and Angular Quantities. All the principles of mechanics which hold for linear motion can be taken over to angular motion by a simple translation of the linear quantities into corresponding angular quantities. The correspondence is shown in Table 13. It is to be understood that the corresponding quantities are not *equal* (for s is not equal to θ), but θ, ω, $\mathcal{T}$, I, etc., play the same role in angular motion as do s, v, f, m, etc., in linear motion. We shall now give several examples of this correspondence.

TABLE 13

CORRESPONDING QUANTITIES IN LINEAR AND ANGULAR MOTION

Linear	Angular	Relation
Distance (s)	Angle (θ)	$\theta = \frac{s_c}{r}$
Velocity (v)	Angular velocity (ω)	$\omega = \frac{v_c}{r}$
Acceleration (a)	Angular acceleration (α)	$\alpha = \frac{a_c}{r}$
Force (F)	Torque ($\mathcal{T}$)	$\mathcal{T} = F_c r$
Mass (m)	Moment of inertia (I)	$I = \Sigma m r^2$
Momentum (mv)	Angular momentum ($I\omega$)	$I\omega = m v_c r$

TABLE 14

LAWS OF ANGULAR MOTION

Constant angular velocity	$\theta = \omega t$
Constant angular acceleration	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
Equilibrium	$\Sigma \mathcal{T} = 0$
Newton's second law	$\mathcal{T} = \frac{\Delta(I\omega)}{t}$
Newton's third law	$\mathcal{T}$(action) $= -\mathcal{T}$(reaction)
Hooke's Law	Torque is proportional to twist
Transverse angular acceleration	$\omega_{\text{prec.}} = \frac{\mathcal{T}}{I\omega}$

173. Uniform and Accelerated Motion. If a wheel is spinning steadily with a speed of 3 radians per second (or 3° per second) it will evidently turn through 12 radians (or 12°) in 4 seconds. When there is no acceleration

$$\theta = \omega t.$$

Examples of such motion are the earth spinning on its axis, a grindstone turned steadily, a ball rolling uniformly.

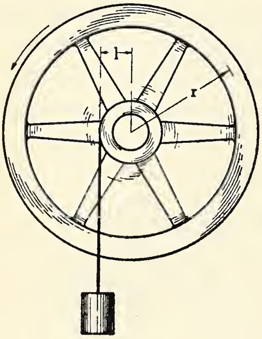

Fig. 2

If a weight is suspended from a wheel as in Fig. 2, the wheel will gradually be set in rotation. Suppose that after 1 second it is turning 1 radian per second. After another second the angular speed will be 2 radians per second; in 3 seconds, 3 radians per second, and so on.

$$\omega = \alpha t.$$

During the first second the *average* angular speed is $\frac{1}{2}$ radian per second and hence the wheel turns through $\frac{1}{2}$ radian (how many degrees)? During 2 seconds the average speed is 1 radian per second; during 3, $1\frac{1}{2}$ radians per second. How far does the wheel turn in these times? In this way we can derive the expression for the angular displacement of a wheel (starting from rest) just as we did for the displacement in linear motion.

$$\theta = \tfrac{1}{2}\,\alpha t^2.$$

Or if the wheel has an initial rotational velocity ω_0, we have

$$\theta = \omega_0 t + \tfrac{1}{2}\,\alpha t^2.$$

(Compare with eq. 3*b*, §92.)

174. Torque and Acceleration. Just as a force produces linear acceleration so a torque produces angular acceleration.

The falling weight in Fig. 2, suspended a distance l from the axis, exerts a torque ($\mathcal{T} = Fl$) on the wheel, and the acceleration is proportional to this. Corresponding to the linear equation ($F = ma$) there is for angular motion the equation[1]

$$\mathcal{T} = I\alpha,$$

---

[1] This equation holds only for constant I (§179).

where I represents the rotational inertia, opposing a change of angular velocity, or, as it is generally called, the *moment of inertia*.

175. Moment of Inertia. Any number of everyday experiences show that this rotational inertia depends not only upon the mass of the body but upon the distribution of that mass as well. In the heavy flywheel of an engine, used to steady the motion, the mass is always concentrated near the rim. It is much easier suddenly to start a meter stick twirling rapidly about an axis along its length than about a perpendicular axis. The hammer thrower cannot whirl his hammer around as rapidly as we can spin an equally heavy ball on its own axis. Actually *each particle of matter in a body* (mass m, distance from axis r) *contributes to the moment of inertia an amount* mr^2. The moment of inertia (I) of the body is the sum of these contributions of its individual elements:

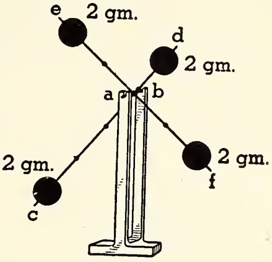

FIG. 3

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \cdots, \text{ etc.} \qquad (14)$$

(This relation will be derived presently.)

For example, in Fig. 3 the moment of inertia of the four bobs about the axis *ab* is $2 \times 1^2 + 2 \times 3^2 + 4 \times 2^2 = 36$ gm-cm.^{2} In a ring, where all the mass can be considered at a distance r from the center, the moment of inertia is simply Mr^2 where M is the total mass of the ring. (This is with respect to an axis through the center perpendicular to the plane of the ring.) In most cases the moment of inertia is less easily computed (requiring calculus), but its value for several simple figures is shown in Fig. 4.

Notice that the value of the moment of inertia depends upon the position and direction of the axis. In Fig. 3 I was computed about an axis perpendicular to the plane of the particles. If *cd* were chosen as the axis of rotation the moment of inertia would have been only $4 \times 2^2 = 16$. If the framework were remounted to spin about this axis its rotational inertia would be decreased. Notice this effect in the ring and the disc shown in Fig. 4.

Examples. What is the moment of inertia of a meter stick about one end (mass 100 grams); of a marble (mass 10 grams, diameter 2 cm); of a nickel as it rolls (mass 5 grams, radius 2 cm); of a nickel as it spins?

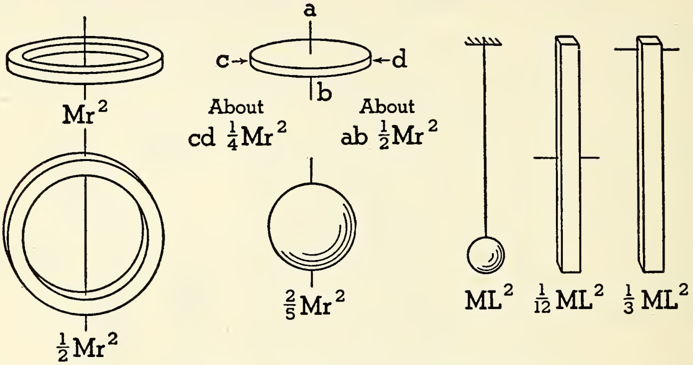

FIG. 4. Moment of inertia for simple figures.

176. Angular Momentum. Analogous to linear momentum (mv) we have angular momentum ($I\omega$). An example is a heavy flywheel with angular velocity of many radians per second. Unless the body is acted on by a torque this angular momentum remains constant. The rate of change of angular momentum equals the torque applied.

$$\mathcal{T} = \frac{\Delta(I\omega)}{t}.$$

For the flywheel the moment of inertia stays constant. Hence we have (analogous to eq. [4′])

$$\mathcal{T} = I\frac{\Delta\omega}{t} = I\alpha.$$

Example. Torque and acceleration. Returning to Fig. 2 let us suppose the radius of the wheel to be 2 ft. and its weight 16 lb.; suppose that the suspended weight is 24 lb. and the radius of the drum 4 in. We shall find the angular acceleration. Consider the wheel as a simple ring.

$$I = Mr^2 = \tfrac{1}{2}\text{ slug} \times (2\text{ ft.})^2 = 2\text{ units},$$

$$\mathcal{T} = F \times l = 24\text{ lb.} \times \tfrac{1}{3}\text{ ft.} = 8\text{ units.}$$

$$\mathcal{T} = I\alpha;\ \alpha = \frac{\mathcal{T}}{I} = 4\text{ radians per sec.}^2$$

177. Newton's Laws for Angular Motion. *First Law. A body not acted upon by a torque continues in its state of rest or if spinning continues to spin with constant angular momentum.* This is the law of equilibrium; in Chapter 8 it was shown that for equilibrium the sum of the torques must be zero. But (as we know) equilibrium does not necessarily mean rest. A rotating wheel would continue to rotate forever except for the friction of the axle which furnishes a couple tending to stop it. The earth spinning on its axis furnishes a good example of a body not acted on by torques (neglecting the effect of tides).

Second Law. The rate of change of angular momentum equals the applied torque. That is,

$$\mathcal{T} = \frac{\Delta(I\omega)}{t},$$

or if I is constant

$$\mathcal{T} = I\frac{\Delta\omega}{t} = I\alpha.$$

This equation has just been discussed.

The change in angular momentum of course occurs about the axis of torque. If the body is spinning about one axis and the torque is applied about a perpendicular axis, there is a change, not in the *magnitude* of the angular velocity, but only in the *direction* of its axis; the results are rather surprising (§185).

Third Law. Corresponding to a torque applied to one body (action) there is always an equal, opposite torque on another (reaction). Angular action equals angular reaction. If body A twists B clockwise, B twists A counterclockwise. When I try to twist a post one way, I must brace my feet, for the post tends to twist me oppositely. . . The airplane engine turns the propeller clockwise; the airplane is pushed counterclockwise. This torque must be compensated by the air thrust against the wings — with twin engines the propellers always turn oppositely and so avoid this torque. . . Can a man, kneeling in a rowboat and holding to the seat, rotate the boat by exerting a torque on the seat — obviously not. A clockwise torque on the seat means a counterclockwise torque on the man, and this he passes on through his knees to the boat. However, if the man simply turns to the right in a rowboat, the rowboat turns to the left.

The angular momentum of the man and of the boat will be equal and opposite

$$I_{\text{man}}\,\omega_{\text{man}} = -I_{\text{boat}}\,\omega_{\text{boat}}.$$

178. Conservation of Angular Momentum. Since angular momentum is changed by torques (second law) and since a torque acting on one body has its reaction on another (third law) it is not possible to change the angular momentum of the universe as a whole. *Indeed, in any complete system* (i.e., a system not acted upon by torques from the outside), *the total angular momentum stays constant.* We add another to the great conservation laws which underlie all nature.

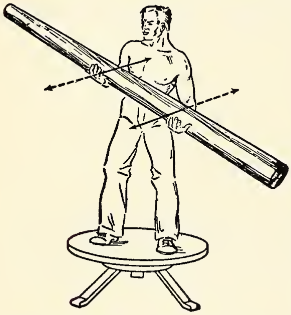

FIG. 5. Action and reaction (broken lines reactive couple).

We have seen how opposite angular momenta were given to man and boat. Imagine several persons on a large turntable with frictionless bearings. Every time one of them turned clockwise the turntable would turn slowly counterclockwise; if someone spun a top, the table would turn very slowly in the opposite direction. The angular velocities of top and turntable would vary inversely as their moments of inertia — the *angular momenta* of top and turntable would be just equal but opposite. Undoubtedly every time we turn or walk or spin a top or roll a ball on the earth, the earth turns[1] (with the same angular momentum, extremely small angular velocity) in the opposite direction.

179. When the Moment of Inertia Varies. By bringing mass closer to the center we change the moment of inertia. If the angular momentum is to be conserved there must be increased angular velocity. A man is standing on a small turntable. He and the table are rotating; the angular motion slows down as he holds his hands out, speeds up as he brings them in. . . An acrobat, turning a somersault, gathers his body together to get increased angular speed. . . The merry-go-round tends to speed up as a man walks to the center. . . We have all noticed the water in the wash basin whirling as it is drained; there was originally a slow rotation of the water, and this is increased as the water is drawn toward the center; the decreased moment of inertia and increased angular velocity tend to preserve the original angular momentum. . . Tornadoes are caused in this way, the air whirling faster as it rushes in. . . The planets move around the sun sometimes closer, sometimes further away, and their speed varies as their distance from the sun varies. When the planet is closer its moment of inertia (relative to the sun) becomes smaller and the angular velocity increases.

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[1] More accurately, *changes* its rotation, for it is already spinning.

In all cases, for a complete system, $I\omega$ remains constant.

180. Cyclones and Anticyclones. If, in the northern hemisphere, a large basin (for example, a swimming pool) is drained, there should toward the end develop a counterclockwise swirl. This assumes that the water was originally at rest in the basin. Basin and water together rotate with the earth; as water converges toward the drain its angular velocity increases. If the drain pipe is small, the outflow can have no appreciable angular momentum and the total angular momentum is preserved in the basin. As the residual water becomes less and less its angular velocity becomes greater and greater.

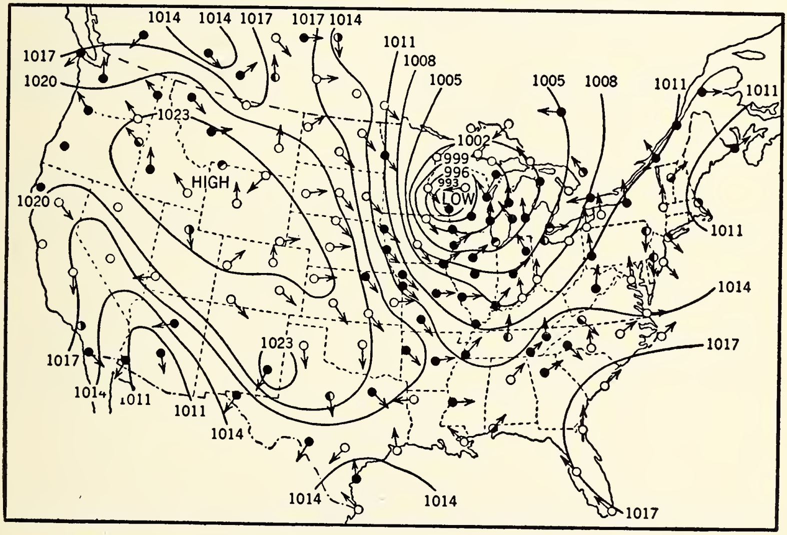


Fig. 6. Weather map showing cyclonic distribution of winds. Barometric pressures (adjusted to sea-level values) are represented by constant-pressure lines (isobars) and are given in kilobaryes. ○ clear; ● rain.

I do not know that this effect has been observed. But a somewhat similar phenomenon, on a much larger scale, occurs in the atmosphere. If we look at a weather map, showing barometric pressures and directions of the wind, we shall in general find the wind moving in toward regions of low pressure. But not only that; we shall in general find the wind moving as a vast whirl about the low-pressure center, moving counterclockwise in the northern hemisphere. This general counterclockwise circulation of air, extending over hundreds or perhaps thousands of miles, is called a cyclone. (This is not to be confused with the smaller, destructive tornado.) The atmosphere is rotating with the earth; as the air is sucked in toward the barometric low, its velocity increases, and the result is this giant whirl of winds, counterclockwise in the northern, clockwise in the southern, hemisphere.

About a barometric high conditions are reversed; the air flow is outward and (in the northern hemisphere) clockwise (*anticyclone*).

The cyclone and anticyclone are examples of an important law in meteorology: If (in the northern hemisphere) one stands with his back to the wind he will have higher pressures to his right, lower to his left (Ballot's Law). This accounts for much of the general atmospheric circulation of the earth. In the winter barometric highs (anticyclone centers) develop in North America, and lows in the north Pacific and north Atlantic. In Asia a low develops in the summer and an intense high in the winter, and the cyclone and anticyclone result in the western and eastern *monsoon* winds of southeastern Asia.

181. Work and Power.

If a torque $\mathcal{T}$ pushes a wheel through an angle θ the work is (§137)

$$W = \mathcal{T} \cdot \theta. \qquad (7'')$$

This follows at once, for (using the relations in Table 13)

$$W = F \cdot s = \left(\frac{\mathcal{T}}{r}\right) \cdot \theta r = \mathcal{T} \cdot \theta.$$

Similarly for power

$$P = \frac{W}{t} = \mathcal{T} \cdot \frac{\theta}{t} = \mathcal{T} \cdot \omega.$$

The power developed by a motor equals the torque exerted times the angular velocity in radians per second.

Example. An electric motor turns 1800 revolutions per minute and furnishes a torque of 7 lb-ft. What is the power? $\omega = 30$ revolutions per second or 190 radians per second.

$$P = 190 \times 7 = 1300 \text{ ft-lb. per sec.} = 2.4 \text{ hp.}$$

This motor does $2\pi \times 7 = 44$ ft-lb. of work during each revolution

$$(\theta = 2\pi, \quad \mathcal{T} = 7).$$

182. Derivation of Expression for Moment of Inertia. Figure 7 represents a body of any shape, rotating about an axis through *c* with angular velocity ω. Consider a small mass (m_1). Its velocity is $r_1\omega$, and hence its kinetic energy is $\frac{1}{2} m_1(r_1\omega)^2$. For mass m_2 the kinetic energy is $\frac{1}{2} m_2r_2^2\omega^2$; and similarly for any other particle. Hence for the whole body the kinetic energy is

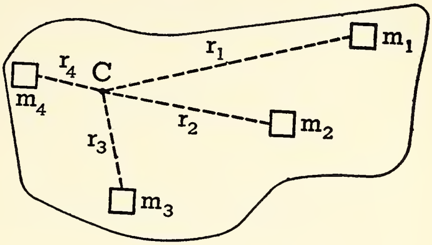


FIG. 7. $I = \Sigma mr^2$.

$$\begin{aligned}\text{K.e.} &= \tfrac{1}{2} m_1r_1^2\omega^2 + \tfrac{1}{2} m_2r_2^2\omega^2 + \tfrac{1}{2} m_3r_3^2\omega^2 + \cdots \\ &= \tfrac{1}{2} (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \cdots)\, \omega^2.\end{aligned}$$

If now we let the expression in parenthesis (which is supposed extended to all the particles in the body) be represented by I we have

$$\text{K.e.} = \tfrac{1}{2} I\omega^2.$$

We call I (which takes the place corresponding to mass in this expression for kinetic energy as in all other expressions in angular motion) the **moment of inertia.**

Moment of inertia $= m_1r_1^2 + m_2r_2^2 + \cdots$, summed over the whole body. (14)

183. Energy of Rotation and Translation Combined. The kinetic energy of a rotating body is $\frac{1}{2} I\omega^2$. When a body has both translation and rotation (as a rolling ball or hoop) the total kinetic energy consists of two parts: (1) due to translation of the center of mass ($\frac{1}{2} mv^2$), and (2) due to rotation about the center of mass ($\frac{1}{2} I\omega^2$).

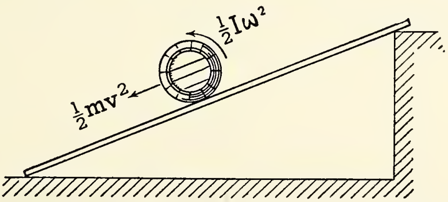


FIG. 8. Energy of translation and rotation.

Example. A hoop 2 ft. in radius is on a platform 16 ft. above the ground. Suppose (*a*) that it slides down a frictionless inclined plane, (*b*) that it rolls down the inclined plane. Compare the velocities as it reaches the ground.

(*a*) Suppose that the mass of the hoop is M slugs. Potential energy at top (Mgh) equals kinetic energy at bottom ($\frac{1}{2}Mv^2$).

$$\begin{aligned} \text{P.e.} &= \text{K.e.} \\ M \times 32 \times 16 &= \tfrac{1}{2} Mv^2. \\ 32 \times 32 &= v^2. \\ v &= 32 \text{ ft. per sec.} \end{aligned}$$

(*b*) The moment of inertia (Mr^2 for hoop) equals $4\,M$. The angular velocity (ω) equals v/r. Hence

$$\text{K.e.} = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} I\omega^2 = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} \times 4\,M \times \frac{v^2}{4} = Mv^2.$$

$$\text{P.e. at top} = Mgh = M \times 32 \times 16 \text{ (as before).}$$

$$\text{P.e.} = \text{K.e.}$$

Hence $M \times 32 \times 16 = Mv^2.$

$$v = \sqrt{512} = 22.6 \text{ ft. per sec.}$$

The kinetic energy at the bottom is the same in either case, but in one case the motion is entirely translation, in the other translation and rotation combined.

184.* Angular Harmonic Motion. When a rod or a spring is twisted the twisting torque is proportional to the angle of twist (Hooke's Law, §63).

Suppose that a disc, with moment of inertia I, is suspended from a vertical wire. If the disc is turned through a certain angle and released, it will rotate back and forth in angular harmonic motion. This is a *torsion pendulum* (another example is the balance wheel of a watch). The angular velocity is greatest in the equilibrium position; the angular acceleration is proportional to the angular displacement from this position. And just as in linear simple harmonic motion we have for the period $T = 2\pi\sqrt{x/a}$, so for angular simple harmonic motion

$$T = 2\pi\sqrt{\frac{\theta}{\alpha}}.$$

It is often convenient to express the period in terms of the moment of inertia and the torsion constant. The torsion constant (κ) is the restoring torque per unit angular displacement ($\mathcal{T}/\theta$). Multiplying under the radical both numerator and denominator by I we have

$$T = 2\pi\sqrt{\frac{I\theta}{\mathcal{T}}}$$

(since $\mathfrak{T} = I\alpha$) or

$$T = 2\pi\sqrt{\frac{I}{\kappa}}. \quad \text{(Compare eq. 12'.)} \tag{12''''}$$

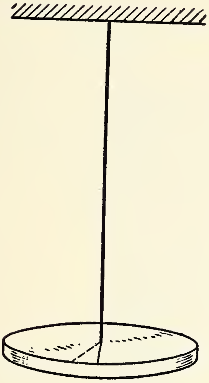


FIG. 9. Torsion pendulum.

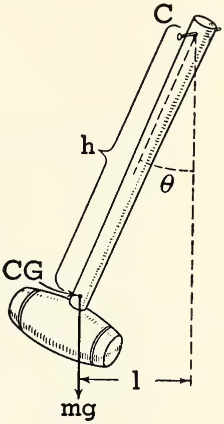
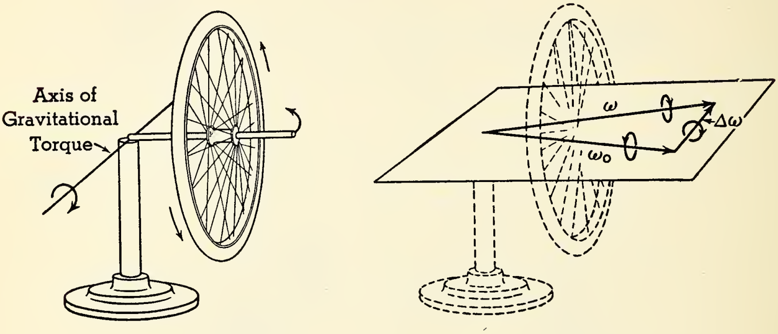


FIG. 10. Physical pendulum.

The physical pendulum (Chapter 15) is another example of angular simple harmonic motion. When the pendulum is displaced by an angle θ the torque is given by mgl. If the angle is small, l is practically the arc of the circle, that is, $h\theta$, where h is the distance from center of support to center of gravity. Therefore $\mathfrak{T} = mgh\theta$ and $\kappa = mgh$. Substituting this in the above equation gives $T = 2\pi\sqrt{\frac{I}{mgh}}$, which is the equation given in Chapter 15.

185. The Gyroscope. We have all seen the spinning top. Why does it not fall? Probably we have amused ourselves with the gyroscope and wondered at the contrary way in which it reacts to forces applied to it. If we push it with our finger, instead of yielding it turns sideways; instead of falling under gravity it "precesses" about in a horizontal plane (Fig. 11). Larger gyroscopes — rapidly turning flywheels, delicately balanced — are widely used as ship compasses, in piloting airplanes, in directing torpedoes, in eliminating the roll in ocean liners.

The angular momentum about an axis in any direction can neither increase nor decrease unless a torque is applied about that axis. Therefore the gyroscope in Fig. 11 cannot fall and spin about a vertical axis for there is no torque tending to create rotation about this axis. The down pull of gravity and the up thrust of the pedestal give a torque tending to change the angular velocity as shown in Fig. 11*b*. The figure shows how the initial rotation plus this added rotation gives rotation about a new axis in the horizontal plane. (We have not studied the composition of angular motions. Each rotation can be represented as an arrow along the axis, pointed so that the rotation is *clockwise* as viewed in the direction of the arrow. The figure shows how the resultant is found by vector addition.)



(*a*) (*b*)

FIG. 11. Precession of rotating wheel under gravity.

186. Gyrocompass and Gyrostabilizer. The magnetic compass becomes unreliable in a steel vessel, and it has been widely replaced by the gyroscopic compass. This is an electrically driven gyroscope so mounted that its axis of rotation is constrained to lie in a horizontal plane. Consider the effect at the equator. Here a horizontal plane is turning over from west to east 360° in the course of a day, 15° an hour. This causes a precession of the gyroscopic axis toward the north. Whatever its original direction of rotation may have been, in a few hours the gyroscope has turned until its axis is parallel with the earth's axis and henceforth it remains there. It gives true north; not (like the ordinary compass) magnetic north. The effect can be shown on the lecture table by gradually turning the gimbals of a small gyroscope from west to east. This is the fundamental principle of the gyrocompass, but there were many other problems to be solved by physicist and engineer — eliminating the effects of friction and of ship roll, correcting for ship velocity and latitude — before it could be used as a practical instrument.

The gyrostabilizer is used to suppress the roll in a ship. The gyroscope may be a 100-ton flywheel turning 15 times a second. Its axis is normally vertical, but by means of an electric motor it can be tipped toward the bow or stern. This is the brawn of the stabilizer; the brain is a smaller gyroscope, which, spinning in a fixed direction, detects any incipient roll of the ship to right or left and closes electrical contacts which causes the motor to tip the stabilizing gyro. As the ship rocks to the right (clockwise) the gyro is tipped (say) forward, so that its rotation (relative to the axis of the ship) is clockwise; when the ship rocks counterclockwise the gyro is tipped back and rotates counterclockwise. So the gyroscope absorbs the angular momentum of the ship. A large ship does not rock like a cork with every wave; it rocks more or less like a pendulum, with a period depending on the dimensions of the boat; the roll is gradually built up. It is because of this, because the waves act upon the ship more or less at random and because the gyro works not at random but always exactly against the motion, that it can, despite its small size (relative to the boat), eliminate perhaps 80 per cent of the roll. (This is exactly the converse of the effect of *resonance* [Chapter 26].)

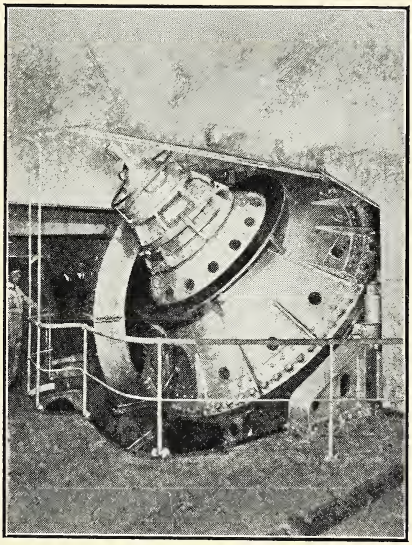


FIG. 12. Gyroscopic stabilizer in ocean liner.

187.* Velocity of Precession. In discussing the gyroscope we have unknowingly completed the survey of the analogies between angular and linear motion. This gyroscopic precession is the analogue of central acceleration. We have for linear motion $F = mv^2/r$ or, substituting ω for v/r, we have $F = mv\omega$ or

$$\omega = \frac{\text{Force}}{\text{Momentum}} \quad \text{(translation with changing direction).}$$

When a force (F) acts perpendicularly to the motion (v), the velocity neither increases nor decreases but the direction of the velocity changes, and the body has an angular velocity ω. Similarly when a torque ($\mathcal{T}$) acts perpendicularly to the angular velocity (ω) of a rotating body (gyroscope) this angular velocity neither increases nor decreases, but the axis of the rotation precesses. And the angular velocity of the precession ($\omega_{\text{prec.}}$) is given by

$$\omega_{\text{prec.}} = \frac{\text{Torque}}{\text{Angular momentum}} \quad \text{(rotation with changing axis).}$$

188.* Proofs of the Rotation of the Earth. There is no way to detect whether the earth and the whole solar system is at rest or moving with constant velocity through space. More stars are approaching than receding from a certain direction in the heavens, and hence it is sometimes said that the solar system is moving in that direction. But it may equally well be said that we are at rest and the stars moving, for all physical phenomena behave the same in a system moving with constant velocity as in one at rest. Indeed, velocity of a body (like position or its size) is purely a relative matter and has a meaning only in its relation to other bodies. This fact became the basis of Einstein's Theory of Relativity (§99).

The case is quite different for changing velocity, such as the velocity in a circle. It can be shown definitely (if the fundamental principles of Newtonian mechanics are accepted) that the earth rotates on its axis. The most celebrated demonstration was by Foucault in the Pantheon in Paris. He mounted in the dome a long pendulum with a heavy bob. During the course of a day the direction of oscillation appeared to change, the direction precessing slowly in a clockwise direction. Actually the earth was turning in a counterclockwise direction while the direction of the pendular oscillation remained unchanged.[1] . . A gyroscope, mounted freely in gimbals, preserves its axis fixed in space while the earth rotates and so betrays its motion. (The gyroscope was invented by Foucault.) The gyroscopic compass, precessing until its axis parallels the axis of the earth, demonstrates the rotation not quite so directly. . . The eastward velocity of the earth's surface decreases as we go to greater latitudes; hence a long-range projectile fired toward the poles drifts to the east (relative to the moving earth). The same general effect is observed in winds (§180). . . Not the least interesting demonstration was one by Michelson in which it was shown that, when a beam of light was caused to travel around a large rectangular area on the earth's surface (reflected by mirrors at the corners), the time required was longer when it traveled in the direction of the earth's rotation than in the opposite direction. (It would take a beam of light about a half millionth of a second longer to go about the earth in the direction of its rotation than against it.)

QUESTIONS

1. What angular quantity is analogous to distance, acceleration, force, mass?

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[1] Strictly speaking, this is the way a pendulum would act if mounted at the north pole. It would precess through 360° in a day. At other latitudes there is only one component of the earth's rotation effective (none at the equa or) and the rate of precession is less.

2. What is the relation between distance, velocity, and acceleration along the circumference of the circle and θ, ω, α?

3. What is the angular velocity of the earth in radians per day?

4. Suppose that when a boy spins a top he pulls off the string at constant speed. Is the angular velocity of the top constant?

5. What is the relation between force and torque? How is moment of inertia computed?

6. Which has the more moment of inertia, a wheel 3 in. in radius mass 1 lb., or one 6 in. in radius mass $\frac{1}{2}$ lb.?

7. A man in a rowboat holds a wheel as it spins about a vertical axis. What happens if he turns it over?

8. What happens in question 7 if he turns the axis parallel to the length of the boat? Does this illustrate the gyrostabilizer?

9. A tight-rope walker carries a long wooden pole. Explain its action as a "stabilizer."

10. A man turning on a turntable extends his arms and doubles his moment of inertia, halves his angular velocity. Compare his angular momenta; his kinetic energies. Was any torque applied? Was any work done?

11. A gyroscopic compass points toward the north because: (*a*) a gyroscope always points in the same direction; (*b*) rotation of the horizontal plane causes a precession toward the north; (*c*) the earth itself is essentially a large gyroscope.

12. Compare a frictionless block sliding and a ball rolling down an inclined plane. The ball reaches the bottom with: (*a*) the greater speed; (*b*) the smaller speed; (*c*) speed equal to that of the block.

Vocabulary: Angular displacement (velocity, acceleration), radian, moment of inertia, angular momentum, (anti-) cyclone, Ballot's Law, Conservation of Angular Momentum, angular harmonic motion, gyroscope, gyrocompass, gyrostabilizer, (velocity of) precession.

PROBLEMS

1. Moment of inertia. Find the moment of inertia of a hoop, sphere, disc, each with diameter of 2 ft. and each weighing 32 lb. (Axis central and perpendicular to hoop and disc.) Also of a rod 2 ft. long, 32 lb., axis central.

2. Acceleration and torque. Find the torque required to give within 1 sec. to each of above objects an angular velocity of 1 revolution per second.

3. Acceleration and distance. If the torque in problem 2 continues to be applied, how far will each turn in 2 sec.?

4. Kinetic energy. What energy did each body in problem 2 possess after 1 sec.?

5. Harmonic motion. It requires a torque of 2 lb-ft. to twist a certain wire through 57°. If the above sphere is suspended from it, find its period of oscillation.

6. The rim of a 4-ft. wagon wheel weighs 16 lb.; it has 16 spokes each weighing 2 lb. What is its moment of inertia? What angular velocity do we give it by exerting a 2-lb. turning force halfway out the spoke for 4 sec.?

7. Suppose that the wheel in Fig. 2 has a moment of inertia of 4000 kilogram-cm^2; suppose a 100-gram weight to be suspended from a drum 5 cm in radius. Find the angular acceleration. How long will it take the wheel to make 1 revolution?

8. A meter stick (moment of inertia 196,000 gram-cm^2) is balanced on a knife edge; a 1-gram weight is then placed on the stick, 2 cm from the support. What is the angular acceleration of the stick in degrees per second? How far will the stick rotate in 3 sec.?

9. In problem 8 a 100-gram weight is placed upon each end of the stick, exactly balancing. Find the moment of inertia and the acceleration due to the 1-gram added weight.

10. A hoop 2 ft. in diameter weighs 4 lb. It is rolling 20 ft. per sec. Find (*a*) its angular velocity; (*b*) its angular momentum; (*c*) its total kinetic energy.

11. A hoop 1 meter in diameter, weight 2 kilograms, rolls down an inclined plane from an elevation of 4 meters. Find its kinetic energy and linear velocity at the bottom.

12. An automobile flywheel is 1 ft. in diameter and weighs 64 lb., the weight being mostly concentrated near the rim. At maximum power it makes 3600 revolutions per minute. What is its angular momentum? What is the change in angular momentum when the machine makes a U-turn? What is its kinetic energy?

13. A turntable with frictionless bearing has a moment of inertia of 20 kilogram-cm^2; a top has a moment of inertia of 20 gram-cm^2. The top spins on the table making 100 revolutions per second. (*a*) Find its angular momentum. (*b*) Find the speed of rotation given to the table as the top comes to rest.

14. Find the period of oscillation of Fig. 3, the framework swinging as a physical pendulum.

15. Show that a meter stick suspended by the end has the period of a simple pendulum $66\frac{2}{3}$ cm long.

16. Find the moment of inertia (I_0) of the bobs in Fig. 3 (*a*) about the axis *ef*; (*b*) about three different axes through the center of gravity.

17. From your results in problem 16 show that the following rules hold:

(*a*) I about any axis equals the sum of I_0 about a parallel axis through the center of gravity and MR^2, where R is the distance from the given axis to the center of gravity.

$$I = I_0 + MR^2.$$

(*b*) For a plane figure, if I_0' and I_0'' are the moments of inertia about two mutually perpendicular axes lying in the plane of the figure and I_0''' the moment of inertia about a perpendicular axis, all axes passing through the center of gravity, then

$$I_0''' = I_0' + I_0''.$$

These rules are very useful in finding moments of inertia about arbitrary axes.

18. Show that the first rule of the preceding example gives the relation between the moments of inertia of the two sticks in Fig. 4, and the second rule gives that between the two rings.

19. A ring 20 cm in diameter hangs on a nail. Find its moment of inertia about this axis (using above rules) and its period of oscillation. (Take any arbitrary mass.)

CHAPTER 17

UNIVERSAL GRAVITATION

Perhaps our pushes and pulls, our pulleys and levers have at times seemed prosaic. Not so the present chapter. It deals with the first great success of Newton's system of mechanics: Newton showed that the same force which pulls the stone to the ground also holds the moon in its orbit; this same force governs the motion of the planets and controls the tides.

Picture the world in which men lived in 1492, before the discoveries of Columbus and the vision of Copernicus. It is a flat world; the sun passes across the heavens by day, the stars by night, for the edification of mankind. To the west it is bounded by an endless ocean; to the east it is populated by half mythical, outlandish peoples. Closer at hand are fairies and goblins. Science means superstition — astrology, alchemy, the search for the fountain of youth. Life is bounded on all sides by the unknown; there are no bounds upon the imagination. Life in such a world, so close to all those unreal wonders and terrors, must for some sensitive souls have been very exciting. Then Columbus discovers a new continent, and Copernicus, Tycho Brahe, Kepler, Galileo, and Newton discover the universe.

189. Copernicus. Ptolemy (in the library at Alexandria, about 130 A.D.) adopting the prevalent doctrines of Grecian astronomy, founded the Ptolemaic system. He taught that the earth stood still, that the sun, stars, and planets went about it. Smaller cycles were geared to the main cycle to account for the motion of the planets. This was the astronomy of the Middle Ages. Copernicus revived a theory of one of the imaginative Greek astronomers, forgotten for nearly 2000 years; he taught that the earth was simply a planet, that the earth and other planets went in circular orbits about the sun. The world lost its place as the center of the universe and became simply a satellite of one of the stars, the sun. His theory was published in 1543, the year of his death; this was 50 years after the discovery of America, 100 years before the birth of Newton.

190. Kepler. Three years later Tycho Brahe was born. He devoted his life to making exact observations of the motions of the planets. It appeared that the motions of the planets could not be accounted for exactly by the circular orbits of Copernicus. Kepler, taking Brahe's data, analyzed the planetary motions. He established the following laws (Kepler's Laws):

(1) (Shape of the orbit.) The planets go about the sun in ellipses with the sun at one focus.

(2) (Velocity in the orbit.) The speed of a planet is greater when it is near the sun. More exactly: The radius vector (from sun to planet) sweeps out equal areas in equal times.

(3) (Periods of different orbits.) The squares of the periods in the different orbits vary as the cubes of their mean radii. Example: The year of Jupiter is 11.8 times as long as that of the earth. Jupiter's orbit is 5.2 times as large as that of the earth ($11.8^2 = 5.2^3$).

191. Newton. Newton, building on the work of Galileo, had developed his system of mechanics. "What," he asked himself, "can be the nature of the force acting on these heavenly bodies?" Now Kepler's *second* law, in modern terms, means simply that the angular momentum of a planet stays constant. Therefore, no torque acts on it — the force acting must be directed exactly toward the sun. And Newton, analyzing the motion, showed that this force must vary *inversely as the square of the distance* from the sun. Only such a force would result in an elliptical orbit (Kepler's *first* law).

Kepler's *third* law verified this. Consider the orbits as circular. If T is the period of revolution, we have

$$v = \frac{2\pi r}{T}.$$

Substituting this in the equation for centripetal force $\left(F = \frac{mv^2}{r}\right)$, we have

$$T^2 = \frac{4\pi^2\, mr}{F}.$$

If in this equation $F \propto 1/r^2$ we have $T^2 \propto r^3$.

192. Universal Gravitation. Newton then reached a daring conclusion: *This force which pulls the planets to the sun is the same as that which pulls the stone to the ground. Matter attracts* matter. *Every particle of matter in the universe attracts every other particle with a force which varies inversely as the square of the distance.* The force is also proportional to the masses (m_1, m_2) of the attracting particles:

$$F = G\frac{m_1m_2}{r^2}, \tag{15}$$

where G is the proportionality constant. There should be a slight attraction between two neighboring balls. For ordinary balls this force is very small (although, as we shall see, it can be measured). But the force between the big ball which we call our earth (m_1) and any neighboring object (m_2) is $980 \cdot m_2$ dynes.

193. Acceleration of the Moon. Newton found the proof of this assumption in the motion of the moon. The moon travels 360° about the earth in $27\frac{1}{3}$ days;[1] the radius of its orbit is 240,000 miles; hence we can compute its velocity and its central acceleration. The value of the acceleration turns out to be a tenth of an inch per second per second. This is the observed acceleration. On the other hand, we know that the acceleration of gravity at the surface of the earth (4000 miles from the center) is 32 feet per second per second. The moon is 60 times as far removed, and the gravitational acceleration at that distance should be 3600 times less. This agrees exactly with the observed value.

In a certain sense the moon is falling — falling off from the straight line just enough to keep in its circular orbit. Its downward acceleration is less than 980 centimeters per second per second — 3600 times less — but this acceleration of the moon is just the acceleration with which any object — a stone, a box, a chair — would fall at that distance from the earth.

194. The Gravitational Constant. G in eq. (15) is called the gravitational constant. To find its value the force of attraction must be measured between two objects of known mass at a known distance apart. (The earth-pull is easily measured, but this cannot be used because the mass of the earth is unknown at the present point in the argument.) Two small gold balls are on the ends of a light cross bar hung by a fine quartz fiber. Two other balls are brought near these, one to the one side of one, the other to the opposite side of the other (*a* and *b*). The gravitational forces now furnish a small couple. When the balls are placed at *c* and *d* an opposite couple results. The angle of twist in each direction can be measured, and, the torsion constant of the suspension fiber being known, the force can be found. The masses and distances being known, the value of G can be found. Its value in c.g.s. units is 6.66×10^{-8}; that is, two gram masses 1 centimeter apart attract each other with a force of 6.66×10^{-8} dyne.

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[1] From full moon to full moon is about 2 days longer than this because of the simultaneous revolution of the earth about the sun.

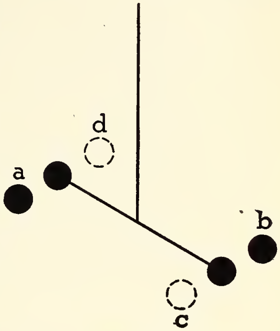

FIG. 1

This experiment was first performed by the eccentric English genius, Cavendish.[1] Cavendish's experiment is often described as "weighing the earth." We know that a 1-gram mass attracts the earth, with its center 4000 miles or 630,000,000 centimeters away, with a force of 980 dynes. Now we find that a 1-gram mass attracts another 1-gram mass, with center 1 centimeter distant, with a force of 6×10^{-8} dyne. By proportion we can find the mass of the earth. (This, a good example in proportion, is left for the student.)

195. Force of Gravity on the Surface of the Earth. By the Law of Universal Gravitation the weight of a body on the surface of the earth is

$$\text{Weight} = G\frac{m_{\text{earth}} \times m_{\text{body}}}{r^2}.$$

Compare this with the familiar expression

$$\text{Weight} = m_{\text{body}} \times g.$$

Evidently

$$g = G\frac{m_{\text{earth}}}{r^2},$$

---

[1] Henry Cavendish (1731–1810), first known for his investigations of gases (discovery of hydrogen), also made fundamental discoveries in electricity, heat, and other branches of physics. But many of his most important results remained unpublished and had to be rediscovered years later. He was of a noble family, one of the richest men of his time, but his main object in life — aside from his scientific interests — seems to have been to avoid contacts with his fellow man.

where r is the radius of the earth. Owing to the flattening at the poles the effective radius decreases as one goes north or south from the equator. There is a corresponding change in g. At sea level on the earth the weight of a gram varies from 978 dynes at the equator to 983 dynes at the poles.[1]

The acceleration of gravity decreases slightly on high mountains. For each mile of altitude the decrease is 2 parts in 4000.[2] In deep mines the gravitational attraction decreases because of the outward pull of the earth shell above — at the center of the earth the force, of course, becomes zero. The value of g also depends somewhat on the density of the earth's crust in the locality — hence gravity measurements of highest precision are used in mineralogical surveys for locating ores and (particularly) petroleum.

TABLE 15

ACCELERATION OF GRAVITY IN UNITED STATES

Pikes Peak	978.954
New Orleans	979.324
St. Louis	980.001
Washington	980.095
Seattle	980.733

196. Tides. As the earth turns on its axis the waters in the ocean tend to pile up on opposite sides of the earth, on the side nearest and the side farthest away from the moon. This is due to the moon's gravitational attraction. The earth pulls upon the moon and the moon pulls upon the earth, and the two revolve about their common center of gravity. But the lunar attraction is slightly greater upon the water on the nearer side (and slightly less upon that on the opposite side) than it is upon the earth as a whole. Hence this piling up of water on these two sides. But because of the continual rotation of the earth equilibrium is not attained. As the earth turns, it has a tendency to carry the tides along with it, and in consequence high tide is not exactly in phase with the moon's motion.

The sun has a similar effect, about a third as strong. Twice a month, when sun and moon are together or opposite in the sky (new and full moon) the effects combine to give unusually large tides (*spring tides*); a week later with sun and moon 90° apart the two effects subtract and give small tides (*neap tides*). The effects of these fundamental tidal forces may be greatly modified by ocean depths and the shore line.

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[1] Part of this change, however, is due to the rotation of the earth — a body at the equator is not strictly in equilibrium but has a centripetal acceleration of 3 cm per sec.^{2} (§123).

[2] Two parts (instead of one) in 4000 because of the inverse square (App. 2).

Most of our energy supply on this earth comes to us directly or indirectly from the sun. Tidal energy is an exception. As the earth spins through the double-crested tidal wave once a day, the waves striking against the continents tend gradually to stop it. Tidal energy is taken from the kinetic energy of our rotating globe and dissipated as heat. There are projects (possibly none too practical) for harnessing some of this energy and driving electrical generators by tidal ebb and flow.

197.* Gravitational Potential Energy. The work required to raise a body through a height (h) has been given as mgh. As the height is made greater and greater does the work increase in exact proportion? If so, it would take an infinite amount of work to remove a body completely from the earth; nothing could ever be so removed. Or, conversely, a meteor falling into the earth would have infinite velocity and infinite kinetic energy. The fallacy in the argument is that g is not a constant and ultimately at great distances from the earth becomes negligible. The work required to carry a body (m_2) from a distance r (measured from the center of the earth) to infinity is

$$W = G\frac{m_{\text{earth}}\, m_2}{r}.$$

(A similar relation is important in electricity [§393], and the proof will not be given here.)

Example. Find the velocity required to shoot a projectile away from the earth. r is now the radius of the earth (630,000,000 cm). To simplify the computation we remember that $\frac{Gm_{\text{earth}}}{r^2} = 980$. Hence

$$\begin{aligned} W &= 980 \times r \times m_2 = 63 \times 10^{10} \cdot m_2 \\ \tfrac{1}{2}\, m_2 v^2 &= 63 \times 10^{10}\, m_2 \\ v^2 &= 126 \times 10^{10} \\ v &= 11.2 \text{ kilometers per sec.} = 7 \text{ miles per sec.} \end{aligned}$$

198.* Orbits. Newton saw in the moon only a large round stone, hurtling through space at such a speed that it whirls completely around the earth instead of falling on it. Like the planets around the sun, it goes in an elliptical orbit, almost circular. Whenever one throws a stone, when the pitcher throws the baseball or when the projectile is fired from the cannon, the stone or the ball or the projectile becomes for the moment a satellite of the earth, describing a brief portion of its elliptical orbit[1] until it strikes the earth and stops. Let us picture these orbits as they would be if they could continue. The stone makes a narrow ellipse; the projectile fired thousands of feet a second makes a wider one — the center of the earth at one focus. Now we hurl the projectile faster and faster. The ellipse widens out, the projectile strikes the ground farther and farther away, until at 5 miles per second the orbit becomes a circle and the projectile goes around the earth $\left(\frac{v^2}{r} = \frac{(5 \times 5280)^2}{4000 \times 5280} = 32 \text{ ft. per sec.}^2\right)$. (A circle is an ellipse with the two foci together.) Still faster, and we get another ellipse, the earth center now at the *nearer* focus. The earth has gained a satellite now, which circles the earth time after time, just as the planets do the sun. And then at 7 miles per second the ellipse fails to close — this is a parabola, and the projectile leaves the earth never to return. (A parabola is an ellipse which never closes, with one focus at infinity.) Still faster, and the orbit widens out into a hyperbola — like some comet which occasionally rushes in toward the sun from outer space and then away in a hyperbolic path never to return.[1]

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[1] See the note to §98.

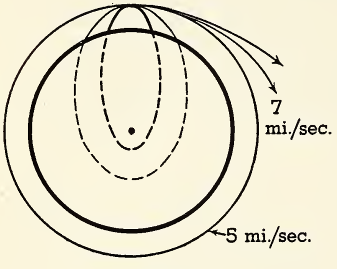

FIG. 2

So — building on the work of Copernicus, Brahe, Kepler, Galileo — Newton was the first to grasp the unity which pervades the universe. Newton was mathematician, astronomer, chemist, as well as physicist. Of him his rival Leibnitz said, " Taking mathematics from the beginning of the world to the time when Newton lived, what he had done was much the better half." A French scientist writes, " Newton was the greatest genius that ever existed, and the most fortunate, for we cannot find more than once a system of the world to establish." And an English historian says, " It [the Law of Gravitation] is indisputably and incomparably the greatest scientific discovery ever made, whether we look at the advance which it involved, the extent of truth disclosed or the fundamental and satisfactory nature of this truth." But Newton himself, looking back upon his years, said: " I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

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[1] The giant German cannon in the first World War fired a projectile with muzzle velocity of a mile per second. Jules Verne wrote about an imaginary " Trip to the Moon " in a giant projectile fired with a muzzle velocity of 7 miles per second. But at such speed a projectile would be heated to incandescence (like meteors) by resistance of the air and any ordinary projectile would be evaporated away.

But at least this Law of Gravitation was a rather large pebble. It revolutionized man's thinking. And some felt a loss when they found the mystery of the planets in their orbits to be no greater than that of the falling stone. They forgot the greater mystery of the falling stone which Newton's Law implies and the greater majesty in this common force which acts alike on all things near and far, and the greater beauty in a universe bound by universal law.

SUGGESTED READING. Read the story of Copernicus, Tycho Brahe, Kepler, Galileo, and Newton in "Watchers of the Sky" by Alfred Noyes (Stokes). Human interest and the spirit of science. This is Vol. I of Noyes' trilogy, "The Torchbearers," with which everyone should (I think) become acquainted.

QUESTIONS

1. What are Kepler's Laws, and how did Newton interpret them?
2. Why are *gold* balls used in Cavendish's torsion balance?
3. What force acts between two bodies, 2 grams each, 2 cm apart?
4. When a body (a meteor) originally at rest relative to the earth falls upon the earth, what is the velocity with which it arrives?
5. Believe it or not: A ship is slightly lighter when the moon is directly overhead than when it is rising or setting. Why? What is the effect when the moon is on the opposite side of the earth?
6. Planets travel in elliptical orbits with the sun at the focus because: (*a*) only elliptical orbits are possible with an inverse-square law of force; (*b*) ellipses are the only possible *closed* orbits; (*c*) only the ellipse has a focus.
7. To estimate the mass of the earth it is necessary to: (*a*) determine its density by geologic exploration; (*b*) measure the force it exerts on the moon; (*c*) determine G by measuring the force between two small balls of known weight.

Vocabulary: Kepler's Laws, Law of Universal Gravitation, gravitational constant, torsion balance, orbit, ellipse, focus, hyperbola.

PROBLEMS

1. What is the force of attraction between two 1-lb. spherical weights 5 cm apart (center to center)?
2. A spherical meteorite with a 1-meter radius would weigh about 25 tons. Imagine such a meteorite in free space. What would be the "weight" of a 1-gram particle upon its surface?
3. What is the acceleration of gravity (*a*) 4000 miles (*b*) 240,000 miles above the surface of the earth?

4. By what per cent did g change in the stratosphere flight (14 miles)?

5. At a distance 240,000 miles from the earth how far would a stone fall (from rest) in a minute? How far does the moon depart from a straight line in a day?

6. The mass of the moon is $\frac{1}{80}$ that of the earth; its diameter is $\frac{1}{4}$ that of the earth. Find the acceleration of gravity on the moon's surface.

7. The mass of the sun is 332,000 times that of the earth, and its diameter is 110 times greater. What is the acceleration of gravity on its surface?

8. Compare the attraction of the moon by the earth and by the sun. The mass of the sun is 332,000 times greater than that of the earth; its distance from the moon 400 times greater.

9. What is the mass of the earth? The volume of the earth is about 10^{27} cc. What is its average density?

10. Compare the acceleration of gravity at the surface of two planets with radii r_1 and r_2, assuming them to have the same mean density.

11. The standardizing balance in the Bureau of Standards has a sensitivity of 1 part in 100,000,000; it will weigh a kilogram mass to 0.01 milligram. It is said that the balance will detect the weight difference when two kilogram weights are placed in a pan, first side by side and then one on top of the other. Is this true? Find the difference in weight assuming the weights to be 7 cm high. (Radius of earth = approximately 7000 kilometers.)

12. A gold sphere 5 cm in diameter weighs about 1 kilogram. Suppose that such a sphere is balanced on the sensitive balance of problem 11. Will the balance be sensibly disturbed by bringing a similar sphere as close as possible beneath the balance? Compute the additional force.

SURVEY OF MECHANICS

This completes our survey of elementary mechanics. Let us now collect the more important principles.

199. Types of Linear Motion. (1) The state of equilibrium; total force on body zero; rest or constant velocity (constant magnitude and direction).

$$s = vt.$$

Examples. Dragging: $F = k \cdot \text{Weight}$
Lifting: $F = \text{Weight}\ (= mg)$.

(2) Constant force (magnitude and direction); uniform acceleration.

$$v = u + at;\ s = ut + \tfrac{1}{2}at^2;\ 2\,as = v^2 - u^2.$$

Examples: Falling bodies ($u = 0$ or vertical).
Projectiles (u not vertical).

(3) Centripetal force (constant magnitude, perpendicular to motion); uniform circular motion.

$$a = \frac{v^2}{r};\ T = \frac{2\pi r}{v}.$$

(4) Force proportional to displacement (opposite in direction); simple harmonic motion.

$$T = 2\pi\sqrt{\frac{x}{a}}.$$

Examples. Pendulum: $\frac{x}{a} = \frac{l}{g}$.

Vibrator: $\frac{x}{a} = \frac{m}{k}$.

(5) Central force varying inversely as r^2; motion in conic sections (circle, ellipse, parabola, hyperbola).

Kepler's Laws

Examples: Planetary motion, electrons in atoms (Bohr's Theory).

200. Types of Angular Motion. (1) Constant angular velocity ($\alpha = 0$); (2) constant angular acceleration ($\alpha =$ constant); (3) constant (gyroscopic) precession (α perpendicular to axis of rotation); (4) angular harmonic motion (α proportional to angular displacement).

201. Newton's Laws of Motion. These laws apply in the first instance to forces and linear motion; in a generalized sense they apply to torques and angular motion.

I First Law (s). When $\mathbf{F} = 0$: velocity is zero or constant (direction and magnitude).

Examples: Body alone in space, book on table, wagon pulled at constant speed, elevator ascending steadily.

First Law (θ). When $\mathcal{T} = 0$: angular momentum is zero or constant (direction and magnitude).

Examples: Body at rest, grindstone turning steadily, earth.

II Second Law (s). $\mathbf{F} = \dfrac{\Delta(mv)}{t} = ma.$

Examples: Falling bodies, elevator starting up, uniform circular motion, simple harmonic motion.

Second Law (θ). $\mathcal{T} = \dfrac{\Delta(I\omega)}{t}$ or (if I is constant) $= I\alpha$.

Examples: Torque applied to wheel, physical pendulum, gyroscope.

III Third Law (s). $F_{\text{action}} = F_{\text{reaction}}$.

Examples: Down pull on body equals up pull on earth; pull up on picture equals pull down on nail; push of baseball equals back push of glove; pull on wagon equals back pull on horse.

Third Law (θ). $\mathcal{T}_{\text{action}} = \mathcal{T}_{\text{reaction}}$.

Examples: Clockwise torque on screwdriver equals counterclockwise torque on hand.

202. The Conservation Laws. 1. *Conservation of Mass.* Mass can neither be created nor destroyed. Established about 1775, when it was shown that mass was not lost in chemical action, burning, etc.

2. *Conservation of Momentum.* The total momentum of a complete system remains constant. Established by Newton. When shell leaves cannon, when balls collide, when man jumps upward (but momentum of earth must be considered).

3. *Conservation of Angular Momentum.* The total angular momentum of a complete system cannot change. Established by Newton. Man on turntable, motion of solar system.

4. *Conservation of Energy.* The total energy of a complete system cannot change (potential, kinetic, heat, electrical, chemical, etc.). This principle extends beyond mechanics. Established in 1845, when heat was shown to be a form of energy. Example: When a ball falls it loses gravitational potential energy (mgh) and it either (1) speeds up ($\frac{1}{2}mv^2$) or (2) stretches a spring ($\frac{1}{2}kx^2$) or (3) descends into water (PV) or (4) is stopped by friction (heat energy) or (5) develops energy in still another form. The total energy remains constant.

203. Development of Mechanics. Greek philosophers developed mathematics, originated an atomic theory, studied sound and light, knew of the rotation of the earth, taught of the motions of the planets, measured roughly the distance to sun and moon. In mechanics Archimedes investigated the principles of statics and hydrostatics. But these scientific beginnings were largely lost.

With the discovery of printing and the discovery of America came the New Learning. Copernicus rediscovered the planetary theory. About 1600 — about the time of Shakespeare and the *Mayflower* — Galileo established the first principles of accelerated motion; he was persecuted for his discoveries. At this time, of all Europe Holland offered greatest freedom of thought. Simon Stevin, a Dutch mathematician, had preceded Galileo in the investigation of motion, but (perhaps because there was no persecution) his work was not well known. Descartes and then Huygens developed mechanics in Holland. In England, Newton developed the concept of mass and stated the laws of motion. The Dutch school emphasized work (Fs) and the change in kinetic energy ("*vis viva*"); Newton emphasized impulse (Ft) and the change in momentum.

In the eighteenth and nineteenth centuries, the mechanics of Descartes and Huygens and Newton were united into a consistent system, and then, in the hands of the Bernoulli brothers, Lagrange, Laplace, Gauss, Hamilton, and others, developed to treat complicated problems. Astronomers applying Newton's law of gravitation computed the positions of the planets and comets, the orbit and eclipses of the moon, with unbelievable accuracy. With the coming of machines the principles of dynamics received their practical application. But withal the fundamental principles of mechanics remained unchanged until the present century.

In 1905 Einstein in his Theory of Relativity showed that Newtonian mechanics was only an approximation, holding for velocities small compared with that of light. And in 1925 Heisenberg and others showed that the ordinary concepts of mechanics — space, time, even the concept of cause and effect — were not strictly applicable to things of atomic dimensions and developed the New (Quantum) Mechanics. But despite these discoveries on the frontiers of science the principles of Newton remain of unshaken validity in the realm of ordinary things about us.

MOLECULAR MOTION AND HEAT

CHAPTER 18

MOLECULAR ATTRACTION AND MOLECULAR MOTION

We now turn our attention from the orderly motion of bodies as a whole — translation, rotation — to the rather haphazard motion of the molecules within the body, and turn from the forces which bodies exert one on the other to the attractions between molecules. This will bring us to the subject of heat. But first we may appropriately ask: what evidence is there for such intermolecular attraction and for molecular motion?

INTERMOLECULAR FORCES

204. Cohesion. In the study of elasticity we have seen how the attraction between atoms or molecules gives the solid body its definite shape and size. Two flat glass plates can be pushed together so closely that they cohere. The same thing is more commonly observed in mica; we can pry the layers in a piece of mica apart or force them together at will. Two pieces of soft rubber pressed together become a single piece. A distinction is sometimes made between the attraction of like and of unlike substances — *cohesion* and *adhesion.* Rubber coheres to rubber; glue adheres to wood. Newton observed: " The parts of all homogeneal hard bodies, which fully touch one another, stick together very strongly."

The adhesion between surfaces is of importance when they slide one over the other. Indeed, adhesion is believed to be the major cause of friction between clean metal surfaces. At points of close contact the metals adhere, and as one surface slides over the other the adhesive forces tear away small portions of the metal. In a dry bearing the seizure may stop the machine.

Let us turn to liquids and gases. In a liquid the molecules cohere whereas in gases the attractive forces are usually negligible. Indeed, this is the fundamental distinction between a liquid and a gas. It is this cohesion which keeps the liquid confined to a definite volume at the bottom of a vessel while a gas fills whatever space is available. As water drips from a faucet we see it form in pear-shaped drops as, for the moment, the attractive forces between molecules struggle against the force of gravity. Water wets my hand. When I pour water from a clean glass the glass remains wet — we explain it rather loosely by saying that the attraction of water molecules for glass molecules is greater than the cohesion of the water molecules among themselves. Mercury does not wet glass, nor does water wet a greasy surface — in these cases cohesion is greater than adhesion (§209).

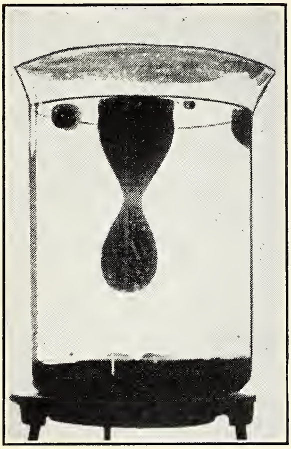

Fig. 1. Aniline drop in water.

205. Surface Tension. Cohesion in a liquid brings the molecules as close together as possible and tends to reduce the free surface to a minimum. Of all geometrical figures the sphere has the smallest surface for a given volume. A falling raindrop, a small droplet of mercury on the table, each assumes a spherical shape. The simplest (though rather artificial) way of viewing the matter is to imagine a tension acting along the surface, tending to make the surface shrink. This effect is called surface tension. The liquid acts as if its surface were a stretched membrane.

There are many familiar examples of surface tension. Our pear-shaped water drop from the faucet is only our raindrop distorted by gravity; one can imagine the weight of the drop supported by surface tension like so much matter suspended in an elastic bag. . . Certain insects skim over the surface of water supported by this fictitious membrane. . . If care is taken not to break through the water surface, a needle can be floated on water, held up by surface tension. . . Water can be held in a sieve if it is poured in carefully. . . Canvas tents shed the rain.

206. Measurement of Surface Tension. The contractive force acting across unit length of the surface is a measure of the surface tension T. Figure 2 will at once elucidate this and explain how surface tension is measured. As the wire frame is drawn upward it carries a film of liquid with it. (With a soap solution a large film can be formed; this is often used for demonstration.) There are two surfaces of the film, front and back, pulling downward upon the wire loop; hence for each centimeter along L there is a force equal to $2\,T$. The pull on the frame is measured by a sensitive balance (dynamometer), and the coefficient of surface tension T found from the relation:

$$F = 2\,TL.$$

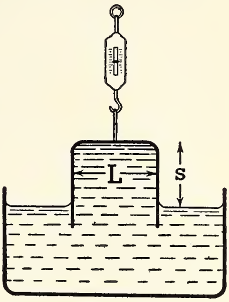

FIG. 2

The surface tension of water is about 75 dynes per centimeter; of mercury, 480 dynes per centimeter. Compared with everything except molten metals, water has a high surface tension. Soap reduces the value four- or fivefold. And the slightest contamination on the surface of water, even if it be only a film a single molecule thick, changes the surface characteristics entirely. This is amusingly shown by sprinkling some small fragments of camphor upon a fresh surface of water. The camphor dissolves, particularly at the points, reducing the surface tension, and the fragments rotate and spin and cavort and chase one another around as if suddenly endowed with a frantic life. They are actually pulled about by the unbalanced tensions in the surface.

207. Condition for Spreading. When a drop of oil (or other insoluble liquid) is placed on the surface of water, it may remain as a flattened drop or it may spread over the surface. At the interface between oil and water the adhesive forces between oil and water partially compensate the cohesive forces, and here the tension ("interfacial tension") is much less than the surface tension of the free water surface. If the drop forms there must be at its edge equilibrium between the surface tension of the water pulling in the one direction and the surface tension of the oil and interfacial tension pulling obliquely in the other. But some oils have such low surface tension and low interfacial tension (i.e., large adhesive force) that these two latter forces together are less than the

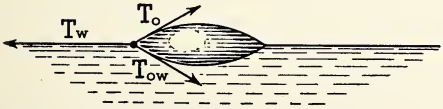

FIG. 3. Oil drop on water; drop forms when $T_w < T_o + T_{wo}$.

surface tension of the water, and the drop is pulled out as a thin film, spreading over the whole surface of the water. Kerosene covers water in a thin film.

208. Surface Energy. The coefficient of surface tension could quite as appropriately be called the coefficient of surface energy. As compared with those within the liquid the molecules on or very near the surface have high potential energy. If the frame in Fig. 2 is lifted a distance s, the work done is $2\ TLs$. This is the potential energy. The area (A) of the (double) film is $2\ Ls$. Therefore p.e. $= TA$, and the coefficient of surface tension (T) may be interpreted as the *potential energy per unit area* of surface.

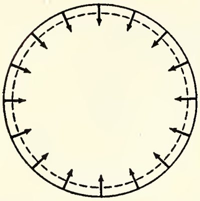

FIG. 4. Surface tension is due to inward forces and high potential energy of surface molecules.

Example. How much work must be done in dividing a water drop 1 cm in diameter into two equal drops (diameter 0.8 cm)?

Original area $= \pi\, 1^2 = 3.14$ sq. cm.
New area $= 2 \times \pi \times 0.64 = 4$ sq. cm.
Work $= T \times \Delta A = 75 \times 0.86 = 64$ ergs.

From this point of view when oil spreads over the surface of a liquid it does so because it thus reduces to a minimum the energy of the surface; the energy of the free oil surface and the energy of the interface are together less than the energy of the free water surface.

$$T_{\text{oil}} + T_{\text{interface}} < T_{\text{water}}. \quad (\textit{Spreading})$$

209. Adhesion between Liquids and Solids. The spreading of a liquid over a clean metal surface or over glass has the same explanation. Although there is no direct method of measuring it, the surface atoms of a solid have high potential energy and the solid has a surface tension. A liquid of comparatively low surface tension and large adhesive force spreads over the surface because it thus reduces the total energy. Because of its very large surface tension mercury does not wet the surface of a glass.

This phenomenon of spreading is important in lubrication. A "penetrating" oil has great adhesive force, comparatively small cohesion, a great tendency to wet and spread in the bearing. (In addition a good lubricant has suitable viscosity and a property of "oiliness" which is less well understood but seems to be associated with the length and flexibility of the molecules.)

210. Capillary Attraction. In a glass tube of small bore (capillary tube) water may rise several centimeters above its general level. The force of adhesion between glass and water being very strong, the water creeps up the glass and by surface tension pulls after it a column of water.

Surface tension is often measured by observing the rise in capillary tubes. The lifting force acts all around the inner circumference of the tube ($2\pi r$); hence the force is $2\pi r \cdot T$ dynes. On the other hand, if the water is lifted a height h its volume is $\pi r^2 h$ cubic centimeters, and its weight $\pi r^2 hdg$ dynes. Hence we have the relation

$$\pi r^2 hdg = 2\pi r \cdot T$$

or

$$h = \frac{2\,T}{rdg}.$$

This derivation assumes that the surface tension forces around the circumference act directly upward. If the liquid does not spread on glass (as do water and the great majority of common liquids), the angle of contact between liquid and glass is not zero, there is only one component of the circumferential force pulling upward and the equation must be modified. Actually, when the liquid does not wet the glass (for example, mercury), the level in the capillary is lowered rather than raised.

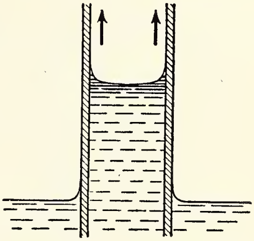

FIG. 5. Capillary attraction.

It is capillary attraction which acts when blotting paper absorbs ink; the ink wets the fibers and is sucked into the interstices between them. It is capillary attraction, again, which causes a sponge or a towel or soil to absorb water, which causes oil to rise in the wick, which plays an important role in the rise of sap in trees. The soil is a sponge, a great reservoir for the water needed by plant life.

211. Adsorption. Although gas molecules do not cohere they may adhere to surfaces (*adsorption*). In evacuating glass vessels, the glass must be heated to drive off the adsorbed gas layer. Charcoal is porous, with great surface area, and adsorbs large quantities of gases and (especially) vapors. A piece of charcoal (at room temperature) will adsorb 30 times its own volume of carbon dioxide, 90 times its volume of ammonia. (The adsorption decreases with rising temperature.) It adsorbs the poisonous gases chlorine and phosgene; hence its use in gas masks. Dissolved substances may be adsorbed out of solution onto bone charcoal, fuller's earth, alumina, etc. This property is used in the decolorizing or deodorizing of oils. Water, filtering through the earth, is purified through the adsorption of organic matter by certain constituents of the soil.

The platinum-like metals (platinum, rhodium, iridium, palladium, etc.; see Table 3) have the notable property of absorbing gases — both by adsorption and by absorption throughout the volume of the metal. Most remarkable is the absorption of hydrogen by palladium; the metal will absorb 600 times its own volume of the gas.

212. Absorption of Gases in Liquids. When a glass of water has remained for some time in a warm room, we see bubbles of air separated out on the glass. Fish depend upon the oxygen dissolved in water for their respiration. During the summer months water loses much of this dissolved air, and fish in shallow ponds suffocate. When water is heated on the stove, bubbles of air form and rise, and the water appears to boil at temperatures much below that at which true boiling occurs.

This shows (*a*) that gases are dissolved in liquids, and (*b*) that the solubility decreases with a rise in temperature. The *volume* of gas absorbed is independent of the pressure; thus the *mass* of gas absorbed is proportional to the pressure. The solubility depends on the nature of the gas and the nature of the liquid. At room temperature water will absorb its own volume of carbon dioxide (as in carbonated beverages); it will absorb about 30 times less oxygen, 60 times less nitrogen.

213. Condensation of Gases. The preceding sections have given varied examples of intermolecular attraction. But perhaps the most important and obvious example is the condensation of gases: under suitable conditions of temperature and pressure, molecules of a gas cohere and the gas turns into a liquid (or a solid). Consider, for example, carbon dioxide at room temperature. At ordinary pressures it behaves like any other gas; it practically obeys Boyle's Law, and the molecules attract each other but do not cohere. But we have seen (§55) that, when the pressure is greatly increased and the gas molecules (on the average) are brought much closer together, the intermolecular forces begin to evidence themselves in an abnormally high compressibility. Finally (at a pressure of about 60 atmospheres) the molecules cohere, the volume shrinks many fold, and we have the carbon dioxide liquid.

Any gas, with suitable pressure, can be condensed to a liquid in this fashion, but only below a certain temperature. We shall return to the subject in our discussion of heat. We must now answer the question: why is it that despite these cohesive forces a gas does not cohere except at suitably low temperature and high pressure?

MOLECULAR MOTION

214. Diffusion. If a little blue copper sulfate solution is placed at the bottom of a tall glass cylinder and the rest of the cylinder is filled with water, we shall find that the copper sulfate molecules wander upwards and in the course of several months the whole liquid is tinted blue. This gradual interpenetration of two substances is called diffusion. It is direct evidence of molecular motion. The atoms in a solid are also in motion, but the motion is only one of vibration back and forth about a fixed position. The molecules in our liquid are nearly as closely packed, and much of their time is spent simply in shuttling back and forth among their neighbors; yet, as the result of this jostling, they gradually wander about — and some of our copper sulfate ions migrate to the top of the liquid in several months.

In gases the molecules are much farther apart; they go much farther between collisions and the diffusion is much more rapid. We may put a little bromine vapor at the bottom of a glass cylinder and in the course of a few minutes the brown vapor will have diffused throughout the whole volume of the cylinder.

215. Brownian Movement. We get a striking picture of what occurs in molecular diffusion when we watch the behavior of very minute particles placed in water or of smoke particles in air. The particles are usually beyond the range of the microscope, but they appear clearly enough as points of light in an ultramicroscope (§662). And as we watch them we see them in incessant motion. They move back and forth irregularly, uncertainly, gradually changing their relative positions. Their erratic motion is caused by the irregular bombardment of the neighboring molecules. We may think of these submicroscopic particles as giant molecules, a billion times larger than ordinary molecules and moving many times more slowly, yet except for this difference in scale acting in every sense as molecules. This is our closest approach to actually witnessing molecular motion.

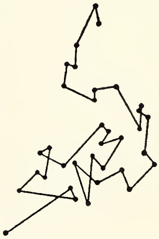

FIG. 6. Brownian movement: successive observed positions of colloidal particle in water.

216. Gas Pressure as a Kinetic Phenomenon. In Chapter 5 we studied the pressure of gases. We must now interpret this pressure in terms of molecular motion.

The atmosphere presses on my hand, but though the pressure depends on the weight of the gas above, I must picture it, not as a dead, static affair, the molecules simply resting on my hand, but as a kinetic, moving process, molecules themselves suffering repeated collisions from those above and, seeking to escape, pounding again and again against my hand. The compressed air in the automobile tire supports the weight of the car, not by a static force as in a compressed spring, but by the repeated impacts of molecules, pounding downward, pounding upward, holding the car on a cushion above the ground. The gas molecules are moving hither and thither, some in this direction, some in that, some fast, some slow. They strike against the walls and rebound. The pressure will depend on the number of molecules and their speed.

If the average speed of the molecules is increased the pressure increases for two reasons: the molecules have more momentum (mv), hence, strike the wall harder; with increasing velocity (v) they also strike the wall more often. Hence the pressure, the cumulative effect of all these blows, varies as mv^2.

217.* Let us find the exact relation. Consider the molecules in a cubical box, 1 centimeter long and high and thick. The area of each face is 1 square centimeter, and the force on this face, equal to the impulse per second, equal to the change of momentum of all the molecules which strike this face (unit area) in a second (§126), is the pressure. Suppose that there are n_1 molecules in this cubic centimeter; let v be their average speed. They move in all directions. But for simplicity suppose that one-third of them are moving in each direction: right and left, up and down, back and forth. Consider the last group ($\frac{1}{3}n_1$ molecules), which, moving back and forth, make repeated collisions against the front face.[1] Each molecule strikes this face with momentum mv (on the average) and rebounds with momentum $-mv$. The change in momentum (impulse) is equal to 2 mv. This molecule travels

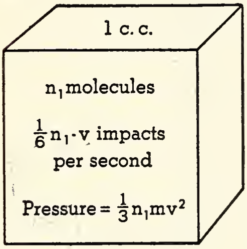
1 c.c.
n_1 molecules
$\frac{1}{6}n_1 \cdot v$ impacts per second
Pressure = $\frac{1}{3}n_1mv^2$

Fig. 7

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[1] We neglect collisions between molecules, but this is immaterial since, if one collision deflects a molecule away from its path to this face, another collision is equally likely to deflect another molecule toward it.

back and forth across the box (2 centimeters) and strikes this face again; hence it makes $v/2$ impacts per second. And there are $\frac{1}{3} n_1$ such molecules. Hence, over this unit area the total change in momentum per second is

Rate of change of momentum = Force (on unit area) =

$$\text{Pressure} = \frac{1}{3} n_1 \times 2\, mv \times \frac{v}{2} = \frac{1}{3} n_1 m v^2.$$

Since n_1 represents the number of molecules in a cubic centimeter and m is the mass of each molecule, $n_1 m = d$, the density of the gas. We thus have

$$\text{Pressure} = \tfrac{1}{3}\, dv^2. \qquad (1)$$

$\frac{1}{2} dv^2$ is the kinetic energy of the molecules in a cubic centimeter. Thus pressure is proportional to (exactly, it equals $\frac{2}{3}$ of) the molecular kinetic energy per cubic centimeter.[1]

From this simple relation we can find at once the average speed of the molecules. For instance at 1 atmosphere (1,013,000 dynes per square centimeter) pressure and 0° C, the density of hydrogen is 0.000090 gram per cubic centimeter. Therefore (from eq. 1)

$$v = \sqrt{\frac{3\,P}{d}} = \sqrt{\frac{3 \times 1{,}013{,}000}{0.000090}} = 184{,}000 \text{ cm per sec.}$$

or 1.84 kilometers per second. The molecules move with different speeds, some fast, some slow, but this is the average speed at this temperature (strictly the " root mean square " velocity).

218.* This is about 1 mile per second. In a later chapter it will be shown that the temperature is proportional to the average kinetic energy per molecule ($\frac{1}{2} mv^2$). At a given temperature the average kinetic energy of a molecule is the same in hydrogen gas as in oxygen gas or any other gas ($\frac{1}{2} mv^2 =$ constant). Hence the average speed of the molecules is inversely proportional to the square root of the mass of molecule. Oxygen molecules are 16 times heavier than hydrogen and (at this same temperature) have an average speed of about $\frac{1}{4}$ mile per second; for water molecules (9 times heavier than H_2) the average speed is $\frac{1}{3}$ mile per second.

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[1] This is the energy associated only with the translational motion of the molecules — diatomic molecules are also rotating, and in larger molecules the atoms may be vibrating.

TABLE 16

Properties of Gas Molecules at Room Temperature and 1,000,000 Dynes per Square Centimeter Pressure

	Mol. Wt.	Av. Speed mi. per sec.	Energy per Molecule, ergs	Number of Molecules per cc	Energy per cc
Hydrogen......	2	1	4×10^{-14}	2.7×10^{19}	1,500,000
Oxygen........	32	$\frac{1}{4}$	4×10^{-14}	2.7×10^{19}	1,500,000
Mercury vapor* .	200	$\frac{1}{10}$	4×10^{-14}	2.7×10^{13}*	1.5 ergs*

* At 1 dyne per sq. cm pressure; at room temperature mercury vapor condenses at the higher pressure.

219. Diffusion through a Porous Wall. The lighter gas molecules move faster than the heavier ones. This can be shown by a simple experiment. The cylinder *a* is of porous earthenware. With air within and about the cylinder, the molecules pass in and out through the walls and the pressure within is atmospheric. But now it is surrounded by hydrogen or by illuminating gas. These lighter, faster-moving molecules diffuse inward through the porous material faster than the air molecules diffuse outward, and the pressure within is increased and water is forced out from the flask below.

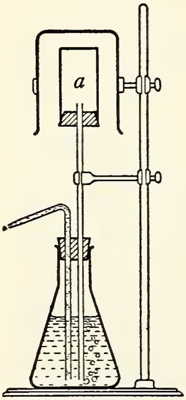

Fig. 8. Diffusion through porous wall.

220. Osmosis. We have here seen how pressure is built up when two different kinds of molecules have different rates of diffusion through porous earthenware.

Certain materials have the property that they will allow one kind of molecule to pass through but will block another almost completely. This permits a kind of one-directional diffusion which is called osmosis.

Water molecules can pass through parchment (an animal membrane — like the skin of a sausage), but the larger molecules of sugar cannot. In Fig. 9 the bottom of the thistle tube is covered with such a membrane. In the tube there is a strong sugar solution and in the beaker distilled water. The water molecules diffuse into the sugar through the "semipermeable membrane"; the sugar cannot come out; the sugar becomes diluted with water, and the level in the tube rises.

There are many familiar examples of osmosis. When link sausages are placed in water they swell as the water passes through the skin to dilute the salt in the meat. Ripening cherries burst after a rainfall, water passing through the skin (semipermeable membrane) into the sugar solution in the cherry. But strawberries left in sugar " turn to water " as the water in the berry passes through the skin out into the concentrated syrup.

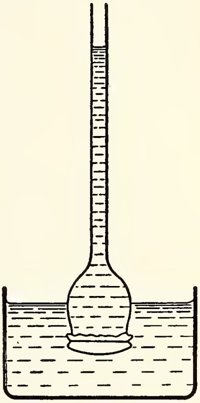

Fig. 9. Osmosis of water into syrup

221. Osmotic Pressure. The pressure which raises the height of the sugar solution is fundamentally due to the bombardment of the water molecules as they pass through the membrane — a bombardment which the sugar molecules cannot return. This is called osmotic pressure. It may be very great — in this case (if the sugar solution is concentrated) several hundred atmospheres, enough to raise the liquid a mile and a half if a membrane could be found to support this weight! The magnitude of the pressure is due to the large numbers of bombarding molecules. The density of population in a liquid is much greater than in a gas, and in both liquid and gas the molecules have the same average speed at the same temperature.

222. Applications to Biological Sciences. The phenomena which have been mentioned briefly in this chapter — cohesion, surface tension, capillarity, diffusion, osmosis — seem to have an almost unique importance to the student of biological sciences. Oxygen diffuses through the walls of the lungs into the blood stream, and carbon dioxide diffuses out. The rates of diffusion of the two gases through the moist membrane are as 1 : 30, in proportion to the relative solubilities of the two gases. The tissues of the body are threaded with capillaries; the tissues of the heart are an extraordinary network of capillaries; the heart is a sponge soaked in blood. The process of nutrition in the body involves capillary absorption and osmosis; digested carbohydrates, fatty acids, proteins, and salts diffuse through the walls of the intestines into the blood stream and from the blood stream through the capillary walls and into the cells of the tissues. The cell seems to be a semipermeable membrane designed to grow through a process of osmosis; and according to many biologists the ultimate explanation of cell behavior is to come in terms of these physical phenomena.

223. Respiration. We choose this single example to illustrate more concretely how these physical effects find application in life processes.

In the lungs a large area (about 1000 sq. ft.) of thin membrane is folded up into a spongelike mass; this membrane separates the air from the blood. The air in the lungs is about 15 per cent oxygen, 5 per cent carbon dioxide, and the remainder mostly nitrogen. The oxygen diffuses through the membrane, is absorbed by the blood stream, and is carried to the tissues. As the blood bathes these tissue cells, oxygen diffuses into the cell through the cell membrane and carbon dioxide diffuses out into the blood stream. The blood returns to the lungs supersaturated with carbon dioxide, which is released to the air. This is the general process of respiration.

But, thinking of the blood fluid as having more or less the properties of water, how can such a process take place? Carbon dioxide is 30 times more soluble than oxygen in water; how can the same number of gram-molecules of gas, absorbed as oxygen, be supersaturating when they return to the lungs as carbon dioxide? The answer is that the blood fluid contains finely divided particles of hemoglobin, which has great adsorbing power for oxygen. The presence of the adsorbent hemoglobin increases the oxygen-absorbing power of the blood 60-fold, allows it to carry the large quantities of oxygen required by the body, and to build up a supersaturating concentration of carbon dioxide.

At ordinary atmospheric pressure the hemoglobin particles are completely loaded with oxygen. They carry no more at higher pressures and not much less at half this pressure; however, for still lower pressures their adsorption decreases more or less proportionally with the pressure. Consequently our respiration mechanism works satisfactorily in high-pressure chambers and at altitudes up to some 3½ miles (Fig. 5–11) except that at high elevations more air is required to ventilate the lungs. But at higher altitudes the mechanism rapidly breaks down. Hemoglobin is an adsorber nicely designed by nature for beings destined to populate a world at elevations less than 3 miles above the sea.

The blood is always saturated with nitrogen. At constant pressure this is of no consequence, but it has very serious consequences for divers and workers in high-pressure caissons. The deep-sea diver experiences no difficulty as he works, blanketed by and breathing air at several atmospheres pressure. But his blood absorbs nitrogen in proportion to the pressure; he becomes charged very much like a bottle of soda pop. If he is brought to the surface quickly, the excess gas bubbles out, blocking the tissue capillaries and resulting in a paralysis which may be fatal. (This is known as " the bends.") It is necessary to raise the diver to the surface very, very slowly while he " decompresses " himself, allowing the excess gas to diffuse out through the lungs. In deep-sea operations today helium has been substituted for nitrogen in the atmosphere supplied to the diver; the lighter gas diffuses out from the blood more rapidly and greatly reduces the decompression period.

QUESTIONS

1. Give examples of cohesion, surface tension, and diffusion, from your own experience.

2. Have you seen a soap bubble contract when the pipe was removed from the mouth? Explain.

3. Define the coefficient of surface tension. What are two methods of determining it?

4. Is the unit of surface tension a dyne per centimeter or erg per square centimeter or both? Explain. Compare pressure: force per unit area or energy per unit displaced volume.

5. It is proposed to make a perpetual-motion machine by lifting water several centimeters above its general level by capillary attraction into a capillary tube bent near the top so that the water may drip out from the upper end of the tube. The falling drops may thus be made to turn a small paddle wheel. Where is the fallacy?

6. Explain Boyle's Law in terms of molecular motion.

7. What are the three kinds of molecules of which the atmosphere is principally constituted? Compare their average speeds.

8. Why is osmotic pressure much greater than customary gas pressure?

9. A good lubricating oil should have a high degree of: (*a*) adhesion; (*b*) diffusion; (*c*) osmotic pressure.

10. Brownian movement is due to: (*a*) bombardment of molecules; (*b*) reduction of surface tension near points of small particles; (*c*) diffusion.

Vocabulary: Cohesion, adhesion, (coefficient of) surface tension, interfacial tension, capillary attraction, absorption, adsorption, solubility, condensation, diffusion, Brownian movement, Kinetic Theory, osmosis, osmotic pressure.

PROBLEMS

1. How much work is required to blow a soap bubble 10 cm in diameter? (T for soap solution = approx. 20 c.g.s. units.)

2. How high will water rise in a capillary tube with a bore 1 mm in diameter?

3. A needle is 2 in. long. Assuming that it is not wetted, how heavy can it be and still float on water?

✓ How much work is required to break a mercury droplet 2 mm in diameter into eight (equal) droplets?

5. Derive the expression for the capillary height of a water film between two parallel glass plates a small distance (l) apart.

6. A vessel contains hydrogen at standard pressure and temperature; how many molecules strike against a square centimeter of surface in a second?

7. At 1 barye pressure and standard temperature how many hydrogen molecules will pass through a millimeter hole in a second?

CHAPTER 19

THERMAL EXPANSION OF SOLIDS AND LIQUIDS

224. Thermometers. The first problem in heat is to measure temperature. Everyone is familiar with several methods of determining the hotness of things. We often use the sense of touch. This obviously is not suited for exact measurement; some phenomenon is needed to act as telltale for hot and cold — something which will at the same time tell the degree of hotness and coldness. There are a number of such phenomena: (1) Usually things expand when heated (mercury thermometer). (2) It becomes somewhat harder to send electricity through a wire when it is heated (resistance thermometer). (3) When a junction of two metals is heated a small electrical current tends to flow (thermocouple). (4) When very hot, a substance becomes incandescent, first red hot, then red-yellow hot, then white hot (optical pyrometer). So there are a variety of ways in which thermometers can be made.

The ordinary mercury thermometer consists of a bulb and a capillary tube partly filled with mercury. Alcohol is sometimes used, but mercury has the advantage that the expansion is very nearly in direct proportion to the increase in true temperature. With rising temperature both glass and mercury expand, but the expansion of the mercury is the greater and it rises in the capillary tube.

225. Temperature Scales. Two temperature scales are in common use. They are compared in Fig. 1. In the centigrade scale the temperature of melting ice is taken as zero, and that of boiling water (under standard atmospheric pressure) is taken as 100°. Room temperature is usually somewhat over 20° C; the temperature of the body is about 37° C. This is the scale generally used in scientific work.

The thermometer in the home has the Fahrenheit scale. Here the melting point of ice is taken as 32° and the boiling point of water as 212°.[1] Hence an interval of 180 degrees on this scale (212°–32°) corresponds to 100 degrees of the centigrade scale; a 9-degree interval (Fahrenheit) corresponds to a 5-degree interval (centigrade). But the zero points are also different; so in changing units it is always best to start from the freezing point of water.

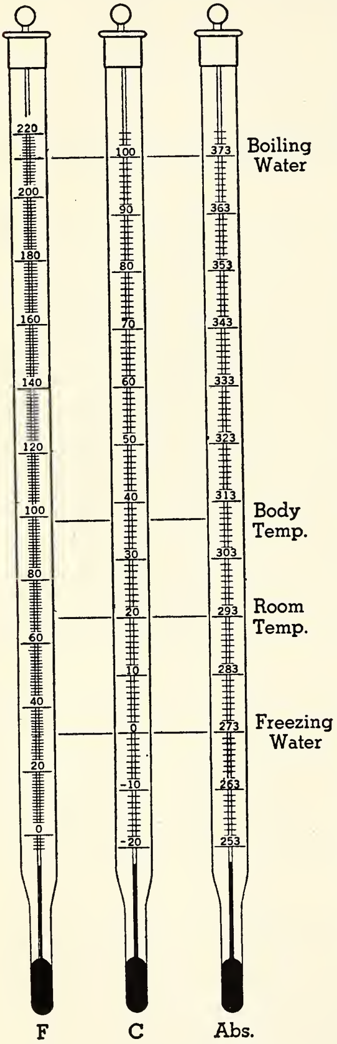

Fig. 1. Thermometer scales.

Derived from the centigrade scale is another, with the same degree but with the zero point 273° lower (the absolute scale).

Examples. 1. Change a temperature interval of 45 centigrade degrees to Fahrenheit. There will be $\frac{9}{5} \times 45 = 81$ Fahrenheit degrees. The zero points are not involved here.

2. Change 45° C to Fahrenheit. This is 45 centigrade degrees above freezing or 81 Fahrenheit degrees above freezing (32° F), hence 113° Fahrenheit.

3. Change 5° F to centigrade. This is 27 Fahrenheit degrees below freezing. $T = -15^\circ$ C.

226. Special Types of Thermometers. Crimson-tinted alcohol is often used in place of mercury, particularly in thermometers of cheaper grade. Alcohol can claim one advantage over mercury — it freezes at lower temperature. Pentane is sometimes used for measuring rather low temperatures (its freezing point is −160° C). Thermometers of the pointer type are actuated by the differential expansion of two metals (§229).

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[1] In his original scale, Fahrenheit used for standard points the temperature of an ice-salt mixture (the coldest temperature which he could obtain artificially) and body heat. He divided this interval into 12 degrees, later into 96. On our modern Fahrenheit scale, body temperature is about 98°.

TABLE 17

Characteristic Temperatures

	Fahrenheit	Centigrade	Absolute
Absolute zero	−459.8	−273.2	0
Liquid helium	−452	−269	4
Liquid hydrogen	−423	−253	20
Liquid air	−314	−192	81
“ Dry ice ”	−108	−78	195
Freezing mercury	− 38	−39	234
Melting ice	32	0	273
Boiling water	212	100	373
Boiling mercury	675	357	630
First red heat	1020	550	823
Melting copper	1970	1080	1353
Melting iron	2780	1530	1803
Melting carbon	6330	3500	3773
Surface of sun (approx.)	11000	6000	6000
Interior of sun and stars (rough estimate)			50,000,000

Resistance thermometers and thermocouples have a wide field of usefulness. They can be used to measure temperature only a few degrees above absolute zero and also to measure the temperature of an incandescent furnace. The compactness of the thermocouple (which is only a junction between two fine wires) is often an advantage; it may be used by the physicist to measure the temperature at different points in his apparatus or by the biologist to measure the temperature in living tissue. In the optical pyrometer the brightness of an incandescent object is matched against that of a standardized glowing wire; its use is obviously limited to high temperatures. It is used by power-plant engineers to measure furnace temperature.

227. Expansion of Solids. With few exceptions, substances expand when heated. Although the change in length is never exactly proportional to the rise in temperature it can usually be considered so to a first approximation. The actual elongation will also be proportional to the original length (L) — a yardstick will expand three times as much as a foot rule. Hence

$$\Delta L = \alpha L \cdot \Delta t, \qquad (2)$$

where Δt (or $t_2 - t_1$) is the change in temperature. The constant of proportionality α depends on the material. It is the change in length per unit length for a 1-degree rise in temperature. It is called the coefficient of linear expansion. A few characteristic values are given in Table 18. The values are given in parts per million. For example, cast iron expands 10 parts in a million for a 1-degree temperature rise, 1 part in a thousand for a 100-degree rise; the value of α is 0.000010.

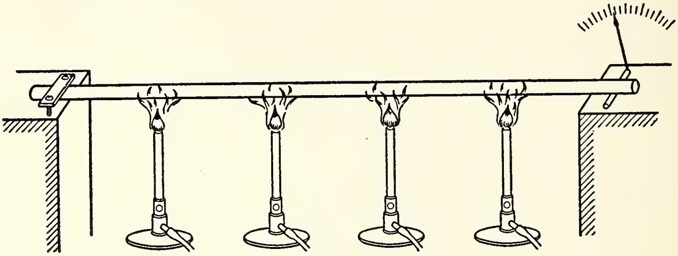

FIG. 2. Demonstration of thermal expansion.

High temperature means increased motion among the atoms and molecules in the body and so causes an increase in its size.

TABLE 18

COEFFICIENTS OF LINEAR AND VOLUME EXPANSION
(in parts per million per °C)

	Linear Coefficient	Volume Coefficient		Linear Coefficient	Volume Coefficient
Copper.....	17	51	Soda glass..	10	30
Iron (cast)..	10	30	Oak parallel to fiber..	5	
Steel.......	11–13	33–39	Oak across fiber.....	55	
Platinum...	9	27	Water.....		See §231
Tungsten...	4.3	13	Alcohol....		1100
Invar......	0.9	2.7	Mercury...		181
Quartz.....	0.5	1.5			
Pyrex glass.	3.6	11			

228. Examples of Thermal Expansion. Cannons are made of concentric steel tubes. Just before being assembled the outer tubes are heated; after being put into place they cool and shrink tightly over the inner tubes. . . In the same way iron tires are shrunk into place upon wagon wheels. . . The top of a fruit jar can often be removed by heating. . . Railroad rails expand in summer, contract in winter; in laying them, room must be left for expansion.

A drinking glass is very likely to break if filled with hot water — this is because of the sudden expansion of the inner surface while the interior of the glass and the outer surface are still cold. On the other hand, red-hot quartz can be plunged into water without breaking — quartz has an extremely small coefficient of expansion. In sealing metals into glass (for instance, the leads in the incandescent lamp) a metal must be chosen with nearly the same expansion coefficient as the glass. Platinum or a special alloy is used with common (soda) glass; tungsten is used with Pyrex.

The student can draw on his experience for many other examples of thermal expansion.

229. Differential Expansion. Advantage is taken of the difference in expansion of two metals in constructing metal thermometers and thermostats. Strips of the two metals are welded together face to face. If this composite strip is now heated it will bend, with the metal having the larger coefficient outside; this curvature gives this metal the somewhat greater length which it demands. In the thermostat, electrical contacts are opened and closed as the composite strip deflects more

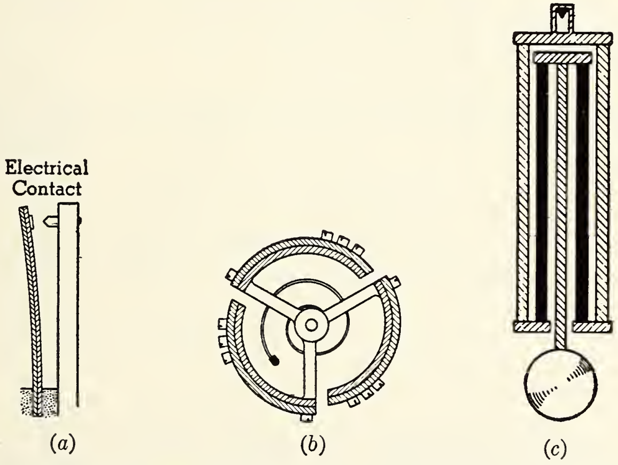

FIG. 3. Examples of thermal expansion: (*a*) thermostat; (*b*) compensated balance wheel; (*c*) compensated pendulum.

or less; thus an electric current is started and stopped, and this current in turn controls the heating source.

When a clock has a pendulum made of any of the common metals the thermal expansion is by no means negligible. There are several methods of compensating for this effect. The pendulum is sometimes made of two different metals, such as iron and brass, so arranged that the one will expand upward, the other downward (Fig. 3). The lengths of the two metals are made inversely proportional to their expansion coefficients; thus the length is independent of temperature. Clock pendulums are often made of invar, an alloy which is, as its name implies, almost invariable with temperature. In certain laboratory timepieces of extreme precision a bar of quartz is used for a pendulum.

In chronometers and in better-grade watches the expansion of the balance wheel is compensated by an ingenious use of differential expansion. The rim of the balance wheel is made of the composite strip described above. The rim is split in several places, and the strip is so arranged that while the spokes are expanding outward the rim is bending inward by just enough to keep the moment of inertia constant.

Example. A grandfather clock has an iron pendulum 1 meter long. (1) Find the change in length for a temperature rise of 10°. From Table 18 $\alpha = 0.000010$;

$$\Delta L = 0.000010 \times 100 \times 10 = 0.01 \text{ cm}.$$

The relative change in length is 1 part in 10,000. (2) How many seconds will this clock lose per day due to this expansion? $T \propto \sqrt{L}$. Hence

$$\frac{\Delta T}{T} = \tfrac{1}{2}\frac{\Delta L}{L} = 1 \text{ part in } 20{,}000 \text{ (approximation; App. 2)}$$

or about 4 sec. in 86,400.

230. Volume Expansion. In general, liquids increase in volume when heated and the volume change is nearly proportional to the rise in temperature.

$$\Delta V = \beta V \cdot \Delta T. \qquad (2')$$

(Evidently the volume change is also proportional to the original volume.) The constant β is called the coefficient of volume expansion.

The same equation holds for the volume expansion of solids.

Since the volume of a solid is a product of length by breadth by thickness, and each of these dimensions expands linearly with rising temperature, the coefficient of volume expansion (β) is three times as great as the coefficient of linear expansion (α). For example, cast iron increases 0.1 per cent in length (Table 18) and 0.3 per cent in volume for a temperature rise of 100°.

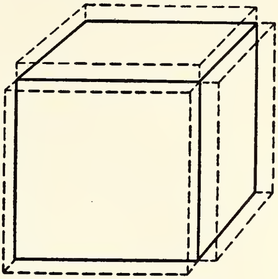

Fig. 4. $\frac{\Delta V}{V} = 3\frac{\Delta L}{L}$.

231. Thermal Expansion of Water. Thermal expansion is never exactly proportional to temperature. For water near the freezing point the proportion breaks down entirely and for several degrees above zero the volume actually decreases with rising temperature. Figure 5 shows how the volume of water changes with temperature. The density of water is greatest at 4° C. The phenomenon is of great importance in nature. At summer temperatures the colder water being heavier is found on the bottom of lakes; but just before freezing sets in, the

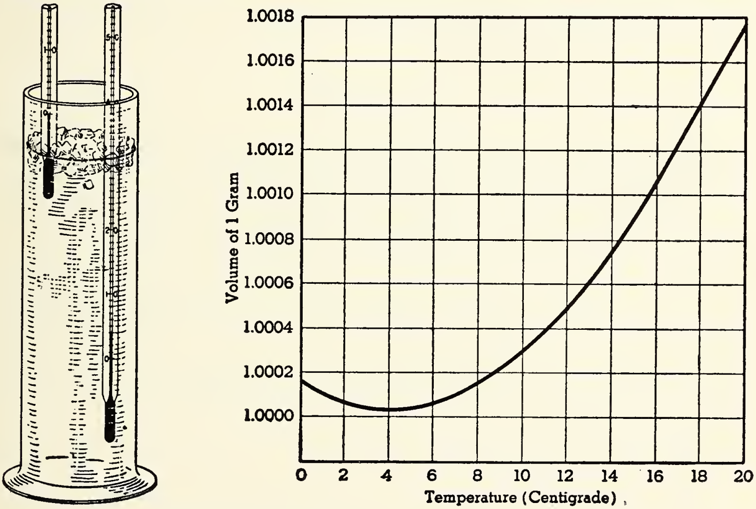

Fig. 5. Thermal expansion of water. Heaviest water (4° C) sinks to bottom.

situation is reversed, and the colder, lighter water is on the surface and freezing sets in from the top. Were it otherwise, lakes would be frozen solid; this would kill many forms of aquatic life, and if it be true, as the biologists affirm, that life started in the sea, would have seriously discommoded our ancestors.

Such contraction with temperature is very exceptional. We shall find that when water freezes (at 0° C), its volume increases about 10 per cent; this anomalous expansion for temperatures just above the melting point must be accounted for by supposing that small clusters of molecules with this larger icelike structure still remain in the liquid.

What an unsatisfactory medium for a thermometer water would be! But is the expansion even of mercury exactly proportional to the true temperature? Indeed, what is the meaning of "true temperature"? And, going somewhat beyond the qualitative concepts of hotter and colder, what is the fundamental meaning of "temperature" itself? We shall approach the answers to these questions in the following chapter.

QUESTIONS

1. What phenomena other than expansion are used to measure temperature? Is expansion of solids ever used for this purpose?

2. In what two respects do the Fahrenheit and centigrade scales differ? What is the historical reason for Fahrenheit's choice of his scale?

3. Why would a water thermometer give ambiguous readings in the range 0°–8° C?

4. If the mercury column in a thermometer lengthens exactly 10 cm when taken from ice into boiling water, does 1 mm correspond exactly to 1 centigrade degree?

5. To what quantities is thermal expansion in length and volume proportional?

6. Will the error in a pendulum clock due to thermal expansion be different for long and short pendulums?

7. Why is the volume coefficient of expansion three times the linear?

8. Give five examples of thermal expansion not mentioned in the text.

9. Show that, if the temperature of a lake is lowered slowly at the surface, all the water will come to the temperature 4° C before any of it is appreciably colder.

10. Three strips of brass form an isosceles triangle with base 3 in. and height 12 in. When the temperature is raised will the angles change, and if so how?

11. To measure the temperature of the human body one could *not* well use: (*a*) an optical pyrometer; (*b*) a thermocouple; (*c*) a resistance thermometer.

12. If a cube expands by 1 per cent in each direction the relative change in volume is: (*a*) 1 per cent; (*b*) 3 per cent; (*c*) 1 part per million.

Vocabulary: Thermocouple, optical pyrometer, coefficient of linear (volume) expansion, invar.

PROBLEMS

1. Change 32° F, 68° F, 0° F to centigrade and absolute.

2. Find how many degrees above or below the melting point of ice each of the following temperatures is: 77° F; 14° F; 20° C; −4° F; −20° C; 900° C.

3. Change each of the above temperatures (problem 2) to the opposite scale. Also express them on the absolute scale.

4. Give temperature of coldest and hottest day of your experience in Fahrenheit, centigrade, and absolute.

5. By how much will (*a*) a 10-meter steel rail, (*b*) a foot rule (steel) expand when the temperature changes from −20° C in winter to 40° C in summer?

6. Find the change in length, diameter, and volume of a copper rod 80 cm long and 1 cm in diameter for a temperature change of 100° C.

7. Find the percentage change in length, diameter, and volume of the rod in problem 6.

8. A 10-cm iron cube is placed first in ice water, next in boiling water. What is the percentage change in (*a*) length of an edge, (*b*) area of a face, (*c*) volume?

9. If the rod in Fig. 2 is of copper, 1 meter long, if the roller is 2 mm in diameter, and the pointer 10 cm long, how much will the pointer move when the rod is heated from room temperature to 200° C?

10. A 1-liter quartz beaker contains an 800-cc block of copper and 200 cc of alcohol. How much will overflow when the temperature is raised 60° C?

11. If a liter jar (soda glass) is exactly full of mercury at 20° C, how much will overflow when the temperature is raised to 100° C?

12. The bulb of an alcohol thermometer has a volume of 0.3 cc. If the bore has a diameter of 1 mm, how far apart will the degree (C) marks be?

13. How much does water expand when its temperature is raised from 32° to 46° F?

14. Find the coefficient of expansion of water near room temperature (Fig. 5). Compare with the coefficient of expansion of alcohol.

CHAPTER 20

EXPANSION OF GASES

The last chapter introduced the concept of temperature; we now ask about its fundamental significance. That chapter treated of the expansion of solids and liquids: some substances expand more, some less; for two degrees the expansion is usually about twice that for one; each substance has its own constant of proportionality. Without going very far indeed into advanced physics this is nearly all that can be said of the expansion of solids and liquids. Important as its applications are in nature, this subject hardly carries us very deeply into the meaning of hot and cold. The situation is very different with gases. We shall find one general law of expansion for all gases, and in this general law we shall find the mechanical interpretation of temperature.

232. Coefficient of Expansion. Either the volume or the pressure of a gas must change when it is heated. When the house is heated the air expands, escaping through the windows and doors; when air in an automobile tire becomes warmer its pressure increases.

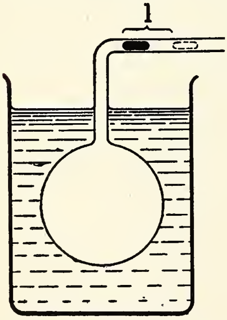

Fig. 1. Thermal expansion at constant pressure.

All gases at constant pressure have nearly the same coefficient of volume expansion. The volume expansion can be measured by means of the apparatus illustrated in Fig. 1. A large bulb contains the gas in question. This bulb is immersed in a vessel of water which can be heated to any desired temperature. Suppose that we start at 0° C. As the temperature is raised, the pressure remains constant (atmospheric), the air expands, and it pushes the mercury slug out a distance l. If A is the cross-section of the small tube the change in volume (ΔV) equals Al.

As the result of such an experiment, *with any gas*, we shall find that the volume increases by 1 part in 273 for each degree that the temperature rises above 0° C. In other words,

$$\frac{\Delta V}{V_\circ} = \frac{1}{273}\Delta t\,,$$

where $V_\circ$ is the volume at 0° C and Δt means $(t - 0)$.

233. Charles' Law. Now this relation can be put into a simpler form. Let us adopt a new zero point for our temperature scale, 273° below the usual centigrade zero. This is *absolute zero*, and temperature measured from this zero point is called absolute temperature (§225). Absolute temperature will be expressed by the capital letter T. Then 0° C becomes 273° absolute, and this equation can be written:

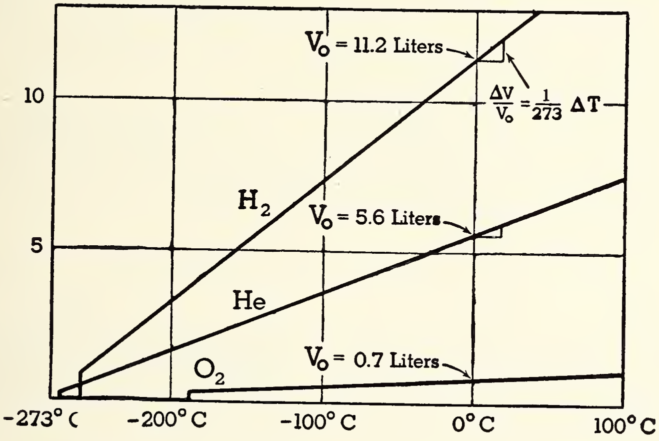

FIG. 2. Charles' Law: Volume of 1 gram of gas at atmospheric pressure.

$$\frac{\Delta V}{V_\circ} = \frac{\Delta T}{T_\circ} \quad \text{or} \quad \frac{V - V_\circ}{V_\circ} = \frac{T - T_\circ}{T_\circ}.$$

Simplifying (adding 1 — that is $V_\circ/V_\circ$ or $T_\circ/T_\circ$ — to each side of the equation) we have

$$\frac{V}{V_\circ} = \frac{T}{T_\circ}.$$

Thus (since the denominators are constant) V varies directly with T. *The volume of a gas at constant pressure is proportional* *to the absolute temperature* ($t + 273°$). This is Charles'[1] Law (first form). The law is usually written as

$$\frac{V}{T} = \text{constant} \quad \text{or} \quad V_1 : V_2 = T_1 : T_2 \qquad (3')$$

(Charles' Law, first form).

Figure 2 shows this proportional relationship between volume and absolute temperature. Each gas at 0° C shows the increase of 1 part in 273 per degree. When the temperature is lowered the volume decreases in this ratio, apparently approaching a zero volume at −273° C. (Each gas condenses into a liquid before this temperature is reached; this is indicated by the sudden volume decrease in the curves.)

234. Pressure and Temperature. If a gas is heated, the volume being kept constant, the pressure increases; and for every gas it is found that the pressure is directly proportional to the absolute temperature (volume constant):

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \text{(Charles' Law, second form).} \qquad (3'')$$

This relation can be logically deduced from eq. (3′) (above) and eq. (3) in §52 (Boyle's Law). Since, from these equations, the volume varies as T and $1/P$ independently, it must be jointly proportional to these quantities.

$$V \propto \frac{T}{P}. \qquad (a)$$

Hence if V is constant $P \propto T$.

Example. If an empty bottle is corked at room temperature and standard pressure, what will be the gas pressure in the bottle when it is placed in boiling water?

$$T_1 = 20° \text{ C} = 293° \text{ abs}; \; T_2 = 100° \text{ C} = 373° \text{ abs}.$$

$$P_1 = 76 \text{ cm (Hg)}; \; P_2 = ?$$

$$\frac{76}{P_2} = \frac{293}{373}; \; P_2 = 97 \text{ cm (Hg)}.$$

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[1] Charles, French mathematician and physicist, may be considered the father of aeronautics — he was the first to inflate balloons with hydrogen (1783). The law is also sometimes associated with Gay-Lussac, chemist, physicist (and another early balloonist), who investigated expansion of gases some years later.

235. The Meaning of Absolute Zero. The pressure which a gas exerts against the walls of a vessel is due to the pounding of the molecules as they continually strike and rebound. But the pressure decreases with decreasing temperature; it would vanish at absolute zero. Evidently absolute zero is the temperature at which the molecules are at rest. This lowest of temperatures can never be exactly reached, for it will never be possible to remove absolutely all the energy from a body. However, physicists, struggling for lower and lower temperatures, have come within a small fraction of a degree of absolute zero.

Many other phenomena show a similar convergence toward this temperature, −273°. The electrical resistance of metals decreases with lowering temperatures very much as do the volumes of gases in Fig. 2. (See Fig. **33**–2.)

236. The Gas Thermometer. Figure 3 represents the apparatus by which Charles' Law for pressure can be verified in the laboratory. It consists of a large bulb communicating with a mercury column. The mercury is raised to a reference mark at *a*, confining the gas to definite volume. If the bulb is heated the gas tends to expand, but the movable column of mercury is now raised (to *b*) until the gas is compressed back to its original volume. The difference between the heights *b* and *a* then gives the pressure above atmospheric.

In a gas like helium or hydrogen the expansion or the pressure increase is very closely proportional to the rise in true temperature. Hence gas thermometers are used as standards. The apparatus just described is the constant-volume gas thermometer. The constant-pressure thermometer (Fig. 1) is less frequently used.

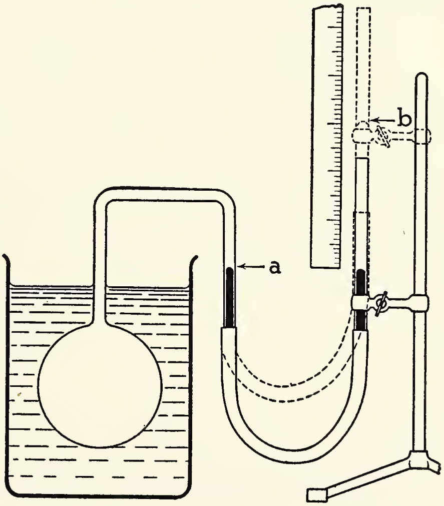

FIG. 3. Constant-volume gas thermometer.

237. The General Gas Law. Since the temperature varies as pressure and as volume separately, it is proportional to their product:[1]

$$PV = RT \quad \text{(General Gas Law).} \qquad (3''')$$

This is the General Gas Law, where R is a constant of proportionality. It is sometimes more convenient to use it expressed as

$$\frac{PV}{T} = \text{constant}; \quad \text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}. \qquad (3'''')$$

This law embodies both Boyle's Law and Charles' Law; for if T is constant, PV = constant (Boyle's Law); if V is constant, P/T = constant (Charles' Law). The General Gas Law gives the expansion or compression of a gas produced by an alteration of pressure and temperature.

Examples. 1. Find the expansion of a liter of air when the temperature rises from room temperature (20° C) to 100° C all at atmospheric pressure. Here P is constant and the law reduces to Charles' Law.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2};$$

$$\frac{1 \text{ liter}}{293} = \frac{V_2 \text{ liters}}{373}$$

and the new volume V_2 can be found. (Temperatures must be changed to the absolute scale.)

2. Find the change in pressure when a liter of gas, originally at a pressure of 70 cm of mercury, is heated from 27° C to 227° C and the volume is halved.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}.$$

$$\frac{70 \times 1}{300} = \frac{P_2 \times \frac{1}{2}}{500}; \quad P_2 = 233 \text{ cm of mercury.}$$

238. The Proportionality Constant in the General Gas Law; Avogadro's Hypothesis. Quite evidently the value of the constant in the General Gas Law depends upon the quantity of the gas chosen. At the same temperature and pressure a kilogram of gas will occupy a thousand times the volume of a gram. As can be seen from Fig. 2, its value will also depend upon the nature of the gas. But a great simplification occurs if it is agreed to compare gram-molecular weights of different gases (2 grams of hydrogen, 4 grams of helium, 32 grams of oxygen, etc., Table 4). By Avogadro's Hypothesis (§51) *a gram-molecular weight of any gas occupies 22,400 cc under standard conditions* (1,013,000 baryes and 273° abs). If these three values are substituted in (3''') we have

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[1] This is the same as eq. (a).

$$PV = R_0T \tag{3}$$

(General Gas Law for 1 gram-molecular weight),

$$1.013 \times 10^6 \times 2.24 \times 10^4 = R_0\, 273,$$

or $R_0 = 83{,}000{,}000$ (R for a gram-molecular weight).

If, then, it is agreed to consider gram-molecular weights of different gases the constant of proportionality assumes a definite value — the General Gas Law can be used not only to get the relative changes in volumes but also to get the actual volume occupied by a gram molecule. R_0 is the *Gas Constant.*[1]

The volume occupied by any other quantity is just proportional to the number (N) of gram-molecular weights in it. For any quantity of gas, consisting of N gram-molecular weights, the proportionality constant (R) in the General Gas Law is equal to NR_0.

Example. Find the approximate volume occupied by a gram of carbon dioxide (CO_2) at 30° C and 3 atmospheres pressure.

Molecular weight of $CO_2 = 44\ (12 + 16 + 16)$; for 1 gram $N = 1/44$.
$R =$ (approx.) 2,000,000. Temperature = (approx.) 300° abs.
Pressure = (approx.) 3,000,000 dynes per square centimeter.

$$V = \frac{RT}{P} = \frac{2 \times 10^6 \times 3 \times 10^2}{3 \times 10^6} = 200 \text{ cc, approx.}$$

239. Van der Waals' Equation. We have seen (§55) that Boyle's Law is never exactly true, that for vapors and for gases at high concentration the deviations from it may be quite large. The same must be said of Charles' Law. There is nothing in

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[1] The exact value of R_0 is 83,136,000. Often in the theory of heat the gas constant for a single molecule (instead of for a gram molecule) is used. It is called Boltzmann's Constant (κ). Since there are 6.02×10^{23} molecules in a gram molecule

$$\kappa = 8.3 \times 10^7 \div 6.02 \times 10^{23} = 1.38 \times 10^{-16}.$$

Charles' Law to account for the sudden breaks in the curves in Fig. 2 (liquefaction). Two factors have been left out of consideration:

(1) The molecules attract one another; hence the pressure (P) which we measure at the boundary of the gas is somewhat less than the internal pressure in the gas. (Compare Fig. **18–4.**)

(2) The molecules themselves have a certain volume and hence reduce the free space available for molecular motion.

Van der Waals introduced these considerations into the General Gas Law. He added to P a term (a/V^2) to take account of molecular attraction; from V he subtracted a term b to allow for the volume occupied by the molecules. (a and b are constants.) Van der Waals' Equation then is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

and this holds with considerable precision not only for gases and vapors, but, less exactly, even for the liquid state.

240. Kinetic Theory. In treating the subject of heat it is customary to forsake the c.g.s. units of force and energy and to introduce " degrees " of temperature and " calories " of heat, as if heat were a thing apart from mechanics. Indeed, so it was for Boyle and Black and Charles. In a later chapter we shall show how Joule, in the middle of the last century, demonstrated that heat was a form of energy and could quite properly be measured in ergs or joules or kilojoules instead of in calories. Then followed (in the hands of Maxwell and Boltzmann) a brilliant interpretation of heat as molecular motion — the kinetic theory of gases. According to this theory the *absolute temperature is proportional to the average kinetic energy per molecule.* (We shall derive this relation in the following section.)

It has already been shown (§217) that the pressure of a gas is proportional to its kinetic energy *per unit volume.* This kinetic energy disappears at absolute zero. As the temperature of a gas rises the kinetic energy of the molecules increases, and therefore, if the volume is kept constant, the kinetic energy per unit volume (pressure) increases — this accounts for the General Gas Law.

The molecules are also in motion in solids and liquids, but here, closely hemmed in by others, for the most part they simply vibrate back and forth. But the temperature is still a measure of their average kinetic energy. As their energy increases the molecules demand more room — solids and liquids (in general) expand as the temperature rises. If two bodies are in contact and one is hotter than the other, heat flows — energy is passed along from the richer to the poorer molecules until there is (on the average) an equal distribution. So the various phenomena of heat can be explained in terms of molecular motion.

241.* Temperature and Kinetic Energy of Molecules. In §217 it was shown that pressure was proportional to the kinetic energy per unit volume of the gas; or, more specifically, $P = \frac{1}{3} n_1 mv^2$. Considering a gram molecule of any gas, we substitute this value in the General Gas Law (3)

$$R_0 T = \tfrac{1}{3} n_1 mv^2 \cdot V.$$

Since n_1 is the number of molecules per cubic centimeter and V the volume of a gram molecule, $n_1 \cdot V = N_0$, the number of molecules in a gram molecule (Avogadro's Number) and

$$\frac{R_0 T}{N_0} = \tfrac{1}{3} mv^2,$$

or, since R_0/N_0 equals Boltzmann's constant, κ,

$$\kappa T = \tfrac{2}{3} \left(\tfrac{1}{2} mv^2\right),^{1} \qquad (1')$$

κT equals two-thirds of the average kinetic energy of the individual molecules.

242.* Thermodynamic Definition of Temperature. Temperature, a measure of the average kinetic energy (translational) of the molecules — this is the interpretation of kinetic theory. But it is after all only an interpretation; closely true, but still, with our growing knowledge of the molecule, needing some qualification. The fundamental laws of heat were established before there was a kinetic theory, and they find an experimental basis in heat phenomena which are independent of any molecular interpretation. Temperature, proportional to the height of a column of mercury — this is a serviceable definition, accurate enough for many purposes. Temperature, proportional to the pressure (at constant volume) of a gas like hydrogen or helium — this is a better definition. But, as is shown in more advanced courses, temperature is exactly defined in terms of entropy and the laws of thermodynamics (Chapter 24); or — what amounts to the same thing — temperature is exactly defined as proportional to the pressure (at constant volume) of an "ideal gas." The "ideal gas" is one with no molecular volume and no cohesive forces, a gas for which the General Gas Law is exactly true. By taking account of the correction terms, such as those in van der Waals' Equation, pressures of the hydrogen gas thermometer can be reduced to these ideal pressures to give the

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[1] Or $R_0 T = \frac{2}{3} (\frac{1}{2} Mv^2)$, where M is the molecular weight. $R_0 T$ equals two-thirds of the kinetic energy per gram molecule. This applies only to translational energy, however.

true temperature. These corrections are small and seldom of practical importance. The important fact is that there does exist an exact definition of temperature, independent of the properties of any particular substance, or of any molecular theory, for which the laws of heat (thermodynamics) are exactly true.

243.* Adiabatic Compression. When a gas is compressed, as by pushing a piston into a closed cylinder, the molecules rebound from the approaching piston wall with increased speed. The increase in kinetic energy of the molecules equals the work expended upon the piston (§283). Compression produces a heating (and expansion a cooling) of the gas. When the conditions are such that the generated heat (or cold) remains in the gas — no loss or gain of heat to the surroundings — the compression (or expansion) is said to be *adiabatic* (meaning "no heat flow"). For air, halving the volume adiabatically increases the absolute temperature approximately in the ratio 3 : 4 (33 per cent) — for example, from 300° (room temperature) to 400° absolute. For an eight-fold compression (halving the volume thrice) the temperature change is in the ratio $\frac{4}{3} \times \frac{4}{3} \times \frac{4}{3} = \frac{64}{27}$. As a lecture demonstration a piece of tinder is placed in a closed cylinder and the air suddenly compressed; the heat generated is sufficient to ignite the tinder. In one form of the combustion engine (Diesel, §293) there are no spark plugs; instead the fuel oil is ignited by the heat generated by the compression of the gases in the cylinder.

In testing Boyle's Law (§53) the gas was compressed slowly in a small tube, the heat generated was lost to the walls, and the temperature remained unchanged (*isothermal* compression). It is, of course, only under such isothermal conditions that Boyle's Law

$$\frac{P_1}{P_2} = \frac{V_2}{V_1} \quad \text{(isothermal compression)}$$

can be applied. For a sudden compression or for the compression of a very large volume of gas the process is adiabatic; the temperature of the gas rises and the pressure therefore increases somewhat more than for an equal isothermal compression. For example, for air, halving the volume adiabatically produces a $2\frac{2}{3}$-fold increase in pressure: $\frac{4}{3}$ (due to the temperature rise: Charles' Law) × 2 (due to the volume decrease: Boyle's Law).

The temperature rise under a given compression depends on the nature of the gas; it can be shown to depend upon the ratio ($\mathbf{r}$) of the specific heats at constant pressure and constant volume (§285), and in general the relation between pressure and volume is:

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\mathbf{r}} \quad \text{(adiabatic compression).}$$

QUESTIONS

1. What law gives the expansion of a bubble rising in a lake? Of air as it enters the house? What law gives the pressure change when a tire becomes heated?

2. At constant pressure what is the relative increase in volume of a gas when the temperature rises from 0° to 10° C? From 0° to 273° C?

3. A can is filled with air at room temperature, sealed, and placed in boiling water. Does the pressure increase approximately 25, 50, or 100 per cent?

4. Three liter flasks contain respectively 1 gram of H_2, of O_2, and of CO_2. Which flask has the greatest pressure and which the least?

5. How does the pressure change if the volume of a gas is doubled and the (absolute) temperature is doubled? If the first is doubled and the second halved?

6. What is the meaning of the Gas Constant? of Boltzmann's Constant?

7. At constant pressure hydrogen expands by about 1 part in 273 per degree centigrade: (*a*) always; (*b*) except near its liquefaction point (−253° C); (*c*) when the initial temperature is 0° C.

8. Departures from the General Gas Law are due (in part) to: (*a*) viscosity of gases; (*b*) Brownian movement; (*c*) cohesive forces between molecules.

Vocabulary: Charles' (Gay-Lussac's) Law, constant-volume (pressure) thermometer, General Gas Law, Gas Constant, Boltzmann's Constant, van der Waals' Equation, ideal gas, isothermal (adiabatic) expansion.

PROBLEMS

1. A bottle is stoppered at the temperature of boiling water and then cooled to room temperature. Approximately what is the decrease in pressure?

2. At 20° C a tire is inflated to 30 lb. pressure (in excess of atmospheric). Find its pressure at 40° C.

3. A house is 30 by 30 by 30 ft. How much air leaves the house when its temperature is raised from 50° to 70° F?

4. If the volume of the bulb in Fig. 1 is 1 liter and the cross-section of the tube is $\frac{1}{2}$ sq. cm, how far will the mercury move when the temperature is increased from 20 to 25° C?

5. A bubble 1 cc in size starts from 34 ft. under water where the temperature is 10° C and rises to the surface where the temperature is 30° C. How large will it be when it reaches the surface?

6. In the 14-mile flight into the stratosphere, the temperature dropped from 0° C to −60° C and the pressure decreased to 3 cm of mercury. How many fold did the volume of the helium gas in the balloon increase?

7. Find the volume occupied by a gram molecule of any gas at a pressure of 100 cm of mercury and at 273° C.

8. If a gas has a pressure of 1 atmosphere when its volume is 10 cu. in. and its temperature is that of melting ice, find its pressure when the volume is 15 cu. in. and the temperature is that of boiling water.

9. Air originally at 27° C and 1 atmosphere pressure is compressed adiabatically to one-eighth of its original volume, the temperature rising to 427° C. What is the final pressure?

10. A gas with volume 20 cc, temperature 320° abs, has a pressure of 160 cm (Hg). Suppose that it expands adiabatically, cooling as shown in the following table. Indicate the corresponding pressures. Plot your values in Fig. 5–10, in contrast to the isothermal expansion.

Volume	Temperature	Pressure
20 cc	320° abs.	160 cm Hg
40 cc	240° abs.	?
80 cc	180° abs.	?

11. Find the pressure in baryes and in atmospheres exerted by 1 gram of helium in a liter vessel at 27° C.

12. Find the average velocity of oxygen molecules at 27° C; of mercury molecules (monatomic) at 300° C.

CHAPTER 21

QUANTITY OF HEAT

I trust that no one has confused temperature and the quantity of heat. Obviously it takes four times as much heat to raise a gallon of water from 0° to 100° as it does to raise the temperature of a quart by the same number of degrees.

It takes the same amount of heat to warm 10 grams of water 1 degree, to warm 5 grams 2 degrees, and to warm 1 gram 10 degrees. And this same quantity of heat will raise the temperature of a gram of iron 100 degrees and a gram of lead 300 degrees.

244. Heat Units. The quantity of heat required to increase the temperature of 1 gram of water 1 degree centigrade is taken as the heat unit; it is called a **calorie.** Since the heat required is jointly proportional to the mass heated (m) and the change in temperature (Δt), the heat used in warming a given quantity of *water* is

$$H \text{ (calories)} = m \text{ (grams)} \cdot \Delta t \text{ (degrees centigrade).} \qquad (a)$$

The "large calorie" is also sometimes used — 1000 calories. Or the large calorie is the heat required to raise a kilogram of water 1 degree centigrade.

The heat capacity of water varies somewhat with temperature (first definitely shown by Rowland). For exactness it is therefore necessary to specify the temperature at which the calorie is defined. In common use are the 15° calorie (the quantity of heat required to raise a gram of water from 14.5° C to 15.5° C), the 20° calorie, the 4° calorie, and the mean calorie (the average value in the range from 0° C to 100° C). These values differ among themselves by as much as 0.6 per cent. In many experiments the precision of the observations is not such as to necessitate a distinction between these values and the difference will be ignored in numerical problems. The heat values given in the tables will be in terms of the 15° calorie.

Examples. How many calories of heat are required to heat a liter of water from the freezing point to the boiling point? How much heat passes into the room when 10 gal.[1] of water cool from the boiling point to room temperature?

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[1] Consider a quart as approximately a liter.

Another heat unit, often used by engineers, is the "B.t.u." (British thermal unit). It is the heat required to raise the temperature of a pound of water 1 degree Fahrenheit. How much heat is required to raise a pint (practically a pound) of water to the boiling point? Observe that eq. (*a*) holds using B.t.u.'s and pounds-mass and degrees Fahrenheit. Since there are 2.2 pounds in a kilogram and $\frac{9}{5}$ Fahrenheit degrees in a centigrade degree, a B.t.u. equals approximately one-fourth of a large calorie:

$$1 \text{ B.t.u.} = \frac{1}{2.2} \times \frac{5}{9} = 0.252 \text{ large calorie.}$$

245. Principle of Heat Exchange (Conservation of Energy). Heat quantities are measured in a "calorimeter." This usually consists of a copper vessel, nickel plated to reduce radiation, shielded from air currents and in general thermally insulated as well as possible from its surroundings. Usually the calorimeter is partly filled with water and its gain or loss of heat measured in terms of the temperature change (eq. *a*).

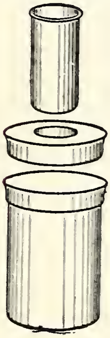

FIG. 1. Calorimeter (with shield).

All measurements of quantities of heat involve the following principle, discovered by Black (1760) but today accepted as a simple consequence of Conservation of Energy: when two bodies of different temperatures are placed in contact, heat flows from the hotter to the colder, and the quantity of heat leaving the hotter body equals that entering the colder. *Heat given out equals heat taken in.*

Example. Suppose that we have in a calorimeter a liter of water at 20° C. Into this we pour 200 cc (grams) of water at 80° C. What will be the temperature of the mixture? Neglect the heat capacity of the calorimeter.

$$\begin{aligned} \text{Heat given out by hot water} &= \text{Heat absorbed by cold water.} \\ m_{\text{hot}}(t_{\text{hot}} - t_{\text{mix.}}) &= m_{\text{cold}}\ (t_{\text{mix.}} - t_{\text{cold}}) \\ 200\ (80 - x) &= 1000\ (x - 20) \\ 1200\,x &= 16{,}000 + 20{,}000 \\ x &= 30° \text{ C.} \end{aligned}$$

246. Heat Capacity and Specific Heat. The quantity of heat (calories or B.t.u.'s) required to raise the temperature of a given body by 1 degree (centigrade or Fahrenheit) is called its **heat capacity.** It depends upon the mass and chemical nature of the body. By eq. (*a*) the heat capacity of a body of water equals its mass.

Most other substances are more easily heated, gram for gram, than water. If two vessels, one containing 100 grams of water and the other 100 grams of mercury, are placed over a Bunsen burner, the temperature of the mercury will rise thirty times as fast as that of the water. (The experiment had best not be tried since mercury vapor is poisonous.) The heat in calories (or B.t.u.'s) required to raise 1 gram (pound) of a substance 1 degree centigrade (Fahrenheit) is its **specific heat.** Specific heats of several common substances are given in Table 19.

If s represents the specific heat of a substance the heat required to raise m grams (pounds) Δt degrees is

$$\text{Heat in} = sm\,\Delta t, \tag{4}$$

and the heat capacity of the body is sm.

When the temperature is lowered, $sm\,\Delta t$ is the heat leaving the body.

TABLE 19

	Specific Heat		Specific Heat	Atomic Weight
Water......	1	Aluminum..	0.21	27
Ice.........	0.5	Sulfur......	0.17	32
Glass.......	0.2	Iron.......	0.11	56
Granite.....	0.2	Copper.....	0.09	63
Concrete....	0.2	Lead.......	0.03	207

Experiment: Determination of specific heat by method of mixtures. Weigh out a quantity of lead shot. Heat to the temperature of boiling water. Then drop the hot shot into a known quantity of cold water and measure the rise of temperature. The specific heat can then be found as in the following numerical example.

Example. In a certain experiment there were 300 grams of lead at 100° C and 100 grams of water at 20° C. The final temperature ($t_{\text{mix.}}$) was 26.8° C.

$$\begin{aligned}\text{Heat out from lead} &= \text{Heat into water}\\ s\,300\,(100 - 26.8) &= 100\,(26.8 - 20)\\ s &= \frac{100 \times 6.8}{300 \times 73.2} = 0.031.\end{aligned}$$

Heat measurements although simple in principle are difficult to carry out with high precision. In the above experiment some heat is lost from the lead while it is being transferred to the water; enough time must be allowed for the lead and water to come to temperature equilibrium, yet during this time the calorimeter, even with the best of insulation, is gaining or losing heat. In the laboratory the student will learn to correct for the more serious errors.

We have in the foregoing examples in calorimetry neglected the heat required to warm the calorimeter from t_{cold} to $t_{\text{mix.}}$ If $s_{\text{cal.}}$ represents its specific heat and $m_{\text{cal.}}$ its mass, then the heat entering the calorimeter is $s_{\text{cal.}}m_{\text{cal.}}\ (t_{\text{mix.}} - t_{\text{cold}})$; this term should be added to the right side of the equation used in this last example. Thus when heated lead is dropped into the water in the calorimeter the complete equation is

$$\text{Heat out} = \text{Heat in}$$

$$s_{\text{lead}}m_{\text{lead}}\,(t_{\text{lead}} - t_{\text{mix.}}) = m_{\text{water}}\,(t_{\text{mix.}} - t_{\text{water}}) + s_{\text{cal.}}m_{\text{cal.}}\,(t_{\text{mix.}} - t_{\text{water}}).$$

$s_{\text{cal.}}m_{\text{cal.}}$ is the heat capacity of the calorimeter, and this is called its " water equivalent." If the calorimeter is of copper and weighs 50 grams what is its water equivalent?

247. Law of Dulong and Petit. Now why is it harder to heat one substance than another? As far as the elements in the solid state are concerned the answer is very easily given but it will be left for the student to find. There is a very simple relation, which the student cannot overlook, between the specific heats of the elements in Table 19 and their atomic weights.

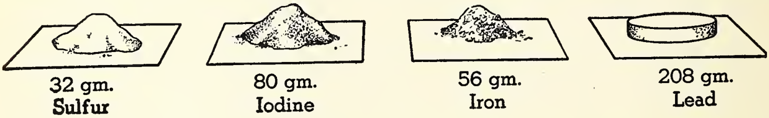
32 gm.
Sulfur
80 gm.
Iodine
56 gm.
Iron
208 gm.
Lead

FIG. 2. Gram-atomic weights have equal heat capacities (Dulong and Petit).

(This relation was first observed by Dulong and Petit.) What is it? Compare the heat capacity at 207 grams of lead and 56 grams of iron. Approximately what is the heat capacity of a gram-atomic weight of each element in the table? Compare the heat required to raise the temperature of a single atom of lead and of copper.

In the solid state the heat capacity of a gram-atomic weight of any element ($s \times$ atomic weight) is approximately the same. (About 6 calories for 27 grams of aluminum or 56 grams of iron or 207 grams of lead, etc.). This means that the heat capacity of each individual atom is the same! The reason that the specific heat of lead is three or four times smaller than that of iron is that the lead atoms are heavier; there are three or four times fewer atoms in a gram of lead than in a gram of iron and the specific heat is three or four times less.

Although the law of Dulong and Petit is not exact, breaking down at low temperatures, and for a few light elements failing even at room temperature, it is nevertheless significant as showing the fundamental reason for differences in specific heat. This law has been very important in the development of modern conceptions of heat. The quantum theory (Chapter 55) has explained the anomalous results at low temperatures.

248. Heat of Combustion. We heat our homes and cook our food by burning coal or wood or gas or oil. The heat obtained in burning a unit quantity of fuel is called its **heat of combustion.**

Figure 3 shows one form of calorimeter used for measuring the heat of combustion of gas. A slow stream of water passes continuously through the apparatus. The hot gas from the flame, passing through copper tubing, gives its heat up to the water. By observing the temperature of the water as it enters and leaves and the rate of flow, we find the number of calories $(m \cdot \Delta t)$ evolved for unit time. The flow of gas is observed on a gas meter, and the heat obtained per cubic foot is determined. Other types of calorimeters are used for measuring the heat of combustion of other fuels, of foodstuffs and other substances.

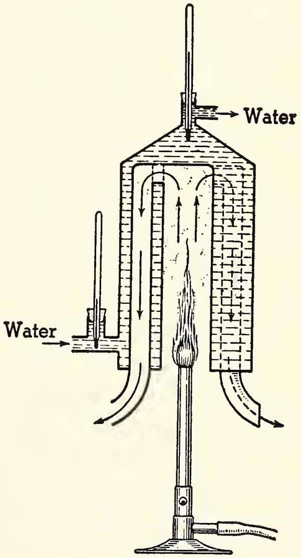

FIG. 3. Gas combustion calorimeter.

Table 20 gives the heats of combustion of common fuels — the values are only approximate since there is large variation within each kind of fuel.

Although the process of nutrition involves much more than the mere burning of fuel in the body, nevertheless the calorific content of food is one very important consideration in dietetics. It has been estimated that the average working man requires 3400 large calories per day. Calorific values (heats of combustion) of different foodstuffs are determined by burning in a suitable calorimeter. Typical values are given in Table 21.

TABLE 20

HEAT OF COMBUSTION OF FUELS

(Approximate)

Hydrogen.........	33,000 calories per gram	....
Coal..............	7,500 calories per gram	13,000 B.t.u. per lb.
Wood............	4,500 calories per gram	8,000 B.t.u. per lb.
Fuel oil...........	11,000 calories per gram	140,000 B.t.u. per gal.
Natural gas.......	....	1,500 B.t.u. per cu. ft.
Artificial gas.......	....	530 B.t.u. per cu. ft.

TABLE 21

CALORIFIC VALUE OF FOODS (calories per gram)

Beef........	2500	Wheat......	3600
Bacon.......	6000	Potatoes.....	700
Fish........	500	Sugar.......	4100

249. Our Energy Resources. Coal, oil, water power, foodstuffs — these are the important sources of energy which heat us, nourish us, work for us. Practically all forms of energy which we, inhabitants of the earth, use, come indirectly from the sun. Picture the "average American family." It owns 75 acres of land. When the sun is overhead, each square centimeter of this land receives 2 calories of solar heat per minute — equivalent to 3 horsepower per square yard. Some acres are barren, but most are green with grasses and grain and trees, storing up a little of this solar heat. Not very efficient (for it takes several acres to support a horse) but in the course of a year 200 bushels of vegetables and grain are gathered by this "average American family" — 12 billion calories or 400 calories per second. Besides this there are pasture land and hay and cotton. And the wood crop is about 500 cubic feet a year — but we use several times as much wood as we produce (I wonder where all this wood goes to). A little stream tumbles through the estate; it has been harnessed to give half a horsepower — enough to light a dozen lights; several times as much waterpower goes to waste.

And, even in this age of mechanical power, our "average family" still owns half a horse.

Of the energy produced in the United States it is estimated that 45 per cent comes from coal, 27 per cent from oil, 9 per cent from natural gas, 8 per cent from waterpower, 6 per cent from wood, 4 per cent from man and beast, and 1 per cent from wind.

All this energy comes more or less directly from the sun. Underground lies a rich trove of solar energy stored in summers long past. There runs beneath these average acres a vein of coal about a foot thick — 100,000 tons of coal; 25 tons a year are used in heating, in cooking, and as power for industry. And there is a buried pool containing 3500 barrels of petroleum — 35 barrels, 1400 gallons (!), are used each year. And each year 60,000 cubic feet of natural gas are used — how much is left no one knows. Here from the ground, buried for a hundred million years, comes the motive power of the twentieth century. Singularly rich is the United States with a sixth of the world's petroleum supply, half of the coal, and practically all the natural gas.

QUESTIONS

1. Which has the greater heat capacity: 1 kilogram of lead or 100 cc of water?

2. What do coal and gas cost in your community? Approximately how many calories can be bought for a cent?

3. Is the specific heat of solid elements: (*a*) proportional to the absolute temperature; (*b*) proportional to their density; (*c*) inversely proportional to their atomic weight?

4. Is the heat of combustion of fuels: (*a*) several calories, or (*b*) several thousand calories, or (*c*) several million calories, per gram?

Vocabulary: (Large) calorie, British thermal unit, heat capacity, specific heat, Law of Dulong and Petit, heat of combustion (calorific value).

PROBLEMS

1. What is the heat capacity of a furnace weighing 600 lb.? Of a concrete house (single room) with 6-in. walls 15 by 15 ft.? (Specific gravity of concrete, 2.5.)

2. What is the heat capacity of 1 cu. ft. of water in B.t.u. per degree Fahrenheit?

3. Three hundred grams of lead shot at 40° C are poured into a 100-gram copper calorimeter at 0° C. Find the resulting temperature.

4. If 10 grams of boiling water are poured into a shallow hole in a kilogram weight (iron) at 20° C, how much will the temperature of the weight rise?

5. A liter of water at 80° C is poured into an aluminum kettle at room temperature. The kettle weighs 600 grams. How much will the water be cooled?

6. How much lead shot, at a temperature of 100° C, must be dropped into a liter of water at 0° C to raise its temperature to 20° C? (Neglect the heat capacity of the vessel.)

7. If 100 grams of lead at 100° C is dropped into 100 grams of water at 20° C, how much will the temperature of the water be raised? (Neglect the heat capacity of the vessel.)

8. Work problems 6 and 7 assuming that the water is in a vessel with a water equivalent equal to 30 grams.

9. What rise of temperature will be observed when a kilogram of copper heated to 150° C is plunged into a liter of water at 10° C? The water is contained in a 300-gram glass beaker.

10. A kilogram of aluminum at 0° C, 200 grams of iron at 30° C, and 100 grams of lead at 100° C are placed in a 300-gram copper calorimeter at 20° C. What is the final temperature of the mixture?

11. An automobile gives 15 miles to the gallon of gasoline. Its efficiency is 20 per cent (80 per cent of the fuel energy going into heat). How much heat must the radiator dissipate per minute when the car is traveling 45 miles per hour? (Heat of combustion, 120,000 B.t.u. per gal.)

12. The automobile engine of problem 11 weighs 400 lb., the copper radiator 50 lb., and the cooling system holds 25 lb. of water. How fast will the cold engine warm up?

13. If coal costs $10 a ton, natural gas costs $1 a thousand cubic feet, and fuel oil costs 7 cents a gallon, compare the costs per B.t.u., assuming the efficiencies of combustion for each fuel to be the same.

14. Assuming 50 per cent efficiency, how much artificial gas is required to raise the temperature of a quart (approximately a liter) of water from the freezing to the boiling point?

15. A swimming pool is 2 meters by 15 meters by 40 meters. Assuming 50 per cent efficiency, how much coal will be required to warm the water 20° C? How much will this cost at $5 a ton?

16. Estimate the value of the specific heat of silver, of gold, of iodine.

CHAPTER 22

TRANSFER OF HEAT

250. Conduction, Convection, and Radiation. Heat can pass from a hot body to a colder one by three processes: conduction, convection, and radiation. Heat passes from one end to the other of a metal rod by conduction. The fast-moving molecules strike their neighbors and share with them their energy; these strike other molecules, and these others, and so the heat is transferred along the rod. In a liquid or a gas, heat is usually transferred by convection; when water is heated on the stove, the heated, lighter, water in contact with the bottom of the pan, rises; a draft occurs in the chimney; the air in the room circulates. In convection the heated liquid or gas moves into the colder region, carrying its heat energy with it. Radiation seems on first thought a more mysterious process: heat from the sun passes through empty space to the earth with no apparent medium to carry it; in a familiar electric heater, heat from the glowing heating element, directed and focused by the reflector, passes as a beam across the room.

A gas-filled tungsten lamp illustrates the three types of heat transfer. Light is radiated in all directions from the filament; along with the light come other rays, twenty times as strong, which we cannot see but which produce warmth — radiant heat. About 10 per cent of the heat flows down the filament leads — conduction. The hot gas near the filament rises — convection; the bulb is much warmer directly over the filament. The heat carried upward by convection remains thus localized in the glass because glass is a poor conductor. (Compare the bulb temperatures in a 40-watt [gas-filled] lamp and a 25-watt [vacuum] lamp to observe the difference due to convection.)

251. Conduction of Heat. The rate of conduction of heat along a rod — that is, the number of calories which pass along the rod per second — depends on four factors: the cross-section of the rod (A), its length (L), the difference in temperature between its ends (Δt), and the material of which it is made. A little thought will convince the student that the rate of flow should be proportional to the cross-section and to the temperature difference and that it should vary inversely with the length.

$$\text{Heat flow per second} = c\,\frac{A \cdot \Delta t}{L}. \tag{5}$$

The constant c depends upon the material; it is called the coefficient of thermal conductivity.

TABLE 22

COEFFICIENTS OF THERMAL CONDUCTIVITY
(Calories per sec. per cm per °C)

Silver	1.00	Common brick	0.001
Copper	0.92	Brick (porous)	0.0005
Aluminum	0.50	Wood	0.0005
Iron	0.16	Sawdust	0.00014
Glass	0.002	Felt	0.00009
Water	0.0014	Rock wool	0.00009
Snow	0.0004	Air	0.00006

Example. A window pane is 1 meter square and 4 mm thick. When the temperature outside is 0° C, how much heat is lost through the pane per hour?

Consider the house at 20° C. Here the "length" of the conductor is 0.4 cm, its area 10,000 sq. cm.

$$\text{Heat transferred per second} = \frac{0.002 \times 10{,}000 \times 20}{0.4} = 1000 \text{ cal. per sec.}$$

or 3600 large calories per hour.

Evidently (from Table 22) metals are much the best conductors of heat. A heavy aluminum frying pan conducts the heat evenly over the surface. . . A soldering "iron" is never made of iron but of copper — the heat must flow readily to the soldered joint. . . Pipes in hot-water heaters are usually made of copper tubing. From hard water, lime — a relatively poor conductor — deposits in the pipes and reduces the efficiency. . . It is a familiar fact that when a piece of wood and a piece of metal have both been heated (or cooled) the metal feels hotter (or colder) to the touch. The temperatures are actually the same, but the metal conveys its heat to the hand much more readily than the wood.

The poorest conductors are substances of porous or fibrous nature. They are important as thermal insulators — in refrigerators, in fireless cookers, for thermal insulation in houses, and, most important, for clothing. Air itself has very low conduction, and air chambers are often used for insulation But if the chambers are large, convection currents occur; to prevent this it is usually best to fill the chamber with sawdust, rock wool, or other porous material.

Gases and liquids (except molten metals) are poor conductors of heat. In them the heat transfer is usually by convection.

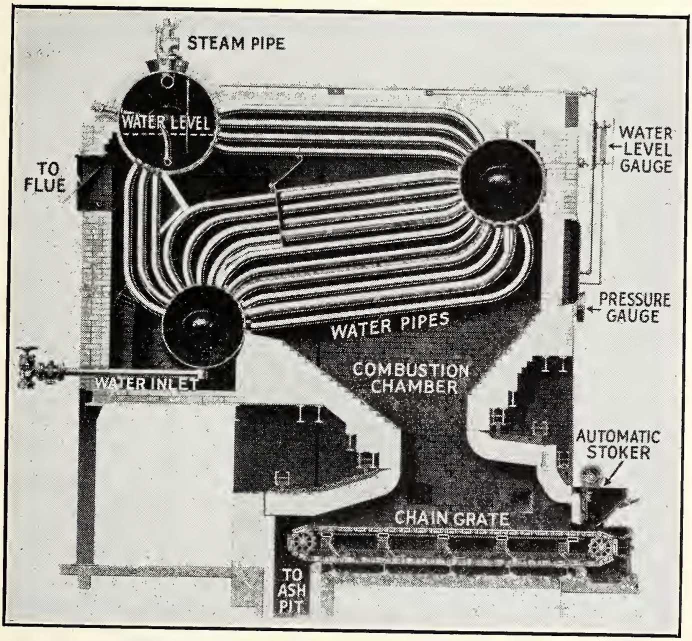

FIG. 1. Boiler in medium-size power plant.

252. Convection. In the hot-air furnace or the hot-water furnace or the steam furnace, heat is carried through the house by convection. Heated air expands and, buoyed up by colder, denser air, rises. The furnace is in the cellar. Hot air rises through the hot-air pipes, cold air returns to the furnace through the cold-air ducts. Hot water rises to the radiators, cold water returns; heat is carried through the walls of the radiator by conduction, and then again by convection into the air and throughout the room. In the steam plant, steam is generated in the furnace and condensed in the radiators; the heated vapor is carried through the pipes by the difference in pressure.

Convection currents on a large scale occur in the oceans and have an important influence on climate — the warm Gulf Stream passing up across the Atlantic gives England moderate temperatures (and fogs); Newfoundland, in the same latitude, is chilled by the Labrador current. In general the oceans tend to moderate the weather, preventing near the coasts the extremities of winter and summer which occur within the continents.

253. Winds. Winds are primarily due to convection currents caused by the unequal heating and cooling of the earth. Solar radiation penetrates the atmosphere and warms the earth; air is heated, like a teakettle of water, mostly from the bottom. Hot air rises, reducing the barometric pressure; cold air rushes in. The continents are warmer in summer, colder in winter, than the oceans (where the temperature is moderated by convection). There result the low-pressure and high-pressure areas which we have seen (§180) are the cause of cyclones and anticyclones.

Near the equator air rises, flows north and south in the upper atmosphere, and settles down near the Tropics of Capricorn and Cancer. In consequence there is a low-pressure belt near the equator and there are high-pressure belts in latitudes of 30° north and south, somewhat modified, particularly in the northern hemisphere, by the continents. In accordance with Ballot's Law (§180) between these belts the winds blow from the east. These are the steady " trade winds." The latitudes 30° north and 30° south are regions of calm (" horse latitudes "). Beyond these latitudes (particularly in the southern hemisphere in the " roaring forties ") the barometric pressures decrease and the prevailing ocean winds are in general from the west.

The temperature difference in hill and valley, forest and field, sea and shore, all tend to promote breezes. Over the ocean the temperature of the air changes little between night and day; the shore air warms by day and cools by night. By day the breeze (in general) comes from the sea, by night from the land.

254. Radiation. I warm my hands before the open fireplace. The heat does not reach my hands by convection — for the hot gases from the flame go up the chimney — and certainly not by conduction. The energy is thrown directly from the hot coals to my hands, through space. The intervening air takes no part in the process. It may be quite cool; heat radiation can pass through a cake of ice without melting it. Indeed, although it comes from the heat of the coal and results in the heat in my hand, during its passage through space this radiant energy is not heat (molecular energy) at all but something closely akin to radio waves, light, and x-rays. In fact light is one important form of radiant heat; however, the greater part of the radiation from ordinary hot bodies is invisible.

Although the effects are less pronounced there are familiar examples of radiation at lower temperatures. All have felt the heat from a brick wall after a hot day. We can feel the heat from the teakettle when the hand is placed a few inches away. In fact, we ourselves and everything about us are receiving heat and radiating heat continually. However, unless the temperatures are different between a body and its surroundings, there is an equality between the heat radiated and the heat received, and we do not notice the radiation. But we feel the cold when our hand is placed near a cake of ice — not because the ice radiates *cold* (which is a mere absence of heat) but because the hand radiates away more heat than it receives. If liquid air (or dry ice in solution) is placed in the flask in Fig. 2 it boils slowly as the result of the heat radiated to it from the walls of the room; it boils more rapidly if a flame or hot iron is placed near by. A thermos bottle is just such a flask as this but silvered to reduce the radiation (§257).

FIG. 2. Unsilvered thermos bottle.

255. Detection of Radiation. Solar radiation is often measured by a mercury thermometer; the bulb is covered with lampblack to absorb the radiation. The radiant light and heat from a star can be measured by focusing its image on a small, blackened thermocouple. Or, instead of a thermocouple, a resistance thermometer (called a bolometer) is often used. Recently photographic plates have been produced which respond not only to visible light but also to a limited region of the near-visible heat radiation.

One of the most interesting devices for detecting radiation is the *radiometer*. Four pieces of mica, each blackened on one side, are mounted to rotate freely as vanes in a partially evacuated glass bulb. When radiation falls upon the vanes the blackened sides (absorbing the radiation) are slightly heated; gas molecules rebound from these heated surfaces with increased momentum, and the reaction sets the vanes spinning.

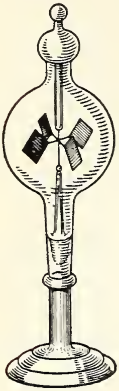

FIG. 3. Radiometer.

256. Radiation Pressure. This is the way the radiometer (which the student has probably seen) usually operates; its action depends simply upon the difference in air pressure. But if the air is removed from the bulb the vanes spin in the opposite direction. The explanation is of great interest. The impinging radiation has momentum, as if it were a material thing! When it is absorbed by the black side, in stopping the radiation gives the vane an impulse; when reflected from the other side it gives a double impulse (§126). The effect, discovered in 1900, was one of the facts which led to the present view that energy of itself has mass (Chapter 43)!

257. Effect of the Surface. If pieces of silver, chalk, glass, iron, and graphite are all heated to the same temperature, say 800° C, we shall find the graphite and iron glowing brightly in the darkened room, the other substances almost invisible. The graphite and iron (covered with a black oxide) are good absorbers of light and heat; the silver and chalk reflect, the glass transmits. A fundamental law of radiation is: *Good absorbers are good radiators* (Kirchhoff's Law).

The law is frequently demonstrated by means of a hollow cube with faces of different kinds — dull black, silvered, and white. The cube is filled with hot water and the emission from each face compared by means of a thermocouple or radiometer. The greatest radiation comes from the black surface.

The thermos bottle is a double-walled evacuated glass vessel such as shown in Fig. 2 but silvered. The silver surface greatly reduces the radiation, both outward and inward. In the thermos bottle we store hot coffee or cold water or liquid air with a minimum of heat transfer — by conduction, convection, or radiation.

Figure 4 shows a block of iron with a silver core and, in the center, a deep hole. The iron is tarnished; it is fairly black in comparison with the silver, but not so black as the hole, which absorbs practically all the light that falls in it. When the block is heated the hole glows brightly, the iron somewhat less so, and the silver surface hardly at all. A deep hole is a perfect *black body;* the radiation from such a black body has been carefully studied and has led (as will be shown in Chapter 55) to one of the most important discoveries of modern physics.

A simple consideration will show why Kirchhoff's Law must hold. If we could increase absorption without increasing emission we could heat water merely by placing it in a blackened kettle. Or we could freeze water by radiation by placing it in a silvered vessel which would reflect any incoming radiation. If this were possible we could make heat flow " uphill," from colder to hotter bodies, and we should soon have perpetual-motion machines. (This will be shown in Chapter 24.)

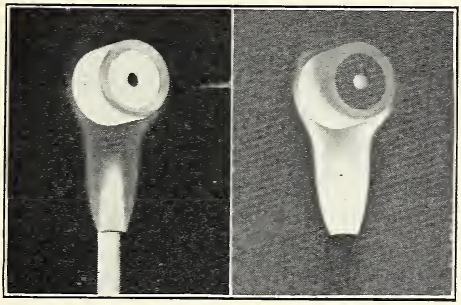

FIG. 4. Kirchhoff's Law: Good absorbers are good emitters. Metal block photographed by reflected and emitted light.

TABLE 23

RADIATION FROM ONE SQUARE CENTIMETER OF BLACK BODY

Approximate Temperature	Temperature		Intensity	Wave-length of Greatest Intensity*
	Centigrade	Absolute		
Dry ice in solution...	−91	182	1 cal. in 10 min.	16 microns
Boiling water.......	100	373	1 cal. in 40 sec.	8 microns
Glass softens.......	477	750	1 cal. in 2½ sec.	4 microns
Working temperature of iron...........	1227	1500	7 cal. per sec.	2 microns
Hottest tungsten lamp	2727	3000	110 cal. per sec.	1 micron
Temperature of sun..	5727	6000	1750 cal. per sec.	½ micron

* Wave-lengths of visible light extend from 0.4 micron (violet) to 0.7 micron (red).

258. Radiation and Temperature. The radiation from a surface varies as the fourth power of its absolute temperature (Stefan-Boltzmann Law):[1]

$$\text{Radiation} = CT^4. \tag{6}$$

This law holds exactly only for black bodies but approximately for other surfaces. Table 23 gives the radiation at several characteristic temperatures — the radiation increases 16-fold each time the temperature is doubled. The enormous radiation at high temperature is illustrated by a comparison: At the temperature of the sun the radiation is about the same as the heat which would flow through an aluminum sheet 2 millimeters thick when one side is at the melting point of aluminum (650° C) and the other at absolute zero!

There is a change in quality as well as quantity of the radiation with rising temperature. At low temperatures the radiation is entirely invisible; at about 600° or 700° C we begin to see a dull red glow which with rising temperature becomes brighter and brighter and is first "red hot," then "red-yellow hot" and then more and more nearly "white hot." The explanation of this simple fact led Planck (in 1900) to the quantum theory of radiation (Chapter 54). But since radiation is not heat but an invisible form of light the further discussion of its nature must be left to a later division of the book.

QUESTIONS

1. Why do teakettles have wooden handles?
2. Why is a porous or fibrous substance usually a bad conductor?
3. Why is thermal insulation improved by filling the dead air space between outer and inner walls of a house with insulating material?
4. Why must a hot-water furnace be placed in the cellar? Steam furnace?
5. Is radiation an important factor in hot-water or steam heating? Is the color of radiators in the house such as to radiate heat most efficiently?
6. Is the automobile radiator cooled by radiation? Of what material are the cooling pipes made?
7. Explain the function of the silvering when a thermos bottle is filled with hot coffee; with liquid air.
8. If silver were a perfect reflector and the vacuum were perfect would a thermos bottle be a perfect insulator?

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[1] The energy radiated in ergs per second from a square centimeter of a black body equals $0.000057 \times T^4$; the constant of proportionality is called *Stefan's Constant.*

9. What are two methods of detecting radiation without first transforming it to heat?

10. Suppose that a surface could be discovered which was at the same time a good absorber and a bad emitter of radiation. Explain how water could be heated, how ice could be made automatically, and how a perpetual-motion machine could be constructed.

11. Eq. (5) can be written: $c = \dfrac{\text{Heat flow per second} \times L}{A \times \Delta t}$. Define the coefficient of thermal conductivity in words.

12. "Trade winds" result from air masses moving: (*a*) with the gulf stream; (*b*) off from the continents; (*c*) toward the equator.

13. A piece of silver and one of charcoal are heated to incandescence. At the same temperature: (*a*) the charcoal is brightest because it is the best absorber; (*b*) the silver radiates most light because it is a metal; (*c*) the two are equally bright.

Vocabulary: (Coefficient of) conduction, convection, radiation, bolometer, radiometer, radiation pressure, Kirchhoff's Law, perfect black body, Stefan-Boltzmann Law.

PROBLEMS

1. How much heat will flow per second through a centimeter cube of copper, iron, glass when the opposite sides differ by 1° C? Compare a 2-cm cube.

2. A copper bar is 4 sq. cm in cross-section and 10 cm long. One end is in boiling water, the other is in ice. How much heat will flow through this bar in a minute?

3. Ice is kept in a wooden box, with sides 1 ft. by 1 ft., and $\frac{1}{2}$ in. thick. If the outside temperature is 30° C, how much ice will melt in a day? (Eighty calories of heat are required to melt a gram of ice.)

4. The intensity of solar radiation with sun directly overhead is about 2 cal. per sq. cm per min. The steel top of an automobile has an area of 3 sq. meters and weighs 200 kilograms; how fast will it heat up in the sunshine?

5. When we start an automobile engine it warms up, rapidly at first, then more slowly, and reaches some equilibrium temperature, say 160° F. Plot in a general way the heating curve (temperature plotted against time), and explain. Plot the cooling curve when the engine is stopped. (Compare the "Negative Compound Interest Law," §54.)

6. The "transmission coefficient" of insulating material is defined as the number of B.t.u. conducted per hour through an area of 1 sq. ft. for a temperature difference of 1° F. Find the transmission coefficient (*a*) of an 8-in. brick wall; (*b*) of a 1-in. wooden ceiling covered with a 4-in. fill of rock wool.

7. Typical values for transmission coefficients (B.t.u. per hour per square foot per Fahrenheit degree) for a frame house are: 0.30 for walls and ceiling-roof, 0.07 for same with rock-wool fill, 1.2 for glass windows. Estimate the coal required per day, when outside temperature is 30° F, to heat a typical

5-room bungalow without allowance for infiltration of air. Assume furnace 40 per cent efficient.

8. What saving in coal is introduced by insulating the bungalow in problem 7 with rock wool?

9. The reciprocal of thermal conductivity is called the thermal resistivity. Compare the thermal resistivities of the metals in Table 22 with their electrical resistivities in Table 36.

CHAPTER 23

CHANGE OF STATE — (MOLECULAR POTENTIAL ENERGY)

259. Melting and Evaporation. Let us start with an experiment. When the thermometer outdoors reads −20° C we gather 100 grams of snow, put it in a beaker, and supply heat at a constant rate — say 1000 calories a minute. In a minute the temperature of the snow will be raised to 0° C. (The snow should be stirred to keep the temperature uniform throughout.) *Then the temperature remains constant while the snow melts.* For 8 minutes we continue to apply heat as more and more of the snow turns to water; the temperature is still 0° C. The temperature then begins to rise again, but now more slowly. (The specific heat of ice is half as great as that of water.) In 10 minutes the temperature reaches 100° C; *the temperature stops rising, and the water begins to boil.* It then takes 54 minutes for the water to boil away.

It takes 10 calories to raise 1 gram of ice 20 degrees in temperature (specific heat of ice, 0.5); it takes 80 calories to melt it without temperature change (*heat of fusion*); it takes 100 calories more to raise the water to the boiling point, and 540 calories more to boil it away (*heat of vaporization*). As we raise the temperature of the snow or water we increase the kinetic energy of the molecules. When ice melts or water boils, there is a change of state and an increase in molecular *potential* energy. It takes nearly as much energy to disrupt the regular arrangement of the ice crystal as to heat the water 100 degrees; it takes more than five times as much energy to pull the molecules away from one another. The **heat of fusion** (or **evaporation**) is the heat required to melt (or evaporate) one gram of a substance (no change in temperature).

All the elements and the stable compounds can exist in the three states — solid, liquid, vapor. The boiling point of helium (at atmospheric pressure) is 4 degrees above absolute zero (at still lower temperature and higher pressure helium has been frozen). Carbon melts at about 3500° C and evaporates freely at this temperature. The melting points and (with some exceptions) the boiling points of other elements lie between these limits. If certain substances (for example, wood and flesh) do not have a liquid and vapor state it is because their molecules decompose before the necessary temperatures are reached.

TABLE 24

Melting Points, Boiling Points and Critical Temperatures
(Centigrade)

	M.P.	B.P.	C.T.		M.P.	B.P.
Helium		−269	−268	Mercury ..	−39	357
Hydrogen ..	−259	−253	−240	Sulfur	119	445
Nitrogen ...	−210	−196	−146	Tin	232	2270
Oxygen	−219	−183	−118	Lead	327	1620
Chlorine ...	−102	− 34	146	Silver.....	960	1955
Carbon dioxide...	− 57	− 78*	31	Copper ...	1083	2300
Ammonia ..	− 78	− 33	130	Iron......	1530	3000
Ether......	−117	35	194	Platinum..	1750	4300
Alcohol	−117	78	243	Tungsten..	3400	5900
Water	0	100	374			

* Sublimation with 76 cm (Hg) vapor pressure (§273).

MELTING

260. Heat of Fusion. The pioneer in calorimetry was Joseph Black, professor of chemistry and medicine in the University of Glasgow. He found that different substances required different quantities of heat to raise them to the same temperature and so evolved the concept of specific heat. He also found that a definite quantity of heat was required to melt or evaporate a substance and introduced the term "latent [i.e., buried] heat." This term, though perhaps somewhat misleading in connotation, is still used to designate the heats of fusion and of evaporation.

Black estimated the heat of fusion of ice by observing the time required for the change of state, much as we have done in introducing the subject. A more accurate measurement of latent heat can be made by the method of mixtures. It follows from the definition of heat of fusion (f) that fm calories are required to melt m grams of the substance.

$$\text{Heat required for melting} = fm. \tag{7}$$

Let us place m grams of ice (at 0° C) in a calorimeter containing a suitable quantity of water at temperature t_w. The water is cooled $(t_w - t_{\text{mix.}})$ degrees; the ice is melted and the ice water raised from 0° to $t_{\text{mix.}}$. The *heat out* from the water (and calorimeter) equals the *heat in* to the ice, which melts the ice and warms the ice water. If m_{w+} is the mass of the water plus the water equivalent of the calorimeter we have

$$m_{w+}(t_w - t_{\text{mix.}}) = fm_i + m_i(t_{\text{mix.}} - 0°), \qquad (7')$$

and from this the latent heat can be obtained.

Examples. 1. In such an experiment 100 grams of ice were placed in 990 grams of water at 30° C; the final temperature was 20° C. Water equivalent of calorimeter was 10 grams. Find the heat of fusion of ice.

$$\begin{aligned} \text{Heat out} &= \text{Heat in} \\ 1000 \times 10 &= f \times 100 + 100 \times 20, \\ 100f &= 8000, \\ f &= 80 \text{ cal. per gram.} \end{aligned}$$

2. Given the value of the heat of fusion, eq. (7′) may be used to find the temperature of the mixture. But observe that this equation applies only when all the ice is melted. For example, suppose that 1000 grams of ice were used in the above experiment; discuss the state of the mixture. Evidently only part of the ice can be melted, for 80,000 cal. are required to melt the ice; the water drops to the freezing point and gives out only 30,000 cal. The mass melted is 30,000 ÷ 80 = 375 grams, and the mixture — 625 grams of ice and the water — is at 0° C.

Table 25 gives the heats of fusion of several common substances and the heats of transformation between different crystalline forms (§280).

TABLE 25

HEATS OF FUSION AND OF TRANSFORMATION (calories)

Melting	Melting Point	Heat of Fusion	Transformation	Heat of Transformation
Mercury...	−39°	3	Ice I → Ice III	3
Sulfur.....	119°	13	Sulfur:	
Water.....	0°	79.7	rhombic → monoclinic	2
Salt (NaCl).	804°	124	Graphite → diamond	14

261. The Laws of Freezing. (1) Under standard conditions of pressure every crystalline substance has a definite temperature at which it melts.

(2) The melting point of the solid is the same as the normal freezing point of the liquid (the liquid may, however, be undercooled, §264).

(3) A given mass of any substance, at the melting point, requires a definite quantity of heat to change it from the solid to the liquid state. It releases this same quantity in freezing.

When the temperature of a liquid — water, molten iron, liquid air — is lowered to the freezing point, crystals begin to form. (We neglect for the moment the effect of undercooling.) Often, as in metals, these crystals are microscopic and combine to form a solid mass. Heat is released in the crystallization (equal to the heat of fusion). Usually the volume decreases (a few exceptions are known, conspicuously water).

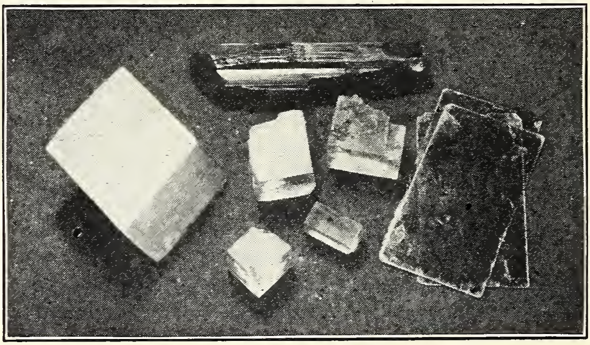

FIG. 1. Crystals: calcite (rhombic); rock salt (cubic); stibnite (elongated with metallic luster); mica (flat sheets).

262. Variation of Freezing-Melting Point with Pressure. The freezing point (which is also the melting point) changes with pressure. Usually the solid has a smaller volume than the liquid. Pressure then aids the freezing; or, when we heat the solid, pressure makes it harder for it to melt and expand; for such substances pressure raises the freezing point. The effect is just the opposite for water, since it expands on freezing. Pressure lowers its freezing point. However, except for very high pressures, the effect is small. (Observe the freezing-melting curve in Fig. 10 or, drawn to exact scale, in Fig. 11.)

263. Effect of Dissolved Substances on Freezing and Evaporation. A larger effect is caused by dissolved substances. When a substance is dissolved in a liquid the freezing point is invariably lowered. Alcohol is placed in the automobile radiator to prevent freezing. Another familiar illustration is salt in water; the freezing point of the brine may be lowered 20 degrees. Salt is mixed with ice in freezing ice cream. The salt lowers the melting point and the ice melts; as the ice absorbs its heat of fusion (changing kinetic energy to potential) the temperature falls; the melting continues, and the temperature is lowered more and more until the freezing point of the salt solution is reached.

A dissolved substance (if not itself volatile) also reduces the vapor pressure and raises the boiling point of the liquid. In fact, Nature rather objects to our separating the solute from the solvent either by crystallization or by evaporation. Nature greatly prefers to have things mixed up. Osmotic pressure (§221) shows the great tendency for different substances to intermingle. We shall see the significance of this more clearly in the next chapter.

264. Undercooling; Amorphous Solidification. Freezing occurs exactly at the normal freezing point only if there is a nucleus for the crystals to form on. Pure water in a clean test tube can be carried 8 or 10 degrees below zero without freezing. This is called undercooling. No similar effect of superheating of the solid ever occurs; when the temperature rises a crystalline solid always melts exactly at the freezing point.

For many substances there is no exact freezing-melting point. As it becomes colder and colder motor oil becomes more and more viscous and ultimately quite hard. If we warm sealing wax or glass it gradually softens without a definite melting point. These must be considered as very viscous liquids, instead of true (crystalline) solids. This is called the *amorphous* solid state. When molten sulfur is suddenly cooled it takes a plastic form; gradually this plastic sulfur crystallizes. The amorphous sulfur is a very viscous undercooled liquid (Fig. 12).

265. Freezing of Water. We have seen that water is in many respects exceptional. It has an unusually large value for specific heat, heat of fusion, heat of vaporization. It expands on freezing, and pressure lowers the freezing point. All these properties are very important in nature. Ice, being lighter than water, forms on the surface; this keeps our lakes and rivers from freezing solid and so has an important influence in aquatic life. . . Water, freezing in crevices, breaks up boulders into stones, stones into soil. . . Any plant cells which contain sap burst with the coming of frost, and vegetation is forced into a yearly cycle in temperate climates.

The high specific heat of water in the ocean and lakes tends to moderate the climate; extreme temperatures are found at places inland from the sea. The heat given out as ice freezes, absorbed as it melts, delays the coming of winter and the return of spring. Icebergs carry the arctic cold down to the coasts of Labrador. Glaciers have had a profound influence upon geology; their motion is conditioned by the fact that at high pressures ice melts.

EVAPORATION

266. Water left in an open vessel gradually evaporates; wet clothes dry as they hang on the line. Alcohol and ether are more volatile, evaporate more rapidly; gasoline evaporates in the carburetor of the automobile. Perspiration evaporates from the body; water from rivers and lakes and oceans evaporates to recondense as clouds and rain.

In evaporation, molecules break away from the liquid to lead a detached existence in the vapor state. The phenomena of cohesion and surface tension are evidence of the forces between molecules, and these forces restrain the molecules as they emerge from the surface. The slower molecules are pulled back; only the faster-moving ones can break away. As the temperature rises there are more and more of these fast-moving particles and the rate of evaporation increases rapidly with rising temperature.

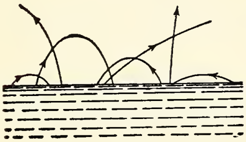

FIG. 2

267. Kinetic Energy (Temperature) in Liquid and Vapor. Since each molecule as it leaves the liquid loses kinetic energy one might suppose that the average molecular kinetic energy in the vapor would be less than in the liquid. On the other hand, it might be argued that, since only the more energetic molecules can leave the liquid, the vapor should have the greater kinetic energy. As a matter of fact, the two factors cancel each other, and the molecular kinetic energy (temperature) in the liquid and vapor are the same. The vapor is richer than the liquid only in potential energy.

268. Sublimation. Solids also may show appreciable evaporation. This is called sublimation. Snow gradually evaporates (sublimes), even at sub-zero temperatures. Camphor, "moth balls," and iodine gradually sublime at room temperature, as does sulfur at somewhat higher temperature; the hot tungsten filament in the incandescent lamp gradually sublimes and (recondensing) blackens the glass. "Dry ice" never melts but evaporates away, even at temperatures far below zero (§273). Theoretically every substance has a certain tendency to evaporate, a certain vapor pressure, but for most solids the evaporation is not appreciable at ordinary temperatures.

269. Vapor Pressure. If the space above a liquid is closed, evaporation will continue and the number of molecules in the vapor state and the pressure of the vapor will increase, until, with increasing numbers, vapor molecules enter the surface and condense at the same rate that molecules from the liquid leave. The vapor is then said to be saturated. If the temperature is increased, molecules leave the surface in greater numbers and the saturation vapor pressure increases. This saturation pressure is taken as a measure of the evaporation tendency of the liquid. Imagine a thin membrane placed in the saturated vapor directly above the liquid (dotted in Fig. 3). The downward pressure is now due to the saturated vapor; the upward pressure is the pressure due to the bombardment of molecules leaving the surface. These are equal: the saturation vapor pressure equals the *vapor pressure of the liquid.*

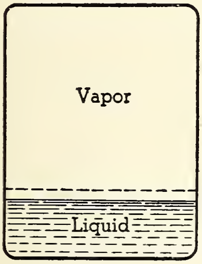

FIG. 3

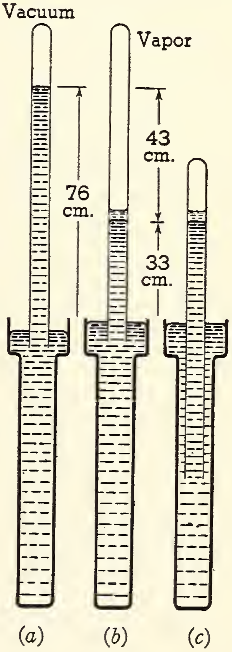

FIG. 4. Vapor pressure of ether.

Vapor pressure can be demonstrated and measured by introducing a few drops of some volatile liquid into a barometer tube. With a vacuum in the tube (Fig. 4*a*) the mercury is elevated some 76 centimeters (depending, of course, on the atmospheric pressure). If a little water is introduced at the bottom of the column, it rises to the top and partly evaporates, and the pressure of the saturated water vapor will depress the mercury column 1.7 centimeters. If it is alcohol which is introduced the depression will be 4.4 centimeters. And ether will drive the mercury more than halfway down the column. All this assumes a room temperature of 20° C. At this temperature the vapor pressure of water is 1.7 centimeters, of alcohol 4.4 centimeters, of ether 43 centimeters of mercury.

If the tube is heated the depression becomes greater. At about 35° C for ether, at 78° C for alcohol, and at 100° C for water the barometric column is depressed to the outer level of the mercury. At these temperatures the vapor pressures are equal to one standard atmosphere.

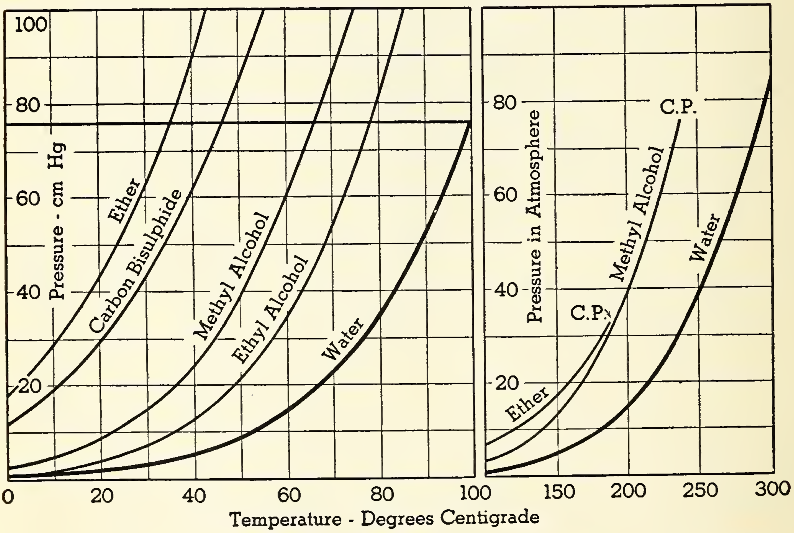

FIG. 5. Saturated vapor pressures. Curves end at critical points (§276).

The pressure in the saturated vapor depends only on the temperature, not upon the volume. This can be easily shown. The barometer tube in Fig. 4 can be lowered in the deep mercury well. As we lower it we decrease the volume of the vapor, but the height of the mercury column, measuring the pressure, remains unchanged. At first this seems surprising to the student familiar with the gas laws. If the volume occupied by the vapor is halved the vapor pressure is not doubled; instead half of the molecules leave the vapor, condensing in the liquid. If we increase the volume the pressure does not diminish but more molecules evaporate. The gas laws do not apply because the quantity of gas is not kept constant. (After the liquid is entirely evaporated any further expansion does result in a decreased pressure; but then the vapor is no longer saturated.)

Vapor pressures of a number of substances are shown in Fig. 5.

270. Dalton's Law of Partial Pressure. We are largely interested in the evaporation of water — not into a vacuum but into the air. If a closed vessel contains air and a little water, water molecules evaporate in the same number and add to the air pressure the same "partial pressure" of water vapor as if evaporating into a vacuum (Dalton's Law of Partial Pressures). Perhaps this may be a little surprising; one might suppose that the air had to a certain extent prepossessed the gaseous region. But the water molecules find their way out into the spaces between the air molecules, add to the total number, and increase the pressure.

271. Humidity. There is always more or less water vapor in the atmosphere. The amount of this vapor — measured sometimes in grams per cubic meter, sometimes in terms of its partial pressure — is the *absolute humidity* of the air. The air is saturated when the partial pressure of its water vapor equals the vapor pressure of water at the existing temperature. Its "relative humidity" is then 100 per cent. The *relative humidity* is the ratio of the partial pressure to the saturation pressure. Or, speaking loosely, relative humidity is the ratio of the water vapor which the air actually contains to the water vapor which (at the given temperature) it can contain. For example, when the temperature of air is 20° C (saturation pressure 1.74 centimeters of mercury by Table 26) and the partial pressure of the water vapor is 0.87 centimeter the relative humidity is 50 per cent. Under these conditions water will evaporate, clothes will dry, half as fast as in completely dry air.

Humidity is one of the important factors in climate. In affecting our personal comfort it is nearly as important as temperature. When the relative humidity is high we say that a room is "stuffy" or (at lower temperatures) "clammy." In air conditioning of houses the air in summer must be dried as well as cooled. In winter in our heated homes the relative humidity is likely to be very low. Furniture dries out and shrinks. Relative humidity depends as much upon the temperature (which determines the saturation pressure) as upon the absolute humidity of the air. The relative humidity may at the same time be 100 per cent in a cool cellar, with water condensing on the walls, and be comparatively low in warmer rooms of the house. Leaves and grass, radiating their heat on a clear night, may cool until the air about them is saturated: dew deposits.

TABLE 26

VAPOR PRESSURES OF WATER AT LOW TEMPERATURES (DEW POINTS)

Temperature (°C) Pressure (cm Hg)	0 0.46	2 0.53	4 0.61	6 0.70	8 0.80	10 0.91
Temperature (°C) Pressure (cm Hg)	10 0.91	12 1.04	14 1.19	16 1.35	18 1.53	20 1.74
Temperature (°C) Pressure (cm Hg)	20 1.74	22 1.96	24 2.22	26 2.50	28 2.81	30 3.16

The humidity of the air can be measured by determining the "dew point," the temperature at which the air is saturated and moisture deposits. The temperature of a metal dish is gradually lowered by the evaporation of ether; soon its polished surface begins to cloud, misted over by the deposit of water droplets. This is the dew point. At this temperature the air is saturated, its humidity 100 per cent. By means of the table of vapor pressures (Table 26) the absolute humidity is determined. The temperature of the surrounding air and the saturation pressure being known, the relative humidity can be found.

Example. What is the relative humidity when the air temperature is 20° C and the dew point is 12° C? From Table 26 the partial pressure of water (saturated at 12°) is 1.04 cm and the saturation pressure is 1.74 cm (Hg). Relative humidity equals $1.04 \div 1.74 = 60$ per cent.

Water evaporates much more rapidly into a vacuum than into air; clothes dry rapidly in a breeze. Why? The rate of evaporation is not *directly* affected by the presence or absence of dry air. But the layer of air over the surface cannot be dry; just above the surface the relative humidity must be practically 100 per cent as molecules leaving the water strike air molecules and wander about in random motion. Water can evaporate only as fast as these molecules can be carried away by diffusion or (more rapidly) by air currents.

272. Heat of Vaporization. Evaporation produces cooling. With the faster molecules continually leaving the surface, the average energy of those remaining decreases. Unless heat is added to the liquid the temperature falls. For water (at the boiling point) 540 calories of heat must be added merely to maintain the temperature while a gram of water boils away. Molecules in the vapor have large potential energy (latent heat); in evaporation, potential energy increases at the expense of kinetic energy.

The human body is cooled by evaporation of perspiration from the skin. . . In dry climates drinking water is stored in porous earthenware jars and is kept cool by evaporation. . . When a substance with high vapor pressure (ether, alcohol, etc.) is in an open vessel its evaporation reduces its temperature very markedly below that of the room. By evaporating ether rapidly in a current of air its temperature can be lowered below 0° C. . . And, as we have seen, heat can be continuously added to water, after it has reached the boiling point, without any rise in temperature.

The heat required to change a gram of liquid to vapor without change of temperature is called the **heat of vaporization** (or latent heat of vaporization, v).

$$\text{Heat required for evaporation} = vm. \qquad (8)$$

TABLE 27

HEATS OF VAPORIZATION (AT THE BOILING POINT)

	calories		calories
Water.......	540	Air.........	50
Alcohol......	204	Mercury.....	65
Ether.......	84	Sulfur.......	362

The value of this latent heat varies somewhat with the temperature. Values for a number of common substances at their boiling points are given in Table 27.

The latent heat of steam (v) can be determined by the method of mixtures. A jet of steam passes into water in a calorimeter; the steam is condensed (heat released: vm_s) and the steam water cooled to the temperature of the mixture (heat released: $m_s[100° - t_{\text{mix.}}]$). ($m_s$ is the mass of the steam; it is measured by the increase in weight of the contents of the calorimeter.) The temperature of the water in the calorimeter is raised from t_w to $t_{\text{mix.}}$ Then

$$\text{Heat in} \qquad = \qquad \text{Heat out}$$
$$m_{w+} (t_{\text{mix.}} - t_w) = vm_s + m_s (100° - t_{\text{mix.}}).$$

273. Boiling. We have described evaporation as taking place at the surface of the liquid, the molecules escaping into the gas above and diffusing away. With rising temperature the vapor pressure increases; when it equals the atmospheric pressure the escaping vapor can push the air above it back bodily and a much more rapid evaporation occurs. Indeed, vapor bubbles can now form in the body of the liquid. This is boiling.

Boiling occurs when the vapor pressure equals the pressure on the surface. It is clear then that boiling temperature depends upon the pressure. Under standard atmospheric pressure water boils at 100° C. Under 15 atmospheres pressure (a typical pressure in the boiler of a steam engine) water boils at 200° C. At a pressure of 64 centimeters of mercury (normal pressure at Denver) the boiling point is about 95° C. (See Fig. 5.) Cooking eggs by boiling is a slow process in the mountains.

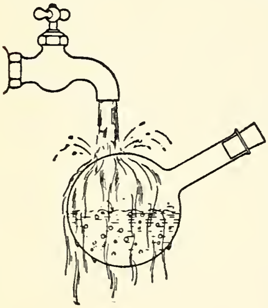

FIG. 6. Boiling at reduced pressure.

A simple experiment demonstrates this lowering of the boiling point. A flask is half filled with water; the water is boiled briskly for a few moments to drive out the air; then the flask is stoppered and removed from the flame. We now pour cold water over the upper part of the flask, and the water boils briskly. Boiling by cooling!

The explanation is simple. In the closed vessel the vapor is saturated. Cooling then condenses vapor on the walls of the flask, reduces the pressure, lowers the boiling point, and boiling occurs. The water gradually cools off as it boils (heat of vaporization). The experiment can be continued until the water is boiling at room temperature, at which the vapor pressure is less than 2 centimeters of mercury. Or if instead of cold water we use a more effective cooling agent (ice-salt mixture or dry ice) we can boil the water until its temperature falls to 0° C and it freezes!

Carbon dioxide furnishes a very interesting example of cooling by evaporation. Carbon dioxide, a gas under ordinary conditions, is stored as a liquid in high-pressure tanks. This liquid freezes at −57° C, and *its vapor pressure at the freezing point is about 5 atmospheres.* So when the tank valve is opened, the carbon dioxide as it issues from the tank boils and cools until it freezes; and still it continues to sublime until its temperature is some 20 degrees below the freezing point (Table 24). We see the carbon dioxide issuing from the tank as a snowlike substance; this is the familiar "dry ice."

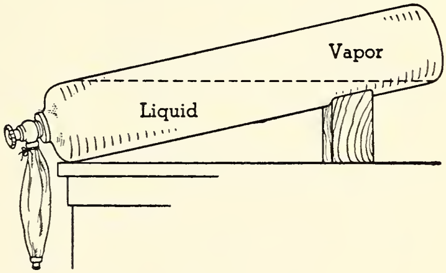

FIG. 7. Freezing CO_2 by evaporation.

274. Superheating and Undercooling. Chemists are often troubled by "bumping." When water is heated in a clean beaker steam bubbles do not form until the boiling point is somewhat exceeded; they then form violently. The vapor needs a small gas nucleus into which it can evaporate. Bumping does not occur as long as air remains absorbed in the water.

On the other hand, the temperature of a vapor may be lowered until it is more than saturated without condensation taking place. The vapor becomes "supersaturated." The vapor needs a nucleus on which to condense. The walls of the vessel, dust particles, electrically charged molecules (§595) may act as centers of condensation.

275. Mechanical Refrigeration. Of itself heat tends to flow from hotter to colder bodies. In the manufacture of ice or in the electric refrigerator this process must be reversed. This can be done, but only at the expense of mechanical energy (Chapter 24).

Fig. 8 shows how heat is transferred from the cold refrigerator to the warmer room. The cooling unit contains a substance of high vapor pressure. When this is compressed by the pump it liquefies and becomes hotter (potential energy into kinetic). The heated liquid is cooled down to room temperature in the radiator. It then passes up to the refrigerator coil which is kept at reduced pressure by the pump; here the liquid boils and cools (kinetic energy into potential). This coil serves as the cooling unit of the refrigerator.

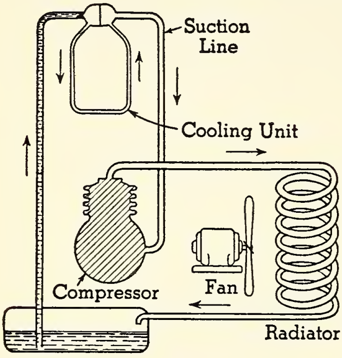

Fig. 8. Electric refrigerator.

In the ice plant the cooling coils lower the temperature of brine and the cold brine is carried through pipes to the freezing tanks where the ice is formed.

276. Critical Temperature. Under ordinary circumstances ammonia, chlorine, carbon dioxide are gases. Their vapor pressures are high; their normal boiling points (at atmospheric pressure) are all far below room temperature. But each of these substances becomes a liquid under sufficient pressure. We need only to increase the pressure until the boiling point (condensing point) is higher than room temperature.

Can all gases be liquefied in this way? Early experimenters tried to liquefy air by subjecting it to pressure, but failed. As a matter of fact, each substance has a *critical temperature* above which it is not possible with any pressure to liquefy it. The critical temperature of air is −140° C; of carbon dioxide, 31° C; of water, 374° C.

It is perhaps misleading to say that the gas cannot be turned to a liquid; above the critical temperature there are no longer two different states — gas and liquid; the properties of the two states have become identical. If we could see into our cylinder of carbon dioxide (§273) as the temperature gradually rose on a warm summer day, we should witness a very interesting phenomenon.[1] The liquid would expand with rising temperature and become less and less dense; more and more would evaporate, and with increasing vapor pressure the vapor would become heavier and heavier; liquid and vapor would become more and more alike; the surface between them would become less and less apparent; and finally at 31° C the liquid becomes the gas and the gas the liquid with a vapor pressure of 73 atmospheres. Above the critical temperature there is only a single state.

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[1] This passage through the critical temperature can be observed in "critical temperature tubes," sealed glass tubes, often containing ether, strong enough to stand the high pressure involved.

277. Liquefaction of Air. Air can be liquefied by pressure alone only below −140° C (its critical temperature). In practice its temperature is lowered to its boiling point (−190° C) and it liquefies at normal pressure. First the air is compressed; this heats it, and the air is brought back to room temperature by cooling water. Then after passing through the inner tube (Fig. 9) it expands and the temperature falls perhaps 100 degrees (§243). This is not cold enough to liquefy it. But this cold gas passes back outside the inner tube, thus cooling the following air far below room temperature before it expands. This process continues, and soon the temperature of the expanding air reaches −190° C and it condenses.

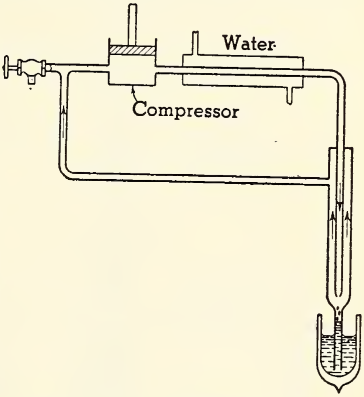

FIG. 9. Liquefaction of air.

All gases have been liquefied. Hydrogen boils at 20° above absolute zero; helium at 4° above absolute zero. In liquefying hydrogen the compressed gas is cooled by liquid air instead of water before expansion; in liquefying helium, liquid hydrogen is used for the pre-cooling.

278. Summary of Laws of Vaporization. 1. Liquids and solids evaporate in greater or less degree at all temperatures. The rate of evaporation increases with the temperature.

2. Vapors condense, and corresponding to every temperature (below the critical) there is a definite vapor pressure at which liquid and vapor are in equilibrium, the saturation pressure. Above the critical temperature the two distinct phases, gas and liquid, no longer exist.

3. The saturation pressure of a vapor in a gas is the same as in

a vacuum. (The ratio of the partial pressure of water vapor present in a gas to its saturation pressure is the "relative humidity.")

4. Boiling occurs at a temperature at which the vapor pressure of the liquid equals the pressure of the atmosphere above it. (There may, however, be superheating.)

5. The change from liquid (or solid) to gaseous state without change of temperature requires the addition of a definite quantity of heat.

279.* Solid, Liquid, and Vapor; Phase Diagram. Figure 10 summarizes many of the facts of this chapter. It shows the vapor-pressure curve for water. At 100° C the vapor pressure is 76 centimeters (Hg); at lower temperatures the vapor pressure decreases, and at 0° C it is 4 millimeters (Hg). At this temperature[1] the water freezes; the vapor-pressure curve continues to the left (at a slightly different slope) as the sublimation pressure of ice. The freezing point at different pressures is rep-

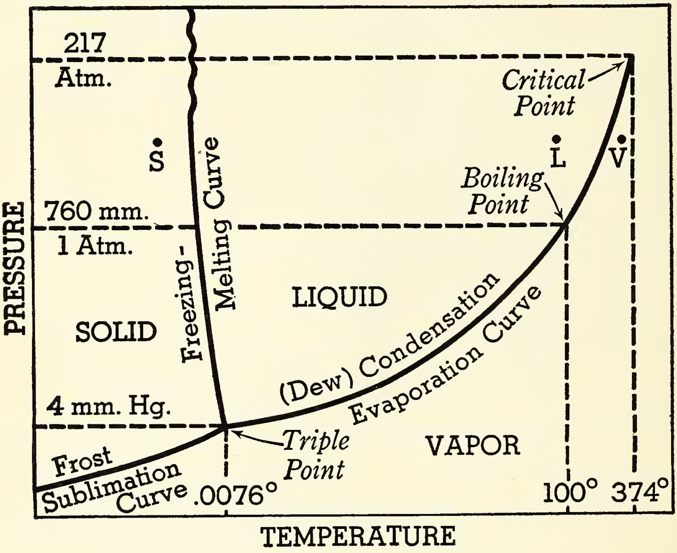

FIG. 10. Three ordinary phases of water.

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[1] More strictly the freezing point at this pressure is 0.0076° C. Water freezes at zero at 1 *atmosphere pressure*. . . Obviously the curve is not drawn to scale.

resented by a curve sloping slightly to the left indicating that the freezing temperature is lowered by an increase in pressure.

At low pressures and high temperatures (point V) the water is in the vapor state. At lower temperature (point L) or higher pressure it is a liquid; at much lower temperature (point S) it is a solid. But for any pressure (below the critical pressure) one can at a suitable temperature (given by the vapor-pressure curve) have liquid water or ice in contact with the vapor. Or for any pressure above 4 millimeters (Hg) at a suitable temperature (given by the freezing-point curve) we can have ice and liquid water together in equilibrium. But only at one particular pressure (4 millimeters Hg) and one particular temperature (0.0076°) can we have solid and liquid and vapor all three in contact. This point on the diagram is called the *triple point.*

280.* Different Phases of the Solid State. It was Willard Gibbs,[1] the distinguished American theoretical physicist of the last century, who first stated these facts clearly and showed that they were quite general. Water can exist in more than three states (or *phases* as they are technically called). There are the gaseous state and the liquid state, but of the solid (ice) there are at least seven fundamentally different crystalline forms. Of these nine phases two can exist together under a variety of conditions (given by curves on a phase diagram); three can exist together at some definite pressure and temperature; but more than three will never be found in stable equilibrium together.

Figure 11 continues the phase diagram of water to vastly higher pressures. As the pressure increases, the melting point of ice is lowered until, at a pressure of about 2 tons per square centimeter (2000 kilograms per square centimeter), it is $-23°$ C. Then the melting point begins to rise. At this temperature ordinary ice (ice I) changes to a form of different crystalline structure, denser than water! Because it is denser than water its freezing-melting point rises with pressure (§262). This is ice III. When its temperature is lowered this ice changes to ice II. Or if we increase the pressure to about $3\frac{1}{2}$ tons per square centimeter it changes to yet another crystalline form of still higher density (ice V). And at a pressure of a little over 6 tons per square centimeter ice V changes to ice VI with a melting point about 0° C. The investigation has been carried by Bridgman to pressures of 50 tons per square centimeter (50,000 atmospheres, 700,000 pounds per square inch!). Two other forms of ice are found, and the melting point of ice VIII is above 100° C!

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[1] Before 1900 America had many distinguished inventors, but few investigators in the fundamental physical sciences. (This was natural in a frontier environment, and the situation is very different today.) The outstanding physicists were Benjamin Franklin, Joseph Henry, Josiah Willard Gibbs, Henry Rowland, and (at the very end of the century) Albert Michelson. Probably most famed among European scientists of his day, least famed in America, was Gibbs, professor of mathematical physics at Yale (1871–1903). He has been called the "founder of chemical energetics."

Rowland, professor of physics at Johns Hopkins (1875–1901), was (like Michelson) a master of precision. He investigated the specific heat of water (§244), made an early determination of the mechanical equivalent of heat of highest precision and is noted for his investigations in electricity and light.

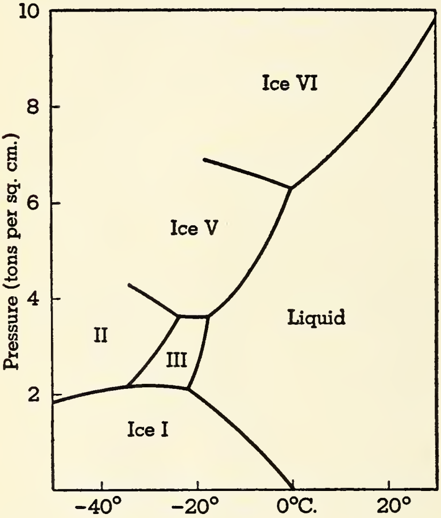

FIG. 11. Phase diagram of water at high pressures. An "Ice IV," once supposed to exist, is now discredited.

In all this the phase rule of Gibbs is followed. Ice II can be in contact with ice I or ice III, or at their triple point (2.2 tons per square centimeter and −35° C) all three can be in contact. But ice II cannot melt without first changing to ice III. Ice I is the only form of ice which can be in contact with its vapor and can sublime (Fig. 10). Ice I, III, V, VI, and VIII all can be in contact with water and have melting points. But never are more than three phases in contact.

Each of these different crystalline forms of ice has different potential energy; heat is absorbed or given out as we pass across a phase line from one phase to another. There is latent heat of crystalline transformation (Table 25), just as there are latent heat of fusion and latent heat of vaporization.

Most substances can exist in several fundamentally different crystalline forms. There are several forms of sulfur; we have yellow phosphorus and red phosphorus; diamond and graphite are different forms of carbon. There is a definite heat of transformation as we go from one form to the other. Each form has a characteristic range of temperature and pressure for which it is stable; these transformation temperatures and pressures and the melting and vaporizing temperatures and pressures are given by a phase diagram such as that which has been shown for water. When, as often happens, two forms exist together under a wide range of conditions (for example, diamond and graphite), one of the forms is (theoretically) unstable, an example of supercooling or superheating.

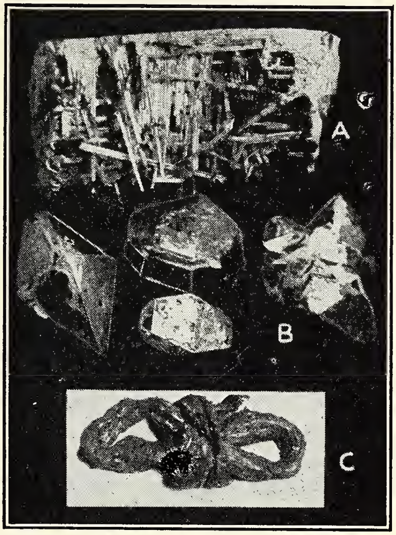

FIG. 12. Different forms of sulfur: Monoclinic, unstable at room temperature (*A*); rhombic, ordinary stable form (*B*); plastic, supercooled liquid (*C*).

If in this chapter we have emphasized melting and vaporization rather than transformations from one solid form to another it is because further consideration of the phase rule would carry us far into a field which really belongs to physical chemistry.

QUESTIONS

1. Under what circumstances may water freeze below zero? Water boil above 100° C?

2. What is the state of water at $\frac{1}{2}$ atmosphere pressure and 100° C? At what pressure does water vapor condense at 20° C, at 80° C, at 150° C? Can water vapor be condensed at 400° C?

3. How can liquid carbon dioxide be seen?

4. What are the boiling points of the liquids represented in Fig. 5?

5. Explain how a temperature below 0° C is obtained by a mixture of salt and ice.

6. How much heat is required (*a*) to melt a gram of ice; (*b*) to raise a gram of ice water to the boiling point and evaporate it?

7. What is the state of H_2O at $\frac{1}{2}$ atmosphere pressure and 90° C? 10° C? 0.0000° C? What are the states of ethyl alcohol under these conditions?

8. Pressure produces a rise in the melting point: (*a*) in the great majority of substances; (*b*) in a few exceptional substances; (*c*) in no substances.

9. To assure that the vapor pressure in a closed vessel has its saturation value: (*a*) there should be no other gas present; (*b*) there should be some of the liquid present; (*c*) the volume should be kept constant.

Vocabulary: Heat of fusion (transformation, evaporation), latent heat, melting (boiling) point, undercooling, evaporation, boiling, superheating, vapor pressure, saturated vapor, partial pressure, Dalton's Law, absolute (relative) humidity, dew point, critical temperature, triple point.

PROBLEMS

1. If a kilogram of iron at 200° C is placed on a piece of ice (at the melting point), how much ice will be melted?

2. If 10 grams of ice (0° C) are dropped into a liter of water at 20° C, how much will the temperature of the water be lowered?

3. A pitcher holds a quart (approximately a liter) of water. We wish to cool it from 77° to 50° F. How many 15-gram ice cubes must be used?

4. In an experiment it is found that 20 grams of ice placed in a calorimeter containing 190 grams of water lowers the temperature from 30° to 20° C. The water equivalent of the calorimeter is 10 grams. From the data compute the heat of fusion of ice.

5. How much ice can be melted by a gram of steam?

6. What is the heat of vaporization of water in B.t.u. per pound?

7. In a steam-heated house (70° F) what is the total amount of heat which can be received per pound of steam condensed in the radiator?

8. A gram of steam is condensed in 100 cc of water, initial temperature 20° C. What is the final temperature?

9. Suppose that 1000 cal. of heat are supplied to a kilogram of ice (0° C); what will be the final temperature? What temperature for 100,000 cal.? for 200,000 cal.? How much liquid water is there in each case?

10. Twenty grams of steam (100° C) pass into a calorimeter containing (*a*) 200 grams of ice; (*b*) 100 grams of ice; (*c*) 20 grams of ice. Find the resulting temperature in each instance. (Neglect the heat capacity of the calorimeter.)

11. How much would it cost to produce a liter of distilled water, heating it with artificial gas, assuming no loss? (Cost of gas $1.00 per 1000 cu. ft.)

12. How much heat is required to boil away a liter of alcohol starting from room temperature? (Specific heat of alcohol, 0.45.)

13. At what temperature would water boil on Pikes Peak? on Mt. Everest? (Fig. 5–11.)

14. Find the relative humidity when the temperature is 26° C and the dew point is 4° C.

15. A few drops of water are placed in a bottle at 20° C and 76 cm (Hg) pressure; after the air has become saturated the bottle is stoppered and then cooled with ice. What is the pressure in the bottle now?

16. A suction pump cannot lift water at 100° C; why? How high can it lift it at 80° C? at 20° C?

17. A barometric mercury column is covered with a little water. How high will the column stand at 20° C? at 50° C? at 80° C? at 100° C? Assume standard atmospheric pressure.

18. Three mercury columns (Fig. 4) are covered with a few drops of ether, carbon bisulfide, and water, respectively. The temperature is 40° C. How high will each column stand?

19. (*a*) A dry fruit jar is sealed at room temperature and placed in boiling water. Find the pressure in the jar. (*b*) What would be the pressure if the jar contained a little water?

20. A loosely stoppered bottle contains some ether. The thermometer reads 25° C; the barometer reads 74 cm. Find the partial pressure of the vapor and of the air in the bottle.

21. A small vial of ether is placed in a bottle. The bottle is stoppered and the vial is broken. What will the pressure in the bottle be?

22. What is the state of water under the following conditions:

−10° C	2 tons per sq. cm	60° C	20 cm (Hg)
20° C	9 tons per sq. cm	80° C	20 cm (Hg)
10° C	1 cm (Hg)	250° C	60 atmospheres

CHAPTER 24

THE LAWS OF THERMODYNAMICS

In describing the phenomena of heat — temperature, expansion, heat transfer, specific heat, and latent heat — we have more or less assumed that heat was a form of energy. We have interpreted it as a manifestation of the random motion of the molecules. But we have not as yet demonstrated that heat is in fact a form of energy. How many ergs are there in a calorie? Our order of presentation has possibly not been the most logical one — certainly not the historical one. In this chapter we shall give the two "fundamental laws of thermodynamics." The chapter will show (1) that heat is a form of energy, but (2) that it is a degraded form which cannot be converted in its entirety into higher forms.

FIRST LAW OF THERMODYNAMICS

281. Mechanical Equivalent of Heat. Until the end of the eighteenth century heat was considered a "subtle fluid." Then Count Rumford, boring cannons for the Elector of Bavaria, and observing that the heat developed by friction was apparently unlimited in amount and proportional to the work done, conceived of heat as a form of energy.[1] However, this view was not generally accepted until the middle of the next century. Then Joule, a student of Dalton, found the exact value of the constant of proportionality.

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[1] Count Rumford was one of the most colorful personalities in physics. He was born in Massachusetts as plain Benjamin Thompson. At 19 he married a rich widow; he was a Tory and at 23 he departed from America, leaving our Revolution and his wife behind. He became a prominent figure in scientific, military, political, and social circles in Europe. After his first wife died he married the (wealthy) widow of the great chemist Lavoisier — later separated. He was for a time a soldier of fortune, then became minister of war, minister of police, and grand chamberlain of Bavaria. He studied gunpowder and cannons and invented a cookstove. He was knighted by George III, and by the Elector of Bavaria made Count of the Holy Roman Empire. In 1800 he founded the Royal Institution of London. In the interim he introduced social reforms; in one day he arrested 2600 beggars and depredators in Munich and gave them each a job. To Rumford we owe the formula: "To make vicious and abandoned people happy, it has generally been supposed necessary first to make them virtuous. But why not reverse the order? Why not make them happy, and then virtuous?"

Joule heated water by stirring it. In Fig. 1 the blades of the paddle shaft turn between stationary blades. As the paddle shaft is rotated the water is violently agitated, churned between the moving and fixed blades. The work done (in ergs) is the product of the applied torque and the angle (in radians — 2π times the number of revolutions). The heat developed (in calories) is computed from the temperature rise of the known quantity of water. Joule found that approximately 42,000,000 ergs of work were necessary to generate 1 calorie of heat.

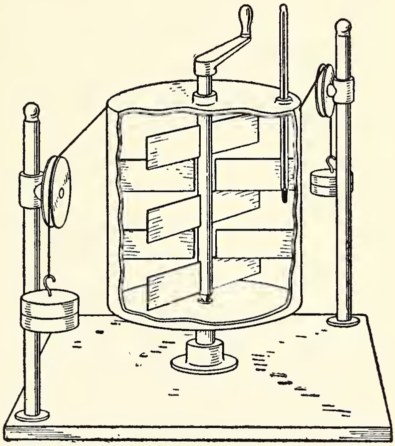
FIG. 1. Joule's experiment.

Ergs and calories are both units of energy; 4.18×10^7 ergs (or approximately 4.2 joules) are equivalent to 1 calorie of heat. This ratio, the number of ergs per calorie, is called the " mechanical equivalent of heat " and is one of the few constants in physics which the student should be expected to remember permanently:

$$4.18 \times 10^7 \text{ ergs} = 1 \text{ calorie.} \tag{9}$$

The mechanical equivalent of heat in British units is the ratio between the B.t.u. and the foot-pound. One B.t.u. = 778 foot-pounds.

Example. An electric mixer rotates 1000 times a minute. Suppose that the torque is 10,000,000 dyne-cm. Approximately how much heat is generated per minute?

$$W = \mathcal{T} \times \theta = 10^7 \times 2\pi \times 10^3 = 6.28 \times 10^{10} \text{ ergs.}$$
$$\text{Heat (in calories)} = W \div 4.2 \times 10^7 = 1500 \text{ cal.}$$

282. Conservation of Energy. Joule showed that the amount of heat developed in his " churn " by a given amount of work was the same irrespective of the liquid. We can measure in the laboratory the heat developed by working against a friction brake. We can measure (though not very accurately) the heat developed as lead shot is dropped from a given height and mechanical potential energy (mgh) transformed to heat. Joule himself measured the heat evolved by electric energy, by chemical energy; from a definite quantity of energy, whatever its form and whatever the process, a definite quantity of heat was developed.

This was the step needed to establish the principle of conservation of energy. In mechanics we have seen potential energy changing into kinetic and kinetic energy into potential. We have studied the potential and kinetic energy of fluids; we shall study the energy of electricity at rest and in motion. But always, because there is friction or resistance, these forms of mechanical and electrical energy tend to disappear. Joule and his successors showed that whenever other forms of energy disappeared a definite quantity of heat was generated.

The law of Conservation of Energy ("First Law of Thermodynamics") may be stated as follows: *The total energy within an isolated* (i.e., "complete" [§115]) *physical system remains unchanged.* Energy can be neither created nor destroyed. It can be changed from one form to another; other forms often change into heat; when they do so it is in the ratio 4.18×10^7 ergs = 1 calorie. But the total energy of the universe remains unchanged.

Examples. 1. How much heat is generated when a 42-gram bullet moving 1000 meters per second is stopped?

$$\text{Kinetic energy} = \tfrac{1}{2} \cdot 42 \cdot 100{,}000^2 = 2.1 \times 10^{11} \text{ ergs.}$$
$$\text{Heat} = 2.1 \times 10^{11} \div 4.2 \times 10^7 = 5000 \text{ cal.}$$

(Observe that if all this heat were generated in a steel bullet [specific heat 0.1] its temperature would be raised more than 1000°!)

2. How fast is heat evolved by a 1-hp mechanical saw? 1 hp = 550 ft-lb. per sec. 550 ÷ 778 = 0.70 B.t.u. per sec.

283. Cooling a Gas by Expansion. These have been examples of the conversion of mechanical energy into heat. We now give the outstanding instance of the reverse process. When a gas expands it must usually push back a piston or push back the atmosphere. In these cases it does work ($W = P \cdot \Delta V$). Hence, if no heat is allowed to flow into it, the gas cools. The work is done at the expense of the kinetic energy of the molecules. . . Gas escaping from an automobile tire is cooler than the surrounding air. . . In cooling by expansion lies the secret of the liquefaction of air (§277). . . When an air mass rises in the atmosphere it expands and cools. When moist air from the ocean passes over mountain ranges the rising air cools below the dew point and there is rain. The air is robbed of its moisture, and the regions behind the mountains are usually arid. . . The outstanding practical example is the steam engine. Here the heat energy of the expanding steam is converted into useful work to drive our trains, to rotate our electric generators, and to revolutionize our civilization.

The converse effect is, of course, the heating of a gas by compression.

We have already discussed these effects of adiabatic expansion and compression from the point of view of Kinetic Theory in §243.

284.* Free Expansion of a Gas. This cooling of the expanding gas occurs as a consequence of the work it does. If it is pushing back a piston each molecule bounces back from the retreating surface with a slight loss of speed. But will there be a change in temperature when a gas expands into a vacuum, without doing work? Suppose that the gas is contained in a small bottle within a larger evacuated vessel, and the small bottle is suddenly broken. Will the molecules lose speed and kinetic energy as they enter the larger volume? Joule asked himself this question and performed the experiment with negative result. Indeed, for most practical purposes we may consider that cooling occurs only when the gas does external work. Yet actually a slight cooling does occur in free expansion into a vacuum owing to the work done against the attractive forces among the molecules. The molecules move into the vacuum with almost unchanged velocity; yet because of the cohesive forces they do slow down slightly as they move apart. (These attractive forces are, as we have seen, very important in a liquid, and great cooling occurs in evaporation.) Although the effect is small (at ordinary pressures) and Joule failed to find it in his first experiment, it was demonstrated in a later indirect experiment which he performed together with Thomson (Lord Kelvin) — the famous Joule-Thomson porous-plug experiment.

285. Two Specific Heats of a Gas. When a gas is heated either its pressure or volume must increase. When the volume is kept constant all the heat supplied goes to increase the kinetic energy of the molecules. Expansion, on the other hand, means external work. *When the pressure is kept constant and the volume increases, energy must be supplied not only to speed up the molecules but also to do the work of expansion.* Consequently there are two commonly recognized specific heats in gases: the specific heat at constant volume, and the larger specific heat at constant pressure.

286.* The work done by the expanding gas as its temperature rises 1 degree at constant pressure equals R ergs;[1] that is, for a gram molecule the work of expansion is 83,000,000 ergs. Experimentally it is found that the difference in heat capacity at constant pressure and at constant volume for a gram molecule of any gas is about 2 calories (exactly 1.986 calories). This relation (83,000,000 ergs = approximately 2 calories) was first found by Joule and furnished him one of his earliest evaluations of the mechanical equivalent.

SECOND LAW OF THERMODYNAMICS

287. Nature of Heat Energy. Heat is the random energy of the molecules. We have seen many evidences of molecular motion. Temperature is a measure of the average kinetic energy of a molecule. As the molecules move faster, hitting the walls harder and more often, the pressure increases. This explains Charles' Law. In the last chapter it was found that molecular potential energy is no less important than kinetic.

This kinetic energy of the molecules is characterized by its perfect disorder. We drop a ball to the floor. Potential energy changes to kinetic energy, and this in turn to heat. While the ball falls the molecules move together downward; the ball hits the floor, and this orderly motion is disarranged; the molecules still move, but now in random directions; the kinetic energy of the falling body is converted to a " scrambled " kinetic energy — this is heat.

Now there are countless ways of producing disorder and only one way of producing order. It is just for this reason that any other form of energy can be entirely converted into heat; but heat can never be entirely reconverted into other forms of energy. It will never be possible to devise an engine which will take the heat from the floor and raise our ball to its original height. The heat energy of the universe forms a sump into which other ordered forms of energy tend to degrade. Without here defining the term strictly we may call the measure of the disorder *entropy*. For mechanical energy or electrical energy the entropy is zero;

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[1] This is easily shown. At temperature T we have $PV_1 = RT$; at temperature $T + 1$, $PV_2 = R(T + 1)$. Subtracting, $P(V_2 - V_1) = R$. By eq. (7′) of §136 this is the work done.

for heat it is greater. And as heat energy is shared by more and more molecules, as the temperature (the average energy of each) becomes less, the entropy increases. Heat energy is high-entropy energy, and the entropy of the heat is greater the lower the temperature.

288. The Second Law of Thermodynamics states: *The entropy of an isolated system cannot decrease.* Order cannot be created from disorder. Heat cannot of itself flow " uphill " (to higher temperature, smaller entropy). This is sometimes called the principle of *degradation of energy.* The total energy of the universe remains constant, but it tends always to degenerate into less and less useful forms — forms with greater entropy.

Considerable ingenuity has been wasted by persons, too wise to violate the first law of thermodynamics and create energy out of nothing, who attempt to make perpetual-motion machines of another kind. They will take heat from the surrounding medium and convert it 100 per cent into work; they will make a steamboat traverse the ocean, using for the boilers the heat from the water and leaving cold water in its wake. (" Perpetual-motion machine of the second class.") But this violates the second law of thermodynamics.

Heat is converted into work in an expanding gas, but only as long as there is a supply of compressed gas. It can be done only when a *difference in pressure* is available and only until the pressure is reduced to that of the surrounding medium (atmosphere). We shall now see how heat, in a continuous process, can be converted into work. This can be done only when a *difference of temperatures* is available.

289. Heat Engines. According to the story that is told, the first steam engine in the coal mines of England had hand-operated valves. It was the duty of a lad named Humphrey Potter to let the steam in and out with each stroke of the piston. The work did not appeal to him. So he rigged up a contraption of wire and gadgets which opened and closed the valves automatically, and one day Mr. Newcomen was surprised to see his steam engine running itself! Humphrey must have been discharged for his laziness, for his name appears no more in history.

Figure 2 shows the descendant of Humphrey Potter's automatic valve as used in the reciprocating steam engine. The steam chest communicates with the boiler, the opening *e* with the exhaust. As the flywheel turns, the slide moves back and forth, allowing the steam to flow first in one side of the cylinder, then in the other. In the position shown, steam is entering the left side and steam from the right side is passing out the exhaust. The incoming vapor (with pressure depending on the boiler temperature) pushes back the piston. In the middle of the stroke (*b*, Fig. 3) the intake closes and the steam expands and cools as it does work.

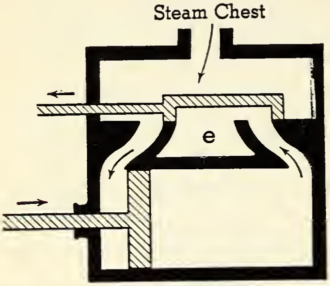

FIG. 2. Slide valve and cylinder of steam engine.

Hot steam enters, cooler steam leaves, heat energy is diminished and work is done.

290. The Condenser. Figure 3 represents the pressure at different parts of the stroke. (This is the " indicator diagram " of an ideal steam engine.) The high pressure steam enters as the piston moves out increasing the cylinder volume from *a* to *b*; from *b* to *c* the vapor expands adiabatically and the pressure and temperature decrease; during most of the return stroke (from *d* to *e*) the exhaust valve is open and the low-pressure steam is expelled. During the forward stroke (*a* to *c*) the steam pushes the piston; during the backward stroke the piston pushes

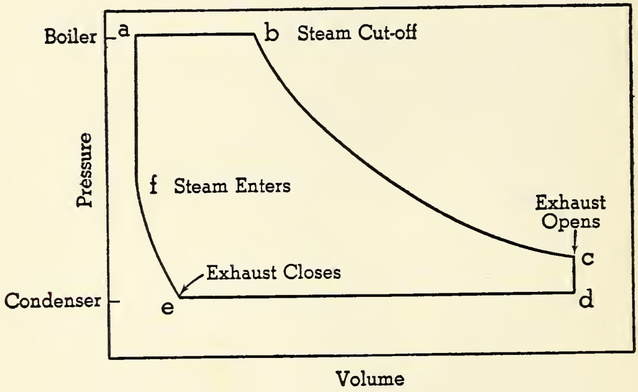

FIG. 3. Ideal indicator diagram of steam engine.

the steam and returns to the steam 70 or 80 per cent of the energy received from it in the forward stroke. This energy return represents wasted energy; it can be reduced by reducing the pressure during the return stroke.

If the exhaust opens to the atmosphere there is a pressure of 15 pounds per square inch from *d* to *e*, opposing the expulsion. For greater efficiency in most steam engines the exhaust steam passes into a vacuum chamber where it is condensed to a liquid on water-cooled pipes (the *condenser*); this reduces the vapor pressure (opposing the return stroke) to a low value. The vapor pressure of the steam fed to the engine depends upon the temperature of the boiler; the back pressure of the exhaust depends upon the temperature of the condenser.

291. Efficiency of a Heat Engine. How can heat be converted into mechanical energy (zero entropy) without violating the second law of thermodynamics? The steam engine takes heat from the boiler, carries most of it to the cold condenser, converts a smaller part of it into work. Figure 4 represents the process diagrammatically. Low-temperature energy is high-entropy energy. The entropy (degradation) of heat at 300° abs. is $\frac{4}{3}$ as much as at 400° abs. We are allowed to promote 1 calorie of heat to mechanical work at the expense of degrading 3 calories of heat from high temperature to low. Observe that the total entropy of the system does not decrease in this process.

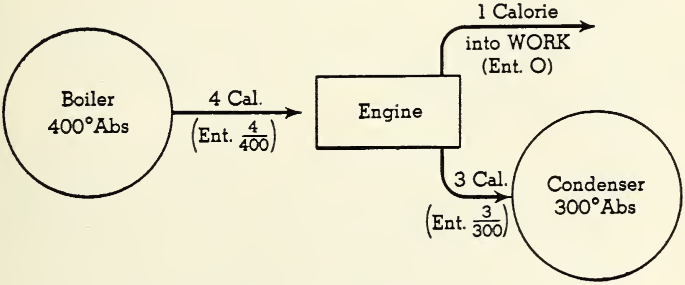

FIG. 4. Thermodynamics of the steam engine.

(1) Energy in equals energy out. (1st Law.)
(2) Entropy gain in condenser equals (perfect engine) or exceeds (actual engine) entropy decrease in boiler. (2nd Law.)

If T_b is the temperature of the boiler and T_c the temperature

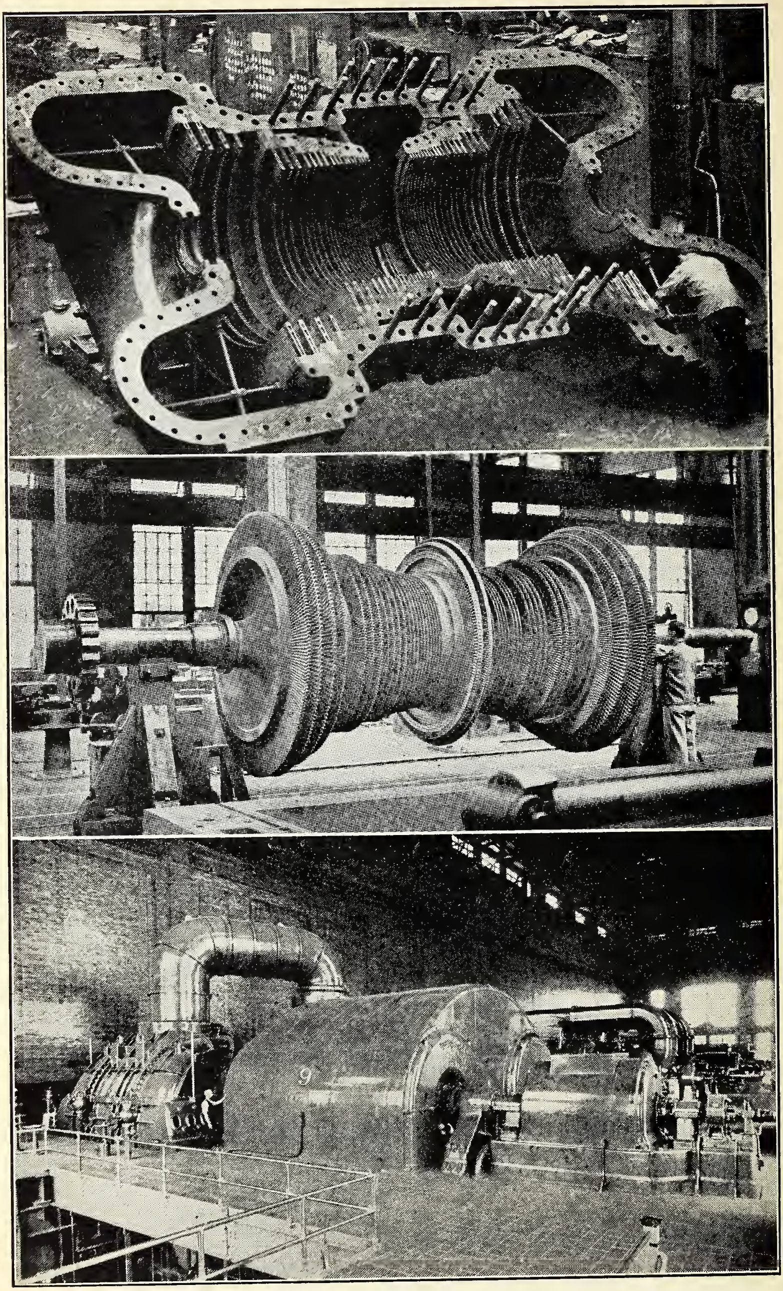

Fig. 5. Steam turbine: stator and casing (*upper*); rotor (*center*); turbine in electric power plant (*lower*) operating a-c. generator at right.

of the condenser, it can be shown that the greatest possible efficiency of a heat engine is

$$\text{Maximum efficiency} = \frac{\text{Work done}}{\text{Heat from boiler}} = \frac{T_b - T_c}{T_b}. \qquad (10)$$

(This law of greatest efficiency is sometimes taken as a mathematical statement of the second law of thermodynamics.)

292. The Steam Turbine. The first steam turbine was built by Hero of Alexandria. Water was heated in a spherical vessel free to rotate. Steam escaped (tangentially) from nozzles in the vessel and the vessel was set spinning, propelled by the reaction of the escaping steam. " Hero's ball " was only a toy. The present-day steam turbine is in appearance more like an intricate windmill, with thousands of blades propelled by the pressure of a steady blast of steam. The modern turbine designed to squeeze every available erg of energy from the steam is truly one of the " marvels of science." The expanding steam writhes its way through a succession of moving and stationary blades, exerts an impulse to the blades of the rotor, then is redirected by the stator to blow against the rotor again and again. A large turbine may take steam at a pressure of 1000 pounds per square inch, rotate 2000 revolutions per minute, deliver 200,000 horsepower, yet is constructed with the delicacy of a watch, running without vibration, with rotor and stator separated by less than the thickness of a piece of tissue paper. The efficiency of the turbine may be 75 per cent of the value given by eq. (10) for the perfect engine. Steam turbines have practically displaced reciprocating engines in electrical power plants.

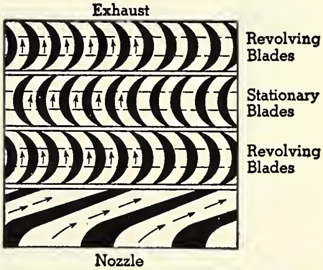

FIG. 6. Illustrating flow of steam through turbine.

The theoretical efficiency of an engine increases with boiler temperature. The low vapor pressure, higher boiling point of mercury gives it an advantage over water as the medium in a high-temperature engine. Mercury-vapor turbines operating at a temperature of nearly 500° C have been introduced into a few central station power plants.

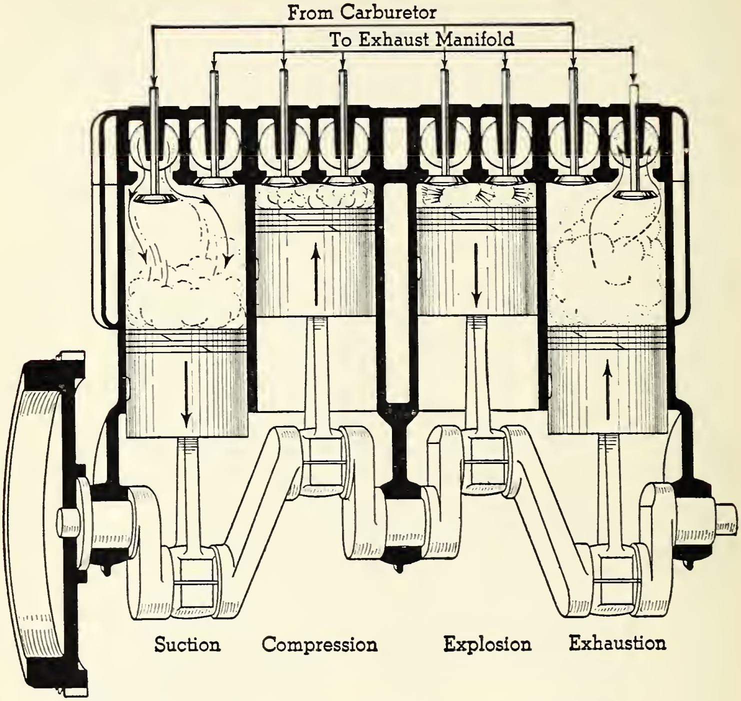

FIG. 7. Operation of internal-combustion engine.

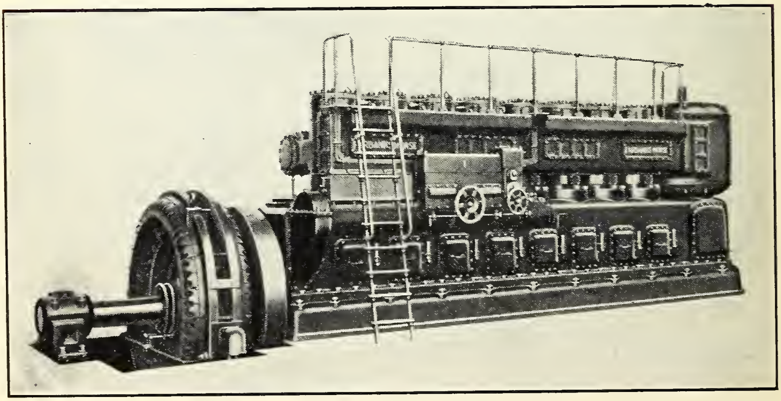

FIG. 8. Diesel engine and (at left) electric generator.

293. Internal-Combustion Engine. Figure 7 represents the operation of a four-cylinder automobile engine. The power comes from successive explosions of the gasoline-air mixture. In the carburetor, gasoline vapor and air are mixed in suitable proportion. Each cylinder has two piston valves, one (inlet) opening to the carburetor and one (outlet) opening to the exhaust. Each cylinder explodes once in every four strokes of the piston. First as the piston moves down (inlet valve open) the explosive gas from the carburetor is sucked into the cylinder (suction stroke); on the return stroke the gas is compressed (compression stroke); the spark then occurs and explodes the mixture (explosion stroke); on the return stroke the outlet valve opens and the exhaust gases are pushed from the cylinder (exhaust stroke).

The large stationary internal-combustion engine is usually of the Diesel type. Here the fuel is oil. There is no carburetor; the fuel oil is burned as it enters the cylinder in a jet. The temperature within the cylinder during the working stroke is quite high. It is designed with a view to efficiency and actually is the most efficient of engines.

The efficiency equation (eq. 10) can be expressed in terms of pressures for internal-combustion engines. It then appears that the efficiency increases with the *compression ratio* (ratio of initial to final volume in the compression stroke). In automobile engines this ratio is between 6 and 7; further increase is limited by the tendency for precombustion and knocking at high pressure. In the Diesel type of engine the compression ratio may be as great as 15; hence its high efficiency.

TABLE 28

TYPICAL ENGINE EFFICIENCIES

Locomotive	9%
High-pressure steam turbine	20%
Turbine with mercury-vapor stage	25%
Diesel engine	35%

294. Refrigerator — a Heat Engine in Reverse. The mechanical refrigerator was described in the last chapter. The refrigerating process is represented by Fig. 4, if we reverse the direction of the arrows: 3 calories of heat are now taken from the refrigerator, 4 calories of heat are turned into the room; we now *do* work (1 calorie) on the system. Heat is taken from the colder source; the heat energy and mechanical energy pass as heat to the warmer region. By a modification of eq. (10) it can be shown that the work required is less the smaller the difference in temperature between the refrigerator and its surroundings.

295. Efficiency of the Human Body. Food provides the substance of the growing body and furnishes the energy for warmth and mechanical work. The principle of conservation of energy has been shown to hold for these processes of metabolism by the results of an elaborate series of experiments in which persons carried on more or less normal activities in a huge, room-size calorimeter. The energy input (the calorie value of the food consumed) and the energy output (the work done, the heat given off, the calorie value of bodily wastes) were measured and were found equal within the experimental error.

Considered as an organism for turning chemical energy into work, the efficiency of the body is remarkable; the efficiency under optimum conditions is about 25 per cent, comparable with that of the best heat engines. If the body were itself a heat engine such an efficiency would be unthinkable, for its temperature is usually only a few degrees different from its surroundings. It is rather a " chemical engine," converting, through the intricate processes of metabolism, chemical energy (molecular potential energy with less entropy than low-temperature heat) into mechanical energy directly; the body is in this respect more nearly analogous to an electric cell than to a steam engine.

Several questions concerning the energetics of the body sometimes puzzle the student. When a gas does work it cools; why does a man warm up as he runs upstairs? It is for the same reason that a steam engine gives out heat when it works. The heat is a tribute to the second law of thermodynamics. The body is 25 per cent efficient; 1 cal. of energy goes into useful work while 3 cal. appear as heat or other high-entropy energy. The physiologist finds that a contracting muscle fiber gives off heat. When one runs upstairs the energy store of the muscles decreases both by the work done and the heat generated. . . And why does the body become fatigued merely by exerting a large force, without doing work? When the muscle is under strain it deteriorates, it loses ability to do work, loses energy; but the energy appears not in useful work done but simply in the energy of heat and of the waste acids which produce the sensation of fatigue. Under these circumstances the efficiency of the body is zero. The efficiency of the body is high when actually doing work. The farmer knows that a team of horses is an efficient motive power for plowing the cornfield but that if left in the barn they will " eat him out of house and home."

SUGGESTED READING. " Behemoth " by Hodgins and Magoun (Doubleday, Doran). The story of the discovery of steam and electrical power, told with gusto; many good pictures. In contrast " Atoms in Action " by Harrison (McGraw-Hill) tells the story of present day advances in all branches of applied physics. Both books are strongly recommended.

QUESTIONS

1. State the two laws of thermodynamics.

2. Is it possible to convert 42,000,000 ergs of mechanical energy into 1 cal. of heat? Is it possible to do the reverse?

3. It is said: " Energy is wealth." What about the vast heat energy all about us in air, water, ground? Is high-entropy energy valuable?

4. Why does the air temperature decrease with altitude?

5. Explain how air is robbed of its moisture in passing over mountain ranges.

6. Is it possible to cool a room by running an electric refrigerator with its door open?

7. Can you devise an engine which will run by ice?

8. Rain drops fall through the air at constant speed. Will they warm or cool? (Discuss the factors involved.)

9. Two drops of mercury coalesce to form a larger drop. Is there a change of temperature?

10. A clock spring is wound up and then dissolved in acid. Is energy conserved?

11. Can you think of any processes other than the expansion of gases by which heat energy can be converted into mechanical energy?

12. A Diesel engine is very efficient because: (*a*) it requires no spark for ignition; (*b*) it burns cheap oil; (*c*) it has a high compression ratio.

13. In an efficient steam engine for each calorie of heat from the fuel approximately: (*a*) 10^7, (*b*) 4×10^7, (*c*) 10^8 ergs of work can be obtained.

Vocabulary: First (Second) Law of Thermodynamics, Mechanical Equivalent of Heat, free expansion, specific heat at constant pressure (volume), entropy, degradation of energy, slide valve, condenser, indicator diagram, reciprocating (internal-combustion, Diesel) engine, steam (mercury-vapor) turbine, compression ratio.

PROBLEMS

1. How much heat is evolved when a kilogram weight falls upon the floor from a height of 10 meters? When a pound weight falls 778 ft.?

2. How much heat is generated when a 50-kilogram weight is dragged 100 meters? (Coefficient of friction, 40 per cent.)

3. By means of a single fixed pulley a 100-lb. weight is lifted 20 ft. per sec. by a force of 110 lb. How much heat is generated in the pulley per second?

4. How much heat is generated in the brakes when a 1-ton automobile traveling 60 miles an hour is brought to rest?

5. How much heat is generated when a lead 20-gm bullet with speed of 700 meters per second strikes a metal plate?

6. If it is assumed in problem 5 that half the heat remains in the bullet, how much will its temperature be raised?

7. Meteors reach the earth's atmosphere with a velocity of at least 11 kilometers (7 miles) per sec. (compare §198). If all the heat generated as it was brought to rest went into the meteor, how much would its temperature be raised? (Assume the specific heat of iron.)

8. The height of Niagara Falls is 170 ft. How much is the water heated as the result of the fall?

9. A copper wire is a meter long and 1 sq. mm in cross section. Applying a force of 20 kilograms (beyond the elastic limit) we stretch it 10 cm. How much does its temperature rise?

10. A 40-watt incandescent lamp is placed in a calorimeter containing 200 cc of water. How much is the temperature raised per minute?

11. If electrical energy costs 3 cents a kilowatt-hour, how much does it cost to distil a liter of water electrically?

12. Suppose that, with a 313° C heat supply, a certain device delivers 2.1 joules per calorie. Is it a perfect engine? (Surrounding temperature 20° C.)

13. In 1910 the average boiler temperature in central station plants was about 200° C; today in the most efficient stations the temperature is above 300° C. What are the pressure and theoretical efficiency at each of these temperatures (condenser at 20° C)?

14. A Diesel engine burns 6 gal. of fuel oil per hour and develops 800 hp. Find its efficiency.

15. For the average man doing no serious work about 2200 large calories are required per day (for " basal metabolism " and incidental activity). It is estimated that a young man can do 1,000,000 joules of work per day without fatigue. The body is about 25 per cent efficient. What is the calorie value of the diet necessary for this man?

16. A spring with spring constant of 1 kilogram per cm is held compressed 5 cm, placed in acid, and dissolved. How much extra heat is evolved by the chemical action in virtue of the compression?

WAVE MOTION AND SOUND

CHAPTER 25

PROGRESSIVE WAVES

296. Longitudinal and Transverse Waves. If I strike sharply against one side of a rubber block, the rubber is compressed locally; the displaced particles then shove against their neighbors and these against their neighbors, and so on, and a wave of compression passes through the block. The same thing happens if I strike the end of a steel rod, except that here (because of the great rigidity of steel) the compression is passed along much more quickly (5000 meters per second). By striking one end of a long helical spring and watching the propagation of the compression one may visualize what is happening (much more quickly) in the rubber and the steel; to the eye the two ends of the steel rod appear to move off simultaneously.

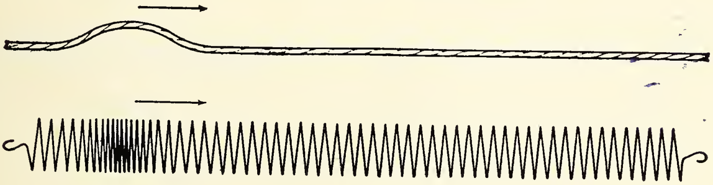

FIG. 1. Progressive pulse, transverse and longitudinal.

These are examples of wave motion. A different kind of wave is produced when I suddenly jerk a long rope or a stretched string sidewise. Here I produce a transverse displacement, and it is this which is now propagated along the rope or string. I can produce a similar wave in the steel rod by striking it on the side. These are *transverse* waves.

The displacement in the compressional waves is along the direction of propagation, and these are called *longitudinal* waves.

These waves — either longitudinal or transverse — are *progressive* waves, as distinguished from the standing waves which will be described in the following chapter.

297. Velocity of Propagation of Waves. The velocity (v) of the wave through the medium depends upon a "stiffness" factor and on an inertia factor. For a compressional wave

$$v = \sqrt{\frac{E}{d}}, \tag{1}$$

where E is the modulus of elasticity (i.e., for propagation along a rod, Young's Modulus, §64) and d the density. For a transverse wave in a string

$$v = \sqrt{\frac{T}{m'}}, \tag{1'}$$

where T is the tension of the string and m' the mass of the string per unit length. E measures the resistance to compression or elongation; T measures essentially the resistance to transverse displacement. The greater this factor E or T is, the more rapidly the distortion is passed on through the medium. The greater the mass per unit volume (d) or the mass per unit length (m'), the smaller the velocity. Although these relations are given here without proof they will appeal to the student as consistent with his experience.

Example. If a light string (fish line) weighing 0.01 gram per cm is stretched with the force of a kilogram (approximately 1,000,000 dynes) the velocity of the transverse wave is (approximately)

$$v = \sqrt{\frac{1{,}000{,}000}{0.01}} = 10{,}000 \text{ cm per sec.}$$

298. Simple Harmonic Wave Motion. The waves which have just been described were simple pulses (longitudinal or transverse) which traveled along the medium; they are hardly typical of most of the waves which we shall study. If, instead of giving the rope a single impulse, I oscillate one end with simple harmonic motion, I produce a train of waves. Every point in the rope will be set into simple harmonic oscillation, but (because of the time required for disturbance to travel along the rope) the phases of this oscillation differ for each successive point. Such a train of waves is shown in Fig. 2 (upper). Curve 1 (drawn heavy) shows the rope at a certain moment, and curves 2, 3, 4 show the progression of the wave from moment to moment.

The left end of the rope (which I shake) is moving downward and upward in simple harmonic motion. In curve 1 this end has no displacement but has downward velocity; a quarter period later, in curve 3, it has maximum displacement and no velocity. Each other point along the rope is executing simple harmonic motion in the same way but at each point the vibration occurs in different phase. Because of this phase difference there is a wave form. Here there is a "hill," shortly beyond a "valley." The distance between two points in the same phase (for example, between successive hills) is called a **wave-length.** As the rope moves up and down the wave form moves to the right: between curves 1 and 3 the end of the rope has gone through a quarter vibration and the wave form has moved to the right by a quarter wave-length. The speed of propagation is given by eq. (1′).

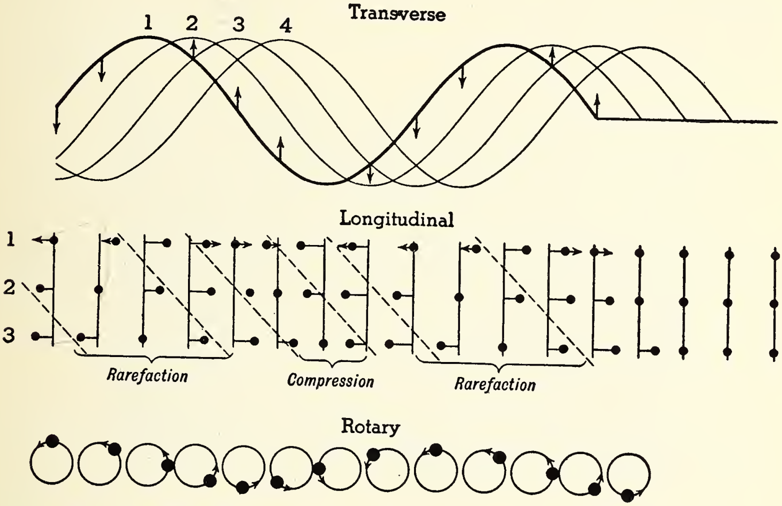

FIG. 2. Progressive waves. Arrows represent velocities. Longitudinal waves are shown as displacement of particles from normal positions.

This is a transverse wave. We may also have longitudinal simple harmonic wave motion. Suppose that, instead of applying a simple pulse to the spring in Fig. 1, we continually push the end of it back and forth with simple harmonic motion. We thus send a series of compressions followed by extensions ("rarefactions") down the spring. Every individual point in the spring vibrates right and left in simple harmonic motion, each successive point with slightly different phase. The wave-length (distance between points of like phase) is now the distance between compressions.

The longitudinal wave motion is represented in Fig. 2. The motion here is somewhat less easy to visualize than the transverse wave motion, and the figure should be carefully analyzed. Imagine beads placed as indicators along the spring at equal intervals; the horizontal rows of dots (1, 2, 3) represent this string of beads at successive moments, the vertical lines giving their normal undisplaced positions. The upper row (corresponding to curve 1 of the transverse motion) represents the displacement when the wave has already progressed three-quarters the length of the spring. The left end of the spring is undisplaced but moving to the left. The next bead is displaced slightly to the right, producing a rarefaction; its velocity (indicated in line 1 with arrows) is to the left, and in (2) this bead has become undisplaced, like the first bead in (1). Thus each particle has simple harmonic motion, compression follows rarefaction, and the wave pattern progresses to the right.

Such longitudinal waves — a train of compressions and rarefactions — can be set up in any medium: solid, liquid, gas, since all of them in greater or less degree resist compression, have a modulus of elasticity. The outstanding example is the waves in air which, tapping on our eardrums with their alternating condensations and rarefactions, produce the sensation of sound (§303).

299. Surface Waves. Most familiar of all progressive waves are those on the surface of the water. There is no progressive movement of the water as a whole; only the wave form is carried along. Possibly the student has observed the motion of a cork floating in the waves. While it is bobbing up and down it is also moving to and fro — its path is practically circular. Each particle in the water wave moves in a circle, a large circle for ocean waves and a smaller one for ripples. Figure 2 shows the circular motion; the difference in phase from particle to particle produces the familiar wave form of the surface.

Gravity tends to make the water surface level; this corresponds to the elastic restoring forces in the bar and the stretched rope. It can be shown that for long waves in deep water the velocity is given by

$$v = \sqrt{\frac{g\lambda}{2\pi}},$$

where λ is the wave-length. $\sqrt{32/2\pi}$ equals approximately 2.2, and hence the wave velocity in deep water is $2.2\sqrt{\lambda}$ ft. per sec.; an ocean wave 100 ft. long travels 22 ft. per sec.

This relation holds when the depth is greater than the wave-length. As ocean waves approach the shore their speed decreases; here the speed is determined by the depth rather than the wave-length. . . For short ripples the velocity depends upon the surface tension rather than the gravitational force.

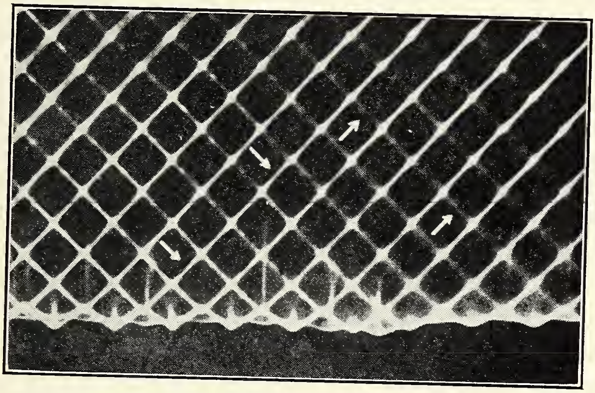

FIG. 3. Reflection of water waves from a straight wall. The white lines are crests in sharp focus.

300. Wave Fronts and Rays. When a stone is thrown into water, waves are sent out in enlarging circles.[1] We may think of the circles in Fig. 4 as drawn along the crests (or equally well along the troughs) of the waves. Such lines drawn through points of like phase are called *wave fronts*. Other lines (*rays*)

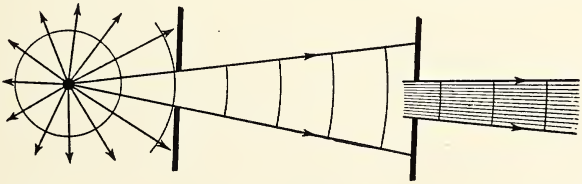

FIG. 4. Wave fronts and rays (neglecting diffraction).

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[1] Not to be confused with the circular path of the particle.

are drawn perpendicular to the wave fronts to indicate the direction of propagation.

As the wave fronts proceed outward making larger and larger circles, the wave-length and frequency of oscillation of the waves remain the same but their amplitude (height) decreases. At great distances a small portion of the wave front appears nearly straight, neighboring rays lose most of their divergence,[1] becoming nearly parallel.

301. Reflection, Refraction, Diffraction, and Interference. A number of fundamental phenomena of wave motion are well illustrated in these water waves. When the waves strike a wall they are *reflected*, leaving the wall, if it is straight, with unchanged divergence. Straight wave fronts are reflected as straight wave fronts (Fig. 3); waves diverging from a source appear to come from a new source an equal distance behind the wall (see Fig. 48–2*b*).

If the wall itself is curved the divergence may be increased or decreased; indeed, the reflected wave may even leave the wall converging and come to a *focus*. For example, if a stone is thrown into the water at the exact center of a circular pool, the waves after striking the circular boundary are reflected directly back and the converging rays come back to a focus at the center. If the source is located slightly away from the center, the reflected waves again converge but now at a focus just beyond the center. The effect can easily be shown in ripples excited in a dish of mercury (Fig. 5*a*). . . In Fig. 6 the curvature of the mirror is such as to reflect the diverging wave as a straight (plane) wave front, without either convergence or divergence.

As ocean waves approach the shore and the water becomes shallower their velocity of propagation decreases. In consequence of this retardation, waves approaching the shore obliquely are bent (*refracted*) so as to turn the advancing wave front more nearly parallel to the shore. (See Fig. 49–1.) It is an effect which anyone who has visited the seashore must have observed; on a shallow beach the waves always come very nearly directly to the shore, never at great oblique angles.

When the waves meet an obstacle they pass by, casting a

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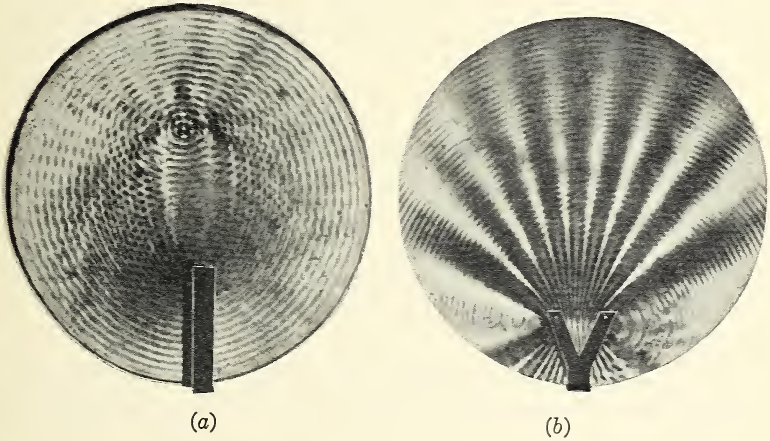
[1] We can measure the "divergence" of the wave as the reciprocal of the distance from the source (§651).

"shadow" of quiet water behind it. But the waves do not move strictly in straight lines; they tend to straggle into the shadow, and the shadow edge is not sharply defined. This is especially true for the longer waves; when ocean waves strike a pile they close in behind it with little trace of a "shadow." This bending of waves about an obstacle is called *diffraction.*

If two stones are thrown into water the expanding wave fronts will cross each other and produce *interference.* When two crests or two troughs coincide a large crest or trough results (constructive interference); when a crest of one wave train coincides with a trough of the other the waves cancel and the water is undisturbed (destructive interference); a point of destructive interference is called a node. The "choppy water" observed near a steep bank is due to the antinodes (constructive interference) and nodes produced as the waves reflected from the bank meet the oncoming waves.



(*a*) (*b*)

FIG. 5. Ripples in mercury. (*a*) Reflection from circular dish showing conjugate foci. (*b*) Interference by two neighboring sources.

All these phenomena — reflection, refraction, diffraction, and interference — are found in all forms of wave motion — waves in metal plates, earthquake waves in the earth, sound waves (longitudinal) in the air, radio or light waves (transverse) in the "ether."

302. Fundamental Formula in Wave Motion. Before we leave these water waves let us derive a formula which is of fundamental importance in all forms of wave motion. In water waves or in simple harmonic wave motion, each particle in the medium revolves or vibrates with a period T and frequency n ($= 1/T$); the wave moves with a velocity v, and there is a wave-length λ. A moment's consideration will show the relation existing between n and v and λ. Imagine yourself bathing in the ocean; the waves are (say) 60 feet apart, and are approaching with a speed of 600 feet a minute; how many waves will pass in a minute? Obviously 10; this is the frequency of oscillation. The frequency is the velocity divided by the wave-length, or as we shall remember the formula

$$v = n\lambda. \tag{2}$$

This is the fundamental formula of wave motion, and the student should memorize it by ear as well as by eye: "Vee equals en lambda."

Example. If one end of a long steel wire is struck 100 times a sec., what will be the distance between the compressions propagated down the wire? Here $n = 100$, $v = 5000$ meters per sec.; from eq. (2) the interval (λ) is 50 meters.

303. Sound. Sound is produced by the vibrating string of a violin, by a vibrating tuning fork, by the vibrating membrane of a drum, and by the irregular vibrations set up in the table top when a stone falls upon it. The sound is usually transmitted to the ear by the air. The vibrating source sets up a train of compressions and rarefactions in the air; these travel rapidly outward in all directions, and, reaching the ear, they set the ear

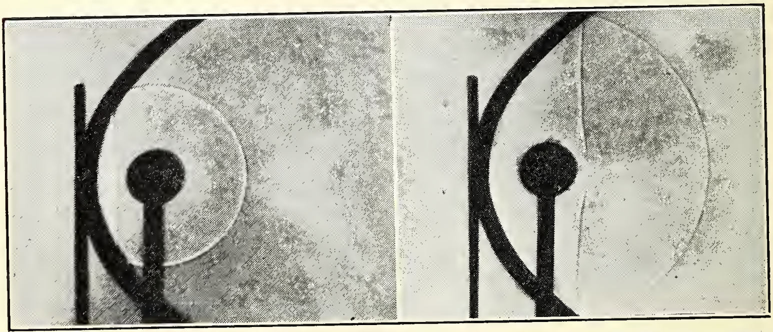


Fig. 6. Successive views of sound wave (from spark) reflected by parabolic mirror. The source is at the " principal focus " of the parabola, and a plane wave is reflected.

membrane into oscillation and we hear sound. The sound wave is longitudinal.

Air is usually but not always the medium which carries the sound. The Indian placed his ear to the ground to hear the gallop of distant horses. . . The hammer sounds carried through the house by water pipes are sometimes annoying. . . Deaf persons can sometimes hear sound by the vibrations communicated to the skull by a stick held in the teeth. . . Most children have made a " string telephone " in which the vibrations are carried along a stretched string. . . Boys who have done much swimming remember with what distinctness certain sounds are heard under water.

Sound waves are longitudinal — pressures and rarefactions. For their propagation there must be some medium which resists compression. Sound will not travel in a vacuum. A common experiment is to ring a bell in a glass jar, and gradually pump out the air; when a fair vacuum is attained the sound is no longer heard. (This experiment was first performed by Boyle.)

Sound waves are propagated outward in all directions from a source in free air. The wave fronts are enlarging spherical surfaces. Usually there is a succession of such wave fronts; in Fig. 6 the sound was the sharp report of an electric discharge which gave a single pulse, appearing in the photograph like a rapidly growing soap bubble.

304. Velocity of Sound. At 0° C the velocity of sound in air is 331 meters per second. For each degree of temperature rise it increases by 60 centimeters per second. At 20° the speed is 343 meters per second, or a little more than 1100 feet per second.

TABLE 29

VELOCITY OF SOUND AT 20° C

Air	343 meters/sec.	Iron	5000 meters/sec.
Hydrogen	1330 meters/sec.	Brass	3500 meters/sec.
Water	1450 meters/sec.	Quartz	6000 meters/sec.

Thus an echo reaches us from a cliff 550 feet away about 1 second after the original sound; the sound of thunder takes about 5 seconds to go a mile. The velocity of sound in other media is given in Table 29.

A not uncommon experience illustrates the difference of sound velocity in air and metal. If a person stands by a long iron rail which is being hammered at the other end, for every blow of the hammer he hears two distinct clangs — the first reaching him through the rail, the second through the air.

When the medium is a solid or liquid the velocity of propagation is given by eq. (1). In a gas at constant temperature the pressure plays the role of bulk elasticity, and one might expect (as did Newton) that the velocity of sound in a gas would be given by $v = \sqrt{P/d}$; but this gives too small a value. The compressions and expansions in the sound wave occur, not at constant temperature, but under adiabatic conditions (§243). Heating and cooling accompany the compressions and rarefactions, and this increases the " stiffness " of the gas. The elasticity coefficient is changed to $\mathbf{r}P$ where $\mathbf{r}$ ($= 1.41$ for air) is the ratio of specific heats at constant pressure and constant volume (§243). The student can show that $v = \sqrt{\dfrac{1.41P}{d}}$ agrees (within experimental error) with the speed given for sound in air. Since, at constant density, P is proportional to T (absolute temperature), $v \propto \sqrt{T}$; this gives the increase of 60 centimeters per second per degree mentioned above.[1]

305. Reflection of Sound. When the pulse in Fig. 1 reaches the end of the coiled spring it is reflected and a pulse travels backward along the spring. Or if this spring is fastened to another spring of different stiffness, the pulse as it reaches the junction is partially reflected; it is partially transmitted as a lesser pulse continuing along the second spring. Similarly a sound wave is partially reflected, partially transmitted at the junction between two media. The rumble of thunder is due to partial reflections of the sharp lightning crash among the clouds, from air masses of different densities.

Because of the great difference in density the reflection is very great and the transmission very small between a gas and a solid or liquid. Sound in the air is almost completely reflected at a solid wall or at a water surface. When fishing we are warned not to frighten the fish by conversation; actually only about a twentieth of one per cent of the sound energy in the air passes into the water; 99.95 per cent is reflected. Similarly for an underwater sound only this same small proportion is communicated to the air.

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[1] A 1° rise in temperature increases P by 1 part in 273 and increases $\sqrt{P}$ by 1 part in 546 (Appendix 2 [6]). $\frac{1}{546} \times 331 = 0.6$ meter.

A flat cliff gives sharp echoes; when we fire a gun the sound wave leaves as an enlarging spherical wave front, is reflected undistorted from the cliff, and reaches our ears as a sharp pulse. From a broken hillside the echo returns as a rumble, owing to reflection at different distances; the wave front is broken up, like a water wave reflected from a rocky shoreline.

Knowing the velocity of sound one can, by timing the echo, determine the distance to the distant cliff. This echo method of measurement is employed in ocean sounding. Sound is communicated to the water, and the echo returning from the ocean bottom is received by a sensitive " hydrophone." The velocity of sound in water being known, the depth can at once be obtained from the echo interval. Echo sounding can be carried on without stopping the vessel and is practicable at all depths beyond about 3 fathoms (6 yards). It has been of great use in charting the ocean bed; at thousands of fathoms the measurement is made in several seconds instead of several hours as with the older method of " passing out the lead line." . . A very similar method is used in oil prospecting. An explosive is buried in the ground and fired; the prospectors observe by means of sensitive recording instruments (seismographs) the time required for the explosion wave to return from the buried " salt dome " where oil is likely to be found trapped. Owing to the varied composition and stratification of the soil, the results are somewhat confused by secondary reflections, but this seismographic exploration has come to have great importance in the location of oil.

306. Refraction of Sound. When different parts of a wave front travel with different speeds the wave front is tipped and the direction of propagation changed. Such refraction (bending) of sound occurs when there is a temperature variation in the atmosphere. In the daytime the air near the ground is usually hottest and the velocity of sound decreases somewhat with elevation. In consequence advancing sound waves tend to be tipped backward (Fig. 7) and the sound ray to be deflected upward, away from the listener on the ground. At night, when the ground is colder, the velocity increases with elevation, the wave fronts are tipped forward, and the sound rays are refracted downward. The intensity at the ground level is then increased. On a lake at night, sounds may be heard at surprisingly great distances.

Often the wind is a more important cause of refraction. Why does the voice carry so much better with the wind than against it? Surely not because the sound cannot make its way against the wind, for the wind velocity is quite small in comparison with the velocity of sound. But wind velocity usually increases with the elevation from the ground. In an opposing wind the sound is retarded, somewhat at the ground but more and more at greater heights. The wave fronts are tipped backward exactly as in Fig. 7 (left), and the sound passes over the listener's head. On a lake a distant sound against the wind may be inaudible at the shoreline but heard quite plainly from a cliff.

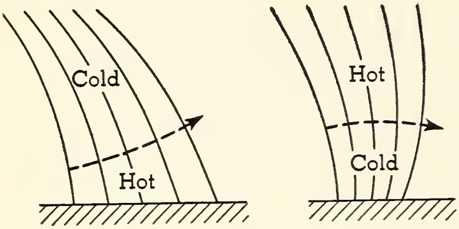

FIG. 7. Refraction of sound waves.

In the atmosphere, particularly in daytime, there may be local irregularities — local temperature variations and air currents — which refract and scatter sound, sometimes toward the listener, sometimes away. The sound from a distant airplane may suffer quite sudden changes in intensity. And after some great explosions observers have reported hearing the sound, focused by some peculiar atmospheric conditions, hundreds of miles away although it was unheard at much nearer points.

307.* Energy in Wave Motion. Progressive wave motion owes its importance in physics to the fact that it is one of two possible methods by which energy may be transferred between two distant bodies. If I wish to attract the attention of a friend some distance away I can do so by bombarding him with waves — light waves, sound waves, or waves sent to him along a rope; or I can bombard him with particles — I can throw stones at him. Energy is propagated from one body to another either as wave motion or as thrown particles.

We must ask, then, how much energy is transported per second by the wave motion. This total outflow of energy per second from the source is called the energy *flux;* the flux per unit area, the energy propagated per second through a unit area of surface, is called the *intensity* of the wave motion. The intensity is jointly proportional to four factors: (1) the density of the medium, (2) the square of the amplitude of oscillation, (3) the square of the frequency of oscillation, and (4) the velocity of propagation.

Consider the sound wave. Any small volume of the medium moves back and forth in simple harmonic motion. Its average velocity of oscillation ($v_{\text{osc.}}$) is evidently proportional to the amplitude (A) and the frequency (n). The average kinetic energy of unit volume is $\frac{1}{2}\,dv_{\text{osc.}}^2$; there is an equal amount of potential energy; hence what we may call the "energy density" in the medium is proportional to d and A^2 and n^2. Then the amount of energy propagated through unit *area* equals this energy density times the velocity of propagation:

$$\text{Intensity} \propto d \cdot A^2 \cdot n^2 \cdot v_{\text{prop.}}$$

We are not here attempting to give a rigorous derivation; we wish only to emphasize the following important fact: The intensity is proportional to the *square* of the amplitude and to the *square* of the frequency.

Consider two sounds one octave apart; the frequencies are in the ratio of 2 : 1. If the two waves are of equal amplitude the higher one has four times greater intensity. In recent work with supersonic frequencies (frequencies too high to be detected by the ear) enormous intensities have been obtained.

QUESTIONS

1. Contrast longitudinal, transverse, and rotary waves, giving examples of each.

2. Upon what factors does the velocity of wave propagation depend? Given two springs, one twice as heavy (per unit length) and 8 times as stiff as the other; compare the speeds of wave propagation.

3. What are the proper foot-pound-second units for use in eq. (1′)? Is it permissible to use meters per second (instead of centimeters per second) for v in eq. (2)?

4. Give examples of reflection, refraction, diffraction, interference.

5. Why is the wave front of the wave in Fig. 6 flattened after reflection? Compare the total distance of travel from the source of various points of the wave front.

6. The speed of sound depends on the temperature of the air but not on the atmospheric pressure. Why?

7. For longitudinal waves in solids the velocity of propagations is: (*a*) greatest for long wave-lengths; (*b*) greatest for short wave-lengths; (*c*) the same for all wave-lengths.

8. It is difficult to hear a person, some distance away, shouting against the wind primarily because the sound waves: (*a*) are refracted upwards; (*b*) are bent down into the ground; (*c*) are retarded by the wind.

Vocabulary: Longitudinal (transverse) wave, progressive (standing) wave, wave-length, wave front, ray, divergence, reflection, refraction, diffraction, constructive (destructive) interference.

PROBLEMS

1. Brass is 7000 times heavier than air and 700,000 less compressible. Compare the speeds of wave propagation.

2. Oxygen is 16 times heavier than hydrogen. Compare the speed of sound waves in these gases.

3. Find the velocity of transverse wave propagation along a string weighing 2 grams per meter and stretched with a force of 1 kilogram.

4. A 100-lb. weight hangs from a 50-ft. length of rope which weighs 25 lb. The rope is struck sharply near its lower end; a kink travels to the top and is reflected. Find the time interval between the blow and the return of the kink. (Neglect the increased tension in the rope due to its own weight.)

5. From Table 5 obtain the value of Young's Modulus for soft rubber (*a*) in kilograms per square centimeter and (*b*) in dynes per square centimeter. (Consider $g = 1000$ cm per sec.^{2}, approximately.) Find the velocity of propagation of a longitudinal pulse along a soft rubber rod (specific gravity 1.0).

6. A tuning fork applies 1000 impulses per sec. to the end of a long brass wire. How far apart are the compressions?

7. What is the frequency of a meter wave in air? in hydrogen?

8. The tone "middle C" has a frequency of 256 oscillations per sec. Find the wave-length in air and in water.

9. A steam whistle gives a frequency of 100 per sec. Compare the wave-length as the sound travels with and against a 20-mile-per-hour wind.

10. What is the period between waves for ocean rollers of 400-ft. wave-length in deep water?

11. Find the velocity of sound in air at 100° C. (Use proportion; the "60 cm per sec. change" rule does not hold for so large a temperature interval.)

12. Compute the velocity of sound in air at 0° C from its pressure and density.

CHAPTER 26

STANDING WAVES
CHARACTERISTICS OF SOUND

308. Standing Waves. The discussion in Chapter 25 dealt with progressive waves. Our water waves progressed away from their source, and we conceived of the waves in the stretched rope as progressing indefinitely far. Actually when the wave reaches the end of the rope it will be reflected, and it is the effect of this reflection which we now discuss. The simplest example of a *standing wave* is shown by a rope tied at one end and shaken back and forth in a single segment.[1] At first glance this motion is very different from the progressive wave. Compare the standing waves in Fig. 1*a* with the transverse progressive waves in Fig. 25–2. Here there is no forward motion of the wave. In progressive wave motion every point in the rope had the same amplitude of vibration but they differed in phase; here every point

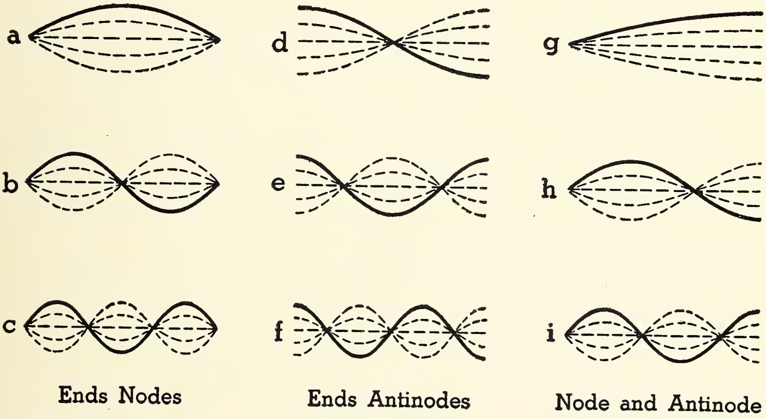

FIG. 1. Standing waves. The waves shown are transverse but they may also be regarded as *representing* longitudinal waves.

[1] Compare a child "turning rope"; this is a transverse *circular* standing wave.

along the rope is in the same phase of vibration but they differ in amplitude. The end of the rope is at rest (node). If we shake the rope more rapidly we can get shorter standing waves with two or more segments (Fig. 1*b* or *c*). A *node* is a point of no motion; an *antinode* is a point of maximum motion. There is a node between each two segments.

Standing waves like these occur in a stretched string, for example in stringed musical instruments where the high-frequency vibration of a tightly stretched string is the source of sound.

A wave-length in the standing wave includes one complete hill and valley — two segments. The mode of vibration of lowest frequency, longest wave-length (Fig. 1*a*), is called the *fundamental*; the other modes of higher frequency are called harmonics or overtones.[1] It is seen from the figure that in the first overtone (*b*) the rope length is just one wave-length; in the next overtone (*c*) it is $1\frac{1}{2}$ wave-lengths; in the fundamental the rope length is a half wave-length.

The rope ends are not necessarily nodes. Under certain conditions (free ends) they may be antinodes (*d*, *e*, *f*). If a rope hangs dangling from a rigid support, the upper, fastened end must be a node, the free end an antinode (compare *g*, *h*, *i*).

A standing wave is not in itself a means of propagation of energy; it is rather to be thought of as a vibration of an extended medium, more complicated than the simple harmonic motion of a single particle because of the differences in amplitude — nodes and antinodes. (A stick swinging as a compound pendulum can be thought of as an example of a [non-sinusoidal] standing wave — quarter wave-length.) Nevertheless, it is possible, and very useful, to conceive of the standing wave as produced by two progressive waves traveling in opposite directions. (Obviously the energy transfer of the two waves cancels.) We shall now show how two such progressive waves produce by their interference the nodes and loops and the characteristic behavior of the standing wave.

309. Interference of Direct and Reflected Wave. In Fig. 2 we must visualize a progressive wave (shown dashed) as moving to the right. When the wave reaches the end of the rope (which is supposed to be fixed) it is reflected with a change of phase. That is, at the moment a crest reaches this end a trough starts back. This is necessary, for at this point, which is fastened, there can never be motion and must always be destructive interference. The figure shows the two oppositely moving waves at successive moments of time. In the second figure the primary wave has moved to the right, the reflected wave to the left until they coincide, giving a large resultant crest and trough (constructive interference). In the fourth figure the two waves are exactly out of phase, giving momentarily a straight string (destructive interference). At certain points (marked by barely perceptible dots) the two waves always meet in opposite phase (nodes); at points midway between the nodes the phases are alike (antinodes). The heavy line shows the resultant standing wave; its amplitude is great at the antinodes, zero at the nodes.

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[1] An *overtone* is a mode of vibration above the fundamental in frequency. In its original sense it applies to sound but is often used for other vibrations. A *harmonic* is a vibration with frequency an exact multiple of the fundamental. For the string (and other simple cases) the overtones are harmonics.

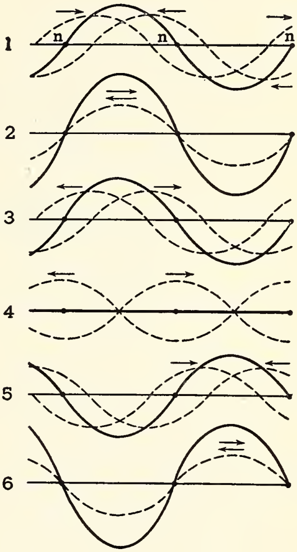

FIG. 2

There is likewise reflection at the end of a rope when it is free (the dangling rope). But the reflected wave now leaves the rope end *in* phase with the direct wave. At the loose end, crest is reflected as crest. To represent this we should simply have to invert the reflected wave curve in Fig. 2. We then find that the nodes and antinodes have interchanged. The end of the rope is now a point of maximum motion. (See Fig. 1*d–i*.)

310. Longitudinal Standing Waves. Let us now consider the longitudinal vibration of a long spring or a metal rod. If one strikes against the end of a metal rod or rubs it with a rosined cloth, the rod is set into longitudinal vibration, the ends oscillating out and back, the center at rest. This is a longitudinal standing wave, produced again by two progressive waves traveling in opposite directions. It corresponds to the transverse standing wave produced in a rope free to move at each end. The direct and reflected waves are out of phase at the center producing a node.[1]

When the two ends of the rod move in opposite phases in this way, with only a single node between them, the length of the rod is half a wave-length. This is the fundamental vibration. The rod may also vibrate in several segments. It may have twice the frequency, half the wave-length, the two ends moving to right or left together, with two nodes between. Or there may be three, four, or more nodes — the only condition is that *each end must be an antinode.*

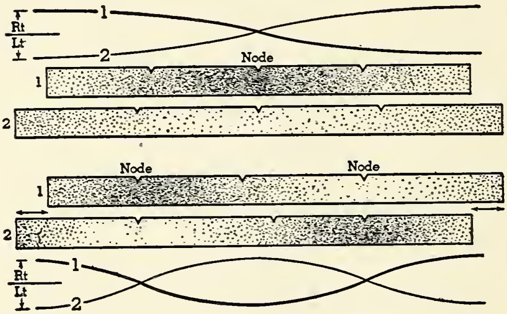

FIG. 3. Longitudinal standing wave in rod: fundamental and first harmonic. (Rod notched to show displacement.)

These waves cannot be depicted as easily as transverse waves; it is often convenient to represent them by plotting the longitudinal displacements transversely in a graph (Fig. 3). The student should simulate the motion of the ends of the rod by motion of his two hands.

Example. A steel rod 1 meter long vibrates 2500 times a sec. (fundamental). Find the velocity of propagation.

$$v = n\lambda;\ \lambda = 2 \text{ meters};\ n = 2500.\quad v = 5000 \text{ meters per sec.}$$

311. Examples of Standing Waves. Incoming water waves striking a steep cliff are reflected back, and fairly well-defined standing waves are formed. In the "choppy water" there are nodes and antinodes as the water simply flops up and down without progressive motion. . . When a rectangular plate of metal is struck or rubbed with a bow the waves reflected from the edges set up a system of standing waves, a more or less complicated pattern of nodes and antinodes. The student should see the interesting vibration patterns which can be obtained; they can be shown by sprinkling sand on the plate; the sand dances away from the points of vibration and settles down at the nodes. Similar vibrations occur in the stretched membrane of a drum. . . A tuning fork is simply a long rod bent and held at the center. Its vibration is merely a standing transverse wave. . . The vibrating string of a violin or piano is an example of a standing wave with the fundamental and overtones of a string tied at each end — with nodes at the ends.

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[1] But though there is no motion here there are changes in condensation; as the ends of the rod move in there is a compression here, as they move out a rarefaction. In a standing longitudinal wave the greatest compressions and rarefactions always occur at points of least motion. Compressions and rarefactions are indicated by the shading in Fig. 3.

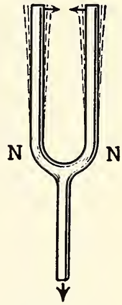

FIG. 4

Standing waves may also be excited in a column of air; we now describe two laboratory experiments which show the nature of these waves.

312. Kundt's-Tube Experiment. When the rod in Fig. 5 vibrates longitudinally it sets the air in the tube into vibration. Waves of compression and rarefaction travel the length of the tube and are reflected; standing waves are set up in the air column. The motion of the air is shown by the dancing of a little cork dust which has been placed in the tube. Every few inches along the tube the dust is strongly agitated; here the air is rushing back and forth. Between each pair of these antinodes lies a node where air and cork dust are at rest.

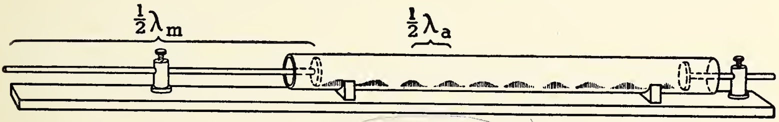

FIG. 5. Kundt's tube.

This is "Kundt's tube." It shows vividly the motion of standing waves in air. It also shows the greatly different velocities of the waves in air and metal. The distance between successive dust piles (i.e., successive antinodes) is a half wave-length; it has been shown above that the half wave-length in the rod is equal to the length of the rod (fundamental). By eq. (2) (§302), for a given frequency the wave-length is proportional to the velocity; the wave travels the length of the rod and the short distance between dust piles in the same time. For a brass rod the distance between piles is about $\frac{1}{10}$ of the rod length; for an iron rod about $\frac{1}{15}$. Compare the velocity of propagation in air, brass, and iron. (Compare your result with Table 29.)

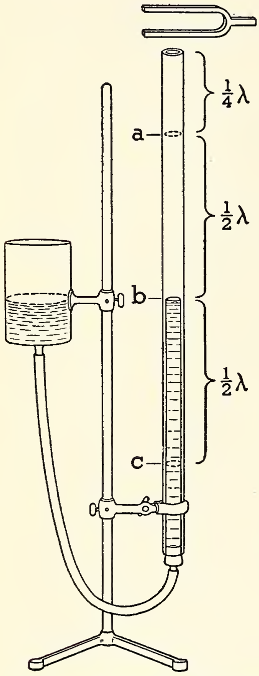

FIG. 6. Experiment of resonant air column.

313. Resonant-Tube Experiment. Figure 6 illustrates a simple method of measuring the velocity of sound in the laboratory. A tuning fork of known vibration period (T) vibrates at the mouth of a glass tube partly filled with water. As the water level is changed we find certain points (a, b, c) at which there is a marked enhancement of the sound. This is due to constructive interference (§301). As the fork vibrates to and fro before the mouth of the tube it sends an air compression (followed by a rarefaction) first downward into the tube, then upward into the air. The waves travel down the air column (length L) and are reflected upward. If the water height is properly adjusted (at a) the compression (or rarefaction) returns to the tube end after a half-period and interferes there constructively with the compression (or rarefaction), which is at this moment starting upward into the room, and enhances the sound. Hence we have

$$2\,L_a = v \times \frac{1}{2}T \quad \text{or} \quad v = \frac{4\,L_a}{T}.$$

There will again be constructive interference when the wave returns (from b, c, etc.), after an interval of $1\frac{1}{2}\,T$, $2\frac{1}{2}\,T$, etc. So with the level at b we have the relation

$$2\,L_b = v \times \frac{3}{2}T \quad \text{or} \quad v = \frac{4}{3}\,\frac{L_b}{T}.$$

314. Standing Waves in Tubes. This explanation is very simple — the length of the tube must be just such that the "echo" returns to the top of the tube in exactly $\frac{1}{2}$ or $\frac{3}{2}$ or $\frac{5}{2}$, etc. period, to produce constructive interference with the fork in its opposite swing. Perhaps this does not go quite to the heart of the matter, however, and we should consider it from a slightly different point of view. The interference of the direct and reflected waves in this experiment produces a standing wave with a node at the water surface, an antinode at the upper, open end. The analogous transverse waves are those for the rope with one end loose (Fig. 1 *g*, *h*, *i*).

More exactly the vibration should be compared to that of a spring with one end fixed. The spring may vibrate in one, two, or more segments. In Fig. 7 (upper), the tube has been inverted, closed end at top. This closed end of the tube must be a node; the bottom, open end, an antinode. Hence, for the fundamental resonance, the length of the tube is a quarter wave-length. (In Fig. 6 this corresponds to the water level at *a*.) There may, however, be one, two, or more nodes in the tube. The vibrations in the first two "overtones" are shown in the figure; the wave-lengths are respectively one-third and one-fifth as long as the fundamental. The student will observe that the possible waves in this "closed pipe" are given by

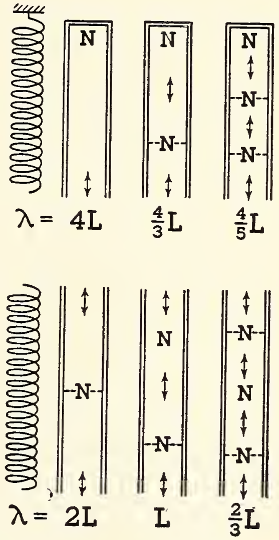

FIG. 7. Nodes and antinodes in organ pipes.

$$\lambda = 4L \text{ (fundamental)};\ \tfrac{4}{3}L;\ \tfrac{4}{5}L;\ \tfrac{4}{7}L;\ \text{etc. (overtones)}.$$

We get standing waves also in a tube open at both ends ("open pipe"). Each end is an antinode. This is like the longitudinal wave in a rod (§310), analogous to the transverse waves in Fig. 1 *d*, *e*, *f*. For the fundamental, the tube measures the distance between consecutive antinodes (out of phase). Hence $\lambda = 2L$ (fundamental). For the overtones, $\lambda = L, \frac{2}{3}L, \frac{1}{2}L$, etc. To show the relation with the values found for the closed pipe the wave-lengths can be written.

$$\lambda = \tfrac{4}{2}L \text{ (fundamental); } \tfrac{4}{4}L,\ \tfrac{4}{6}L,\ \tfrac{4}{8}L, \text{ etc.}$$

315. Organ Pipes. Horns, flutes, organs, and other wind instruments make practical use of standing waves in pipes. We have an impromptu organ pipe when we blow across the lip of a bottle; a toy whistle is an organ pipe on a small scale. The organ pipe is excited by an air jet directed at a sharp lip; as this jet trembles back and forth across the lip alternating air pulses are sent down the pipe.

The lip may be regarded as an open end; the other end may be either closed or open (Fig. 7). Both closed and open organ pipes are used. The discussion above brings out the essential differences between the two:

1. For a certain fundamental pitch the closed pipe is half as long as the open pipe. The wave-length in air of middle C is about 4 feet (Table 30). Thus for middle C:

$$\text{Open pipe length} = \tfrac{1}{2}\lambda = 2 \text{ ft. (approximately).}$$
$$\text{Closed pipe length} = \tfrac{1}{4}\lambda = 1 \text{ ft. (approximately).}$$

2. The possible overtones are different. Open pipe: Overtone wave-lengths = $\tfrac{1}{2}$, $\tfrac{1}{3}$, $\tfrac{1}{4}$, $\tfrac{1}{5}$, etc., times fundamental; all harmonics. Closed pipe: Overtone wave-lengths = $\tfrac{1}{3}$, $\tfrac{1}{5}$, $\tfrac{1}{7}$, etc., times fundamental; alternate harmonics with frequencies 3, 5, 7, etc., times the fundamental. The absence of the even-multiple harmonics makes the quality of the sound of the closed pipe somewhat less rich.

Sounds usually originate in standing waves — in an organ pipe, a violin string, the vocal cords, or a vibrating board or metal plate.

PROBLEMS

1. I shake a 12-ft. rope; one end is tied, and the hand can be regarded as practically at rest. What are the wave-lengths of the three longest waves which I can produce?

2. A 12-ft. spring hangs from the ceiling. What is the wave-length of its fundamental mode of longitudinal vibration? Assuming that a pulse is propagated along the spring 24 ft. per sec., what is the period of vibration?

3. What are the wave-length and the period of the first overtone of the spring in problem 2?

4. If the second resonant point *b* in Fig. 6 is 30 cm from the end of the tube, what is the wave-length? What is the frequency of the fork?

5. An open organ pipe is 1 meter long. What is the wave-length and frequency of (*a*) the fundamental and (*b*) the first overtone?

6. A 6-in. test tube is used as a whistle. Find the wave-length and frequency of (*a*) the fundamental and (*b*) the first overtone.

7. Find the change in the fundamental frequency of a 1-meter open organ pipe when the temperature rises from 20° to 30° C.

8. What is the fundamental frequency of a closed organ pipe 1 ft. long, filled with hydrogen gas?

9. A brass rod, used in the Kundt's-tube experiment, is 1 meter long. What is the frequency of vibration? What is the distance between dust piles in the tube?

10. The vibrations of short quartz rods are used to produce very high-frequency (supersonic) vibrations. What is the fundamental frequency of a quartz rod 1 in. long? (Still shorter rods, or rather thin plates, are used to control radio frequencies.)

CHARACTERISTICS OF SOUND

316. Loudness, Quality, and Pitch. Sounds differ one from the other in loudness, in quality, and in pitch.

Loudness. The loudness of a sound wave depends on its amplitude and frequency. The effect of the amplitude is rather obvious; the greater the amplitude the greater the compressions and rarefactions and the greater the impulses upon the eardrum. The relation between loudness and frequency is more complicated. We have shown (§307) that the energy of wave motion (intensity) increases with frequency, but the loudness depends also upon a psychological factor — the sensitiveness of the ear for different frequencies (Chapter 27).

Quality (*or Timbre*). The tones of a piano and of a violin differ in quality; and both differ in quality from the vowel sound $\bar{e}$ or $\overline{oo}$, and these in turn differ from the explosive sound of the consonant *b* or the hissing sound of an *s*. Such differences in quality make conversation possible. The quality of a sound depends upon the *shape* of the wave. In pure *noise* the condensations and rarefactions follow one another at random with no semblance of a period or wave-length. In a simple *musical tone* there is a well-marked periodicity (fundamental wave-length) although there may still be complex overtones. In the pure tone the vibration is a single simple harmonic frequency and the wave has a simple sinusoidal form.

Pitch. In the siren, air is blown as a rapid succession of puffs through the holes in a spinning disc. As the disc spins faster and faster and the frequency of the puffs increases, the siren note rises in pitch. This shows in very direct manner that the pitch of a sound depends upon the frequency of the vibration.

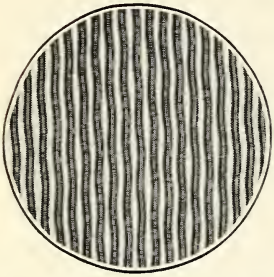

FIG. 8. Magnified portion of phonograph record.

In general, pitch is determined by the frequency of the fundamental. The pitch is very pronounced for most musical sounds. Tapping a drum or rapping on a table gives a less pronounced pitch because of the presence of overtones and random oscillations; pure noise with purely random oscillation has no pitch.

317. Frequency and Wave-Lengths of Sounds. When the frequencies of two notes are in the ratio of 2 : 1 their interval is called an octave. The frequency of middle C (as commonly adopted in physical laboratories) is 256 vibrations per second. The range of human voices is from about two octaves below this (64 vibrations per second) for a low bass to about two octaves above (1024 vibrations per second) for a soprano. The range of audibility is between about 20 and 20,000 vibrations per second.

TABLE 30

CHARACTERISTIC FREQUENCIES AND WAVE-LENGTHS (approximate)

Lowest organ.........	16	64 ft.	2 octaves
Low bass............	64	16 ft.	2 "
Middle C............	256	4 ft.	2 "
High soprano.........	1024	1 ft.	2 "
Highest note on piano..	4096	3 in.	2 "
Highest audible.......	16000	$\frac{3}{4}$ in.	

What are the wave-lengths of typical sounds? Taking the velocity in air as 1100 feet per second, the wave-length for any frequency can be found from our fundamental eq. (2):

$$\lambda = \frac{1100}{n}.$$

For middle C $$\lambda = \frac{1100}{256} = \text{approximately } 4 \text{ ft.}$$

Several other characteristic frequencies and wave-lengths in air are given (in round numbers) in Table 30.

318. Consonant Intervals. When the frequencies of two sounds stand in simple ratio the ear finds the combination particularly pleasing. The frequency ratio (usually expressed as an improper fraction) is a measure of the musical interval.

The simpler intervals occur in the notes of the major triad. These are the notes of the bugle: *do, mi, sol, do*. Their frequencies stand in the ratio of 4 : 5 : 6 : 8. Among these notes occur all the simplest intervals: $\frac{2}{1}$; $\frac{3}{2}$; $\frac{4}{3}$; $\frac{5}{4}$. These are the frequencies of the simpler chords; these notes to a great extent give us our simple melodies; melodies usually start and almost invariably end on *do*. Why these simpler intervals are especially pleasing to the ear is not a question for physics but for psychology or esthetics.

The student, even if not especially interested in music, should remember so much at least of the frequency relations in the scale.

TABLE 31

MAJOR DIATONIC SCALE

		9/8	10/9	16/15	9/8	10/9	9/8	16/15
Tonic	do	re	mi	fa	sol	la	ti	do
C_3	256	288	320	341.3	384	426.7	480	512
G_{-1}	24	27	30	32	36	40	45	48
	←5/4→		←6/5→		←4/3→			

319. Major Diatonic Scale. Presumably everyone is familiar with the musical scale: *do, re, mi,* etc. The octave (interval $\frac{2}{1}$) is divided into seven smaller intervals. The frequencies of the successive notes are proportional to the whole numbers 24, 27, 30, etc., given in Table 31. Observe that the first interval (*do-re*) is $\frac{9}{8}$; the next is $\frac{10}{9}$; the next $\frac{16}{15}$. Since there are five larger intervals ($\frac{10}{9}$ or $\frac{9}{8}$) and two smaller ones ($\frac{16}{15}$) we can describe the scale approximately (or for the tempered scale, §326, exactly) as consisting of five whole and two half intervals in proper sequence.

320. Absolute Pitch. The diatonic scale implies no definite pitch. This is introduced with the lettered notes. For some reason hidden in antiquity the natural scale begins with C. Because of the convenience of having it a power of 2, a frequency of 256 has been adopted by scientists for their middle C. Musical instruments are pitched slightly higher; the standard " international pitch " gives middle A a frequency of 440 (instead of 426.7 as in Table 31) and middle C a frequency of 261.6.

The successive octaves are often designated by subscripts: C_0, C_1, C_2, etc., beginning with the third octave below middle C. Middle C is then C_3. (This nomenclature is not universally adopted.)

Example. What are the fundamental pitch (" pedal tone ") and first two overtones of a French horn (open pipe) 17 ft. long? By §314, for open pipe

$$\lambda_{\text{fund.}} = 34 \text{ ft.}$$

$$n = \frac{v}{\lambda} = \frac{1100}{34} = 32 \text{ vibrations per sec.}$$

This is C_0, three octaves below middle C. This pedal tone is not actually played. The overtones (open pipe) have double and triple this frequency, hence are C_1 and G_1 (two octaves below *sol* of Table 31).

321. Quality. " Quality depends on the shape of the wave." Seldom does a sounding body vibrate in exactly simple harmonic motion. When one strikes a tuning fork, along with the fundamental tone comes a high-pitched clang. When we strike a metal plate or a drum head the vibration is very complex indeed. The sound wave does not have the simple sinusoidal form. But, whatever its shape, any wave can be considered as a superposition of a fundamental simple (sinusoidal) wave and others (harmonics or overtones) with wave-lengths one-half, one-third, one-fourth, etc.,[1] as long. Figure 9 represents a rather simple wave composed of a fundamental and two harmonics.

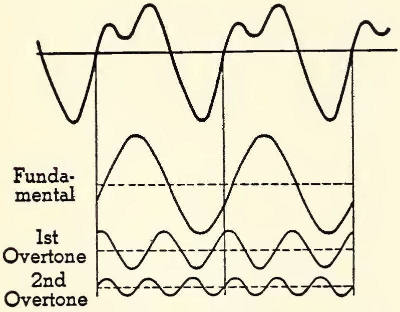

FIG. 9. Analysis of sound from open organ pipe.

A great deal of study has been given to sound analysis in recent years. The first and largest problem is to get a correct representation of the wave shape; the oscillograph (§579) receives the rapidly fluctuating pressure changes and, with as little distortion as possible, depicts this as a transverse wave form. With the aid of special analyzing machines the physicist or engineer or psychologist breaks up this complex wave into its components of fundamental and overtones. Less exactly, the vibration frequencies present in a sound wave can be detected by listening to the resonance in selected organ pipes or " resonators " of proper size. The pipe picks out from the wave the oscillation of its own natural frequency and emphasizes it. This explains the singing of the seashell; out of the general noise which is usually about us the shell selects certain frequencies and brings them to our ear. The song of the seashell appeals to our poetic fancy, but we can hear this same singing at the end of a tube or bottle.

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[1] That is, with double, triple, quadruple, etc., the fundamental frequency.

The shape of a wave depends not only upon what harmonics are present but also upon their relative phase relations. (The wave form in Fig. 9 would look quite different if the first overtone began with a valley instead of a hill.) The ear, acting like an assemblage of resonators, analyzes sound into its harmonics without regard to phase. To this extent it is not quite correct to say (as has been done in introducing the subject) that quality depends on wave shape; more exactly, *quality depends upon the relative intensity of fundamental and the various overtones.*

322. Resonance. We have just given an example of resonance. If two neighboring tuning forks have the same frequency the sound from one will set the other into sympathetic vibration. Striking one string of a piano excites the first overtone (i.e., the same frequency) in the string an octave lower. When the frequencies are exactly the same, successive impulses of the sound wave, though each of seemingly negligible importance, continue to be applied always in exactly the right phase of vibration and ultimately produce a strong response. In the same way the child in the swing, by repeated impulses properly timed, works up his motion.

There are many examples of resonance in other fields of physics. Our radio set is excited by only a single one of the many " radio waves " in the " ether." It responds by resonance to the particular wave for which the set is " tuned "; we select our station by changing the frequency of electrical oscillations in the receiving set. . . A vibration develops in the automobile, most noticeable at some particular speed; probably some part of the body comes into resonance with the engine when it is rotating at this speed. . . It has been shown how the gyroscopic stabilizer (§186) by its properly timed reactive torques, always exactly *out* of phase with the motion, damps out the roll of a vessel. These "anti-resonant" impulses compensate for the much greater impulses of the waves applied at random intervals.

323. Interference. The prongs of a tuning fork move in opposite directions; if the vibrating fork is held a short distance from the ear and slowly rotated, one hears a marked variation in intensity. When one prong is hidden behind the other the sound is loud; at an angle where the opposite waves from both prongs reach the ear there is comparative silence. This is an easily exhibited example of destructive interference.

If a whistle is blown at some distance from a wall, and we explore about in the neighborhood with the ear, we shall find that as we leave the wall we pass through alternating zones of sound and quiet. These are the antinodes and nodes formed by interference of the direct and reflected sound, quite like the standing waves in the organ pipe.

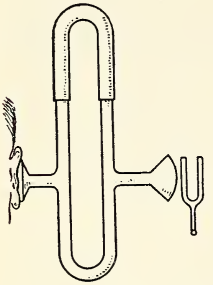

FIG. 10. Simple interference experiment.

Figure 10 illustrates another simple device for showing interference. The sound travels to the ear by two paths; if the additional length is $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, etc., wave-lengths, the two waves will interfere destructively and the ear will hear little sound; when the paths are the same length or differ by a whole number of wave-lengths there is constructive interference and a sound maximum. The difference in path length we shall call the *relative retardation.*

When one claps his hands before a long flight of steps, the echo returns as a distinct musical tone. Its wave-length is twice the step width. The reflections come from the successive steps with greater and greater retardation. Observe: the sound echoed from the second step has a retardation of twice the step width relative to that from the first; from the next step the retardation is 4 times the step width, 6 times from the next, and so on. Hence for sound whose wave-length is twice the step width the retardations are exactly one, two, three, etc., waves; all the returning waves are in phase and give constructive interference. (Such experiments cannot be performed indoors because of reflections from the walls of the room.)

324. Beats. Perhaps the most striking example of interference is the effect when two sounds of slightly different pitch are sounded together. Periodically the two sources will be in phase and out of phase, and we hear successive maxima and the minima of intensity (beats). The number of beats per second equals the difference in the frequencies; why? The phenomenon is often used to compare exactly the frequencies of sounds of nearly the same pitch.

325. Doppler Effect. When we rapidly approach a source of sound the pitch is slightly raised. This is for the same reason that a man finds the waves dashing over him more frequently when he swims into them than when he is at rest. Effectively he adds his velocity to the velocity of the waves; since λ is unchanged this means a greater frequency. Similarly when we recede from a source of sound the pitch is lowered.[1] Perhaps the student when riding in a train has heard the sudden lowering in pitch of the roadside signal bells as they passed by. The effect is almost (but not quite) the same when the sound source, rather than the observer, is moving. This change in pitch due to motion is known as the Doppler effect.

The Doppler effect is usually quite small. When the observer is moving the relative change in frequency is given by the ratio

$$\frac{\Delta n}{n} = \frac{\text{Velocity of the observer}}{\text{Velocity of sound}},$$

and this is seldom more than a few per cent. The Doppler effect will be met again in the subject of light; the small Doppler shifts in the light from the stars has given the astronomer his knowledge of their speeds of approach or recession.

QUESTIONS

1. How is sound produced in a phonograph, a telephone receiver, an automobile horn?

2. Which of the transverse standing waves shown in Fig. 1 are possible: (*a*) in a string tied at each end and plucked in the middle; (*b*) in a metal rod struck at the middle?

3. Regarding Fig. 1 as representing longitudinal waves, which of them may be excited in the rod of Fig. 5? (Note that the rod is clamped.)

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[1] If we could travel away from the source with the velocity of sound the frequency would be zero — the waves could not catch up with us. What if we traveled still faster?

4. Why cannot the sound wave represented in Fig. 9 be produced in a closed pipe?

5. If the frequencies of two tuning forks are respectively 256 and 258 vibrations per sec. how many beats will be heard per second?

6. If these forks are placed on a table several feet apart the sound maxima and minima will not be heard simultaneously in all parts of the room. Explain.

7. The greatest sound wave on record was due to the explosion of the volcano Krakatoa in 1883 and was audible as sound for thousands of miles. The atmospheric pressure wave passed completely around the earth three and one-half times converging to a focus each time at the antipodes (near Panama). Explain this by interference. How long did it take for the wave to reach the antipodes?

8. For a closed organ pipe the first overtone has a frequency: (*a*) twice, (*b*) three times, (*c*) four times, the fundamental.

9. Consider two notes with frequencies in the ratio 1 : 5; if the first note is called *do*, will the second be: (*a*) *do*; (*b*) *mi*; (*c*) *sol*?

Vocabulary: Progressive (standing) wave, intensity, quality (timbre), pitch, fundamental, overtone (harmonic), relative retardation, beats, consonant interval, major diatonic scale, absolute pitch, resonance, Doppler effect.

PROBLEMS

1. The strings of a violin are G_2, D_3, A_3, E_4; what are the frequencies?

2. Inspect a piano keyboard and determine the lowest and highest notes. (Keys are labeled in Fig. **27–1**; middle C is in almost the exact center of the board.) What are the frequencies?

3. Give the frequencies for the scale of G_2 (tonic 192); how many of these frequencies are found in the scale of C?

4. What is the fundamental frequency of an open organ pipe 67 cm long? What note of the scale does this frequency represent? What are the frequencies and notes of the first two harmonics? (Temperature 20° C.)

5. What are the fundamental and first two harmonics of a closed pipe 67 cm long?

6. How long must an open organ pipe be to have a fundamental pitch of $G_3\sharp$?

7. How long a whistle (closed) will give (as fundamental) the highest audible pitch? the lowest?

8. An iron rod 20 cm long is set into longitudinal vibration (Fig. 5). Show that the fundamental will be audible but none of the overtones.

9. Suppose the additional length around the upper tube in Fig. 10 to be 2 ft. Contrast the interference for a C_3 and a C_4 tuning fork.

10. How many beats are obtained per second between two closed organ pipes, 50 and 51 cm long, respectively?

CHAPTER 27

USE OF SOUND

326. The Tempered Scale. In musical instruments it is necessary to modify somewhat the true diatonic scale. A piano, tuned to the true diatonic scale, could play only the scale of C (*do* natural) and could not play "accidental" notes (sharps and flats). The addition of the black keys would bring in the accidentals, but still the piano could be used only for the scale of C, because the full interval between C and D in this scale is not quite the same as that between D and E. On the piano the octave is divided, not into the five full intervals ($\frac{9}{8}$, $\frac{10}{9}$) and two half intervals ($\frac{16}{15}$) of the true diatonic scale but into twelve exactly equal half-tone intervals (white and black keys).

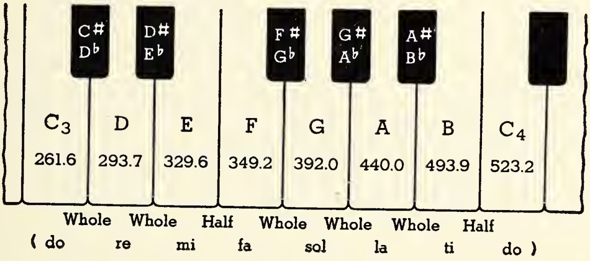

FIG. 1. Tempered scale on piano.

This is the tempered scale. It is found on any instrument with fixed notes. Below Fig. 1 the scale of C is indicated, but any other note can be used for the tonic (*do*) with the proper sequence of whole, whole, half, etc., intervals — for example, the scale of G, with one sharp.

327. Stringed Instruments. In stringed instruments the sound originates in the standing wave of a stretched string. The vibration must be communicated to the air, and the air easily eludes the vibrating string. So in every instrument — violin, banjo, piano — the string is mounted on a sounding board. The wooden shell of the violin and the heavy frame of the piano and the stretched membrane of the banjo respond differently to the various overtones. Also different means are employed to excite the string — bowing, hammering, plucking. Such factors account for the quality difference among the different stringed instruments.

The pitch of the tone depends upon the fundamental frequency of the string. This frequency in turn depends upon its length (L) and tension (T) and mass per unit length (m'). The equation for wave velocity in a stretched string was given in §297: $v = \sqrt{T/m'}$. And when a string vibrates as a single segment $\lambda = 2\,L$. And from the wave equation $n = v/\lambda$. Hence:

$$n = \frac{1}{2\,L}\sqrt{\frac{T}{m'}}. \tag{3}$$

Thus the pitch of a string can be changed by altering either its length, its mass, or the applied tension. In the piano lower strings are heavy, long, not tightly stretched; the high strings are short, light, and very taut. In fingered instruments (as the violin and guitar) different notes are obtained by varying the string lengths.

328. Wind Instruments. In fundamental principle a bugle is like an open organ pipe. The wave-length of its fundamental is twice the length of the tube, but this low tone (called the pedal tone) is not actually used. By "overblowing" the harmonics are excited, with 2, 3, 4, 5, 6, (7), 8 times the pedal frequency, giving do, sol, do, mi, sol, (la sharp), do. Omitting the undesired seventh multiple, this gives the tones of the major triad. The conical (rather than cylindrical) shape of the tube permits this large range of harmonics; flaring the end increases the sound output (corresponding to a sounding board).

The trumpet and French horn resemble the bugle, with different pitch and timbre. In other instruments the effective length of the tube can be altered: by openings in the flute and cornet, or by a sliding extension in the trombone; these instruments give the complete scale.

329. The Voice and the Ear. Undoubtedly the most remarkable sound instruments are those which nature, experimenting by the trial-and-error method through millions of years, has developed — the voice and (especially) the ear. The larynx and the vocal cords act somewhat as a valve in the throat. As we speak, the air from the lungs forces its way through the closed larynx as a rhythmic succession of pulses; when we whisper the breath streams through the open larynx. In either case the air entering the vocal cavities stirs them into oscillation much as air does an organ pipe. The tongue twisting around between lips and palate changes the shape and size of the resonating chambers to form the vowels; the breath hisses or explodes through the teeth or lips or escapes through the nose to form the consonants.[1]

Perhaps this is not such a wonderful instrument; perhaps the wonder should go to the man, who upon such a simple instrument can play so many tunes. But let me describe the ear! The visible parts of the ear are (in man) mostly ornamental. The sound waves fall upon the eardrum, and thence through a system of leverlike bones the vibration is transmitted to the liquid in the *cochlea*. This lever system has a mechanical advantage of 30 or 40, stepping up the pressure, stepping down the displacement, in just the ratio which an engineer would have prescribed for transmitting vibration from a light medium like air to a heavy one like the liquid of the cochlea.

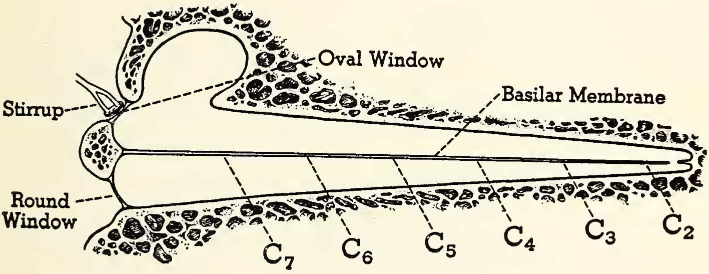

FIG. 2. Cochlea of human ear (straightened).

The cochlea (Latin: *snail*) is a small bony tube, about an inch and a half long but coiled into a spiral somewhat resembling a snail shell. (For the sake of clearness in Fig. 2 the cochlea has been represented as straightened out.) Along the middle of this tube runs a membrane, stiffened by many thousands of tiny fibres which stretch across it with different lengths and different tensions somewhat like the strings of a harp. Except for a small hole at the further end this membrane divides the cochlea into two parts. The liquid flows back and forth through this hole when a single pulse or a slow succession of pulses is applied to the "oval window." But for higher frequencies the oscillation is carried through from one chamber to the other by vibration of the flexible membrane, the maximum motion in the membrane occurring at a point which depends on the frequency. High-frequency (short-wave-length) sounds hardly penetrate the tube at all, being passed at once through the membrane. So for each pitch there is a point in the membrane of maximum motion. Along the membrane are distributed some twenty thousand hair cells, the nerve cells which on being moved give the sensation of sound. The perception of pitch depends on the localization of the excited cells along the membrane.

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[1] In speaking or singing the fundamental pitch is given by the vibration of the vocal cords, higher for the lighter cords of the woman or child, lower for the man. With open larynx the pitch cannot be changed; try to sing in a whisper.

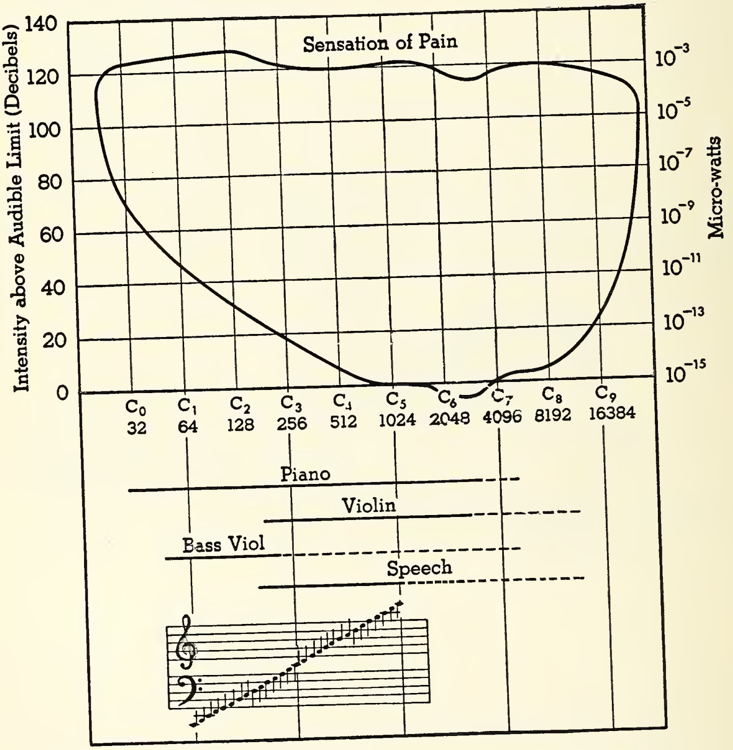

FIG. 3. Sensitivity of normal ear and range of musical instruments. (Harmonics in broken lines.)

330. Range of Ear and Voice. The ear is a very sensitive organ. It can detect sound having amplitude less than 10^{-8}

centimeter. Between the faintest sounds of rustling leaves and the deafening blast of the steam whistle it can distinguish 300 or 400 gradations in loudness. It can hear frequencies as low as 20 per second and as high as 20,000 per second and can distinguish 1000 or 2000 gradations in pitch.

The sensitivity of the ear varies greatly with pitch; it is a maximum for frequencies about 2000 per second. In the complex sounds of speech most of the energy lies in the range below 500 vibrations per second, comparatively little in the harmonics above 1000 per second. Yet because the ear is so sensitive to higher frequencies it is just these higher overtones that are most important in articulation. This is a fact of great importance in the design of acoustic apparatus.

331. Acoustics of Rooms. Only in the past few years has architectural acoustics received the consideration that it deserves. The chief problems are how to make the lecture room and auditorium suitable for speech and music, and how to reduce the noise in busy offices and public rooms.

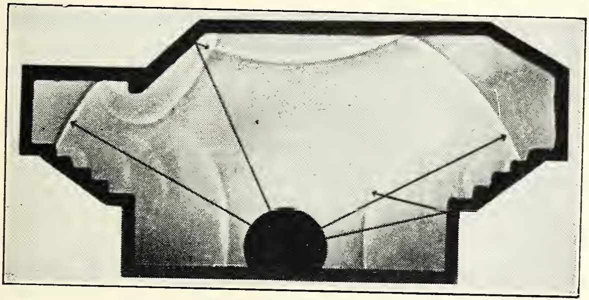

FIG. 4. Reflection of sound waves in model auditorium.

Sound waves travel out from the source and are reflected again and again from the walls of the room. At each reflection some of the sound is absorbed. But when one claps his hands in a large room with stone walls it may be a number of seconds before the intensity has been reduced below the range of audibility — a millionfold reduction. This is called the reverberation time. A certain amount of reverberation is desirable for speech, somewhat more for music. The voice does not carry well out-of-doors; it carries better in a large room with low ceiling. Reverberation adds to the intensity and for music enriches the tone. But it should not be so great that successive syllables of speech are confused. For large halls a reverberation time of 1 to 2 seconds is considered best.

Extensive investigations have recently been made of the absorbing powers of different materials. Ordinary brick and stone absorb only 1 or 2 per cent of the incident sound. In a stone vault hundreds of reflections are necessary before the sound intensity is reduced to extinction. On the other hand, an open window serves as a perfect absorber. An audience as usually seated absorbs more than 90 per cent of the sound that falls on it. There is a very real difference between speaking to an empty and a full house. As a result of these investigations, wall materials of relatively high absorbing power are now on the market.

TABLE 32

SOUND ABSORPTIVITY OF DIFFERENT MATERIALS

Brick, marble, glass......	1–3%	Light curtains........	20%
Ordinary plaster........	3%	Heavy curtains in folds.	50–100%
Acoustic plasters and tiles	20–30%	Open window.........	100%

332. Regular Reflection of Sound. All forms of wave motion tend to travel more or less in straight lines (§301). But sound waves, because of their comparatively long wave-length, are usually strongly diffracted. Nevertheless, large objects in the sound path cast acoustic shadows, and a sound beam is regularly reflected from a large plane surface and can be focused by large spherical mirrors. These effects are always most marked with high-pitched sounds. The high-pitched note of a "Galton whistle" or the hissing sound of an air jet can be reflected from a wall like light from a mirror. . . When a watch is placed before a large concave mirror the reflected light forms an image which we see a short distance in front of the mirror. The image not only looks like the watch but also ticks like the watch. The high-frequency components of the sound are reflected in the same way as the light. . . The dome of the Transportation Building of the 1933 World's Fair in Chicago was a hemispherical shell. It served as a beautiful acoustic mirror. When a pistol was fired on the floor at the exact center of curvature of the dome, the report was reflected back to the center a half second later, reflected back again — and so (while the building was still empty) as many as 40 echoes could be heard! When fired a few feet to one side of the center the echo returned a half second later at the conjugate focus a few feet to the other side and then again after another half second at the original point (compare Fig. 25–5*a*). This is the secret of whispering galleries, where conversation at one point of the room can be clearly heard focused at another.

Such focusing of echoes sometimes occurs in auditoriums and is highly objectionable. In the design of auditoriums sometimes a small model is made and the reflection of waves is studied by means of spark photography (Fig. 4).

333. Noise. Noise has been defined as " sound out of place." We live in a sea of noise. What noise do you hear now as you read this? On the average busy corner of a large American city the noise is such as to make the normal person from one-third to two-thirds deaf. Most of the noise of offices and city streets is the result of scientific inventions — typewriters, horns, whistles, streetcars — but science is also doing much to reduce the evil — noiseless typewriters, absorbing walls, mufflers, ball bearings, paved streets, rubber tires. In air transportation the passengers must be isolated from engine noise; in apartment houses the tenants wish to be acoustically insulated from their neighbors; in the broadcasting studio there must be no " sound out of place." Each of these problems can be solved, and the solution depends upon the achievements of the acoustic engineer.

334. Unit of Sound Intensity. A unit has been introduced for comparison of sound intensities. Intensity is measured in terms of the acoustic energy passing through unit area per second. When one sound has (in this sense) 10 times the intensity of another they are said to differ by 1 " bel "; a hundredfold difference is 2 bels, a thousandfold difference 3 bels, etc. In actual practice the tenfold smaller unit, decibel, is more generally used. One decibel represents (roughly) the least intensity difference which the ear can distinguish. When a person in a noisy room stops his ears with his fingers the sound intensity is reduced about 10^5-fold — a difference of 5 bels or 50 decibels.

Table 33 gives the relative intensities of familiar sounds. The range of the ear from the least perceptible sounds to those producing the sensation of pain is about 13 bels (ratio 10^{13}).

TABLE 33

Characteristic Noise Levels (in decibels)

Threshold of hearing	0	
Average residence	30	Whisper
Noisy restaurant	65	Ordinary conversation
Noisy factory	85	Lion in zoo
Boiler factory	100	Loud automobile horn
	110	Pneumatic riveter

335. The foundations of present-day acoustic engineering were laid in the last century. There were several outstanding figures: Helmholtz in Germany and Lord Rayleigh in England, both great men with interests running throughout the range of physics but centered chiefly in sound. America furnished the inventors: Edison of the phonograph and Bell of the telephone. But today, as often as not, the hero in the development is not some individual man but the research department of some corporation: a collection of trained executives, scientists, engineers — perhaps a few geniuses — each a specialist in his chosen field, with minds coordinated, working together toward a common goal like the differentiated cells in some complicated biological organism. It is only so that a telephone or radio system — or an automobile — can be perfected in a generation.

Within a few years our acoustic environment has radically changed. Not only is there a completely new bedlam of noises in the city. We do business, shop, or visit over the telephone; we dance to the phonograph more often than to the piano; we listen to the radio rather than sing; and we get our drama from the sound-film. This has happened largely because of the intense investigation of practical acoustics during the last few decades. But for the most part it has been acoustics combined with electricity, and the discussion of the more remarkable of these devices must be left to later points in this book.

QUESTIONS

1. What is the sequence of whole and half tones in the tempered scale?

2. What notes are sharped in the scale of D? (Pick out the fingering in Fig. 1 or, better, play it on a piano if available.)

3. Explain why a piano with a true diatonic scale could not give the true scale of D.

4. How is the frequency of a string affected if length, tension, and mass per unit length are all doubled?

5. When a violin string is plucked in the middle the odd harmonics are missing. Why?

6. What is the difference between a violin and a bass viol? Why does a harp have the shape it does? The lower strings are wound with wire; why?

7. Explain the function of the box of the violin; why is the sound from a tuning fork enhanced when its handle rests on the table?

8. Sometimes we cup our hands around our mouths when we shout; a megaphone strengthens the sound in front but not at the sides. Explain.

9. Why is the reverberation time greater in a large room than a small one? What is the reverberation time of the room in which you sit?

10. If the length of a string is doubled and the tension is quadrupled the pitch of the note: (*a*) remains unchanged; (*b*) is raised one octave; (*c*) is raised eight octaves.

11. When the intensities of two sounds are in the ratio 100 : 1 they are said to differ by: (*a*) 1.4; (*b*) 10; (*c*) 20 decibels.

Vocabulary: Tempered scale, vocal cords, cochlea, reverberation time, absorptivity, decibel (bel).

PROBLEMS

1. A 1000-lb. weight hangs on a 50-ft. rope which weighs 10 lb. Find the fundamental frequency of oscillation when this rope is struck in the middle.

2. A string on the violoncello is 70 cm long and weighs 1.4 grams. Find the fundamental frequency when the string is stretched with a force of 19.4 kilograms. What note is this?

3. The strings of a violin are G_2, D_3, A_3, E_4. Their weights are respectively 18, 12, 6, 4 milligrams per centimeter length (typical values). The length of the vibrating string is 33 cm. Compute the tension in each string.

4. What is the pedal note in a bugle which gives the notes C_2, G_2, C_3, E_3, G_3, C_4? What is the length of the tubing?

5. If a bugle has a fundamental frequency of 100 vibrations per sec. at 20° C, what is its frequency at 0° C?

6. How much energy of the sound wave will remain after five reflections from a light curtain?

ELECTRICITY AND MAGNETISM

CHAPTER 28

MAGNETS

336. Probably at some time or other everyone has handled a toy magnet and wondered at the force which enabled it to pick up tacks and nails. Perhaps some have seen large electromagnets picking up huge chunks of iron. Certainly almost everyone has ridden in streetcars; their wheels are turned by this same magnetic force.

Pieces of the iron ore, magnetite (Fe_3O_4), have this magnetic property. Such natural magnets, "lodestones," were known to the early Greeks; Thales thought the attraction due to the soul of the iron. A thousand years before Christ the Chinese were using lodestones for compasses in journeys over the land. In one of the Arabian Nights tales we hear of a lodestone so strong that it pulled the nails from a ship. Many were the absurd ideas of magnetism. Rubbing a lodestone with garlic was supposed to quench its magnetism, for "Garlick is the lodestone's proper Opium, and by it that spirituell sensation in the magnet is consopited and layd asleep." (Van Helmont, 1621!) Gilbert, physician to Queen Elizabeth, made the first scientific study of magnetism. Gilbert put into practice the principles of scientific investigation which Francis Bacon so gravely philosophized about. Yet the greatness of his work was not appreciated. Bacon, ridiculing his theories, said that Gilbert was endeavoring "to build a ship out of materials not sufficient to make the rowing pins of a boat."

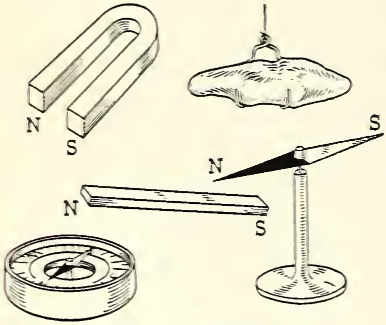

FIG. 1. Magnets.

337. Magnet Poles. In 1600 Gilbert showed that iron rubbed with the lodestone became magnetized. If a knitting needle is stroked with a magnet it becomes magnetized and will pick up small iron bits. There appear to be points near either end of the needle which are the seats of attraction; these are called the *poles* of the magnet. Iron filings cling near the ends of the needle, not to the middle. Such poles always exist in pairs. Break the needle in the middle and each half is still a complete magnet, new poles being formed at the broken ends.

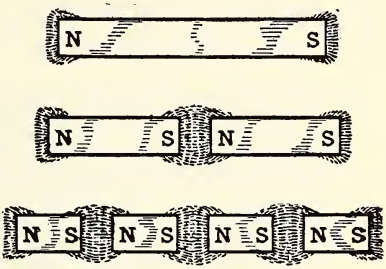

FIG. 2. Poles in broken magnet.

338. North and South Poles. The two poles of a magnet act differently. If a magnetized needle is suspended by a fine thread so as to swing horizontally, it will point approximately north and south. This is the simplest form of compass. The north-pointing end is called the north pole; the other, the south pole. Let us suspend a second needle and likewise determine its north pole and south pole. If now the two north poles are brought close together it is found that they repel each other. Similarly for two south poles. On the contrary, a south pole and a north pole attract each other. *Like poles repel, unlike poles attract.*

The ordinary compass is a small magnetized strip of iron mounted on a delicate pivot bearing. To explain the behavior of the compass needle we suppose that the earth as a whole is a large magnet, with one pole in the northern hemisphere attracting the north pole of the needle, another in the southern hemisphere attracting the other pole. Since unlike poles attract each other, it is evident that the pole which is located in the northern hemisphere is a magnetic pole of south magnetism.

339. Magnetic and Non-Magnetic Substances. Magnets can be made of cobalt and of nickel, and these substances are also attracted by magnets, though the force is several times less than for iron. There is indeed a small magnetic effect for every substance, but except for those mentioned and for a few magnetic alloys the effect is millions of times less than for iron, and it can usually be disregarded.

Lay a magnet on a sheet of glass and observe that iron filings or tacks will be attracted through the glass and will cling to the under side. The same thing will happen through a sheet of paper or mica or copper, but not through a sheet of iron. The magnetic force acts through material substances such as glass or paper as well as it does through air. Indeed, any substance except one which is itself notably magnetic can be interposed without appreciably affecting this magnetic action through space. The region about a magnet, where bits of iron are attracted, magnetic poles attracted or repelled, compass needles turned, is called the *magnetic field.* The strength of the field is not appreciably affected by the interposition of non-magnetic substances.

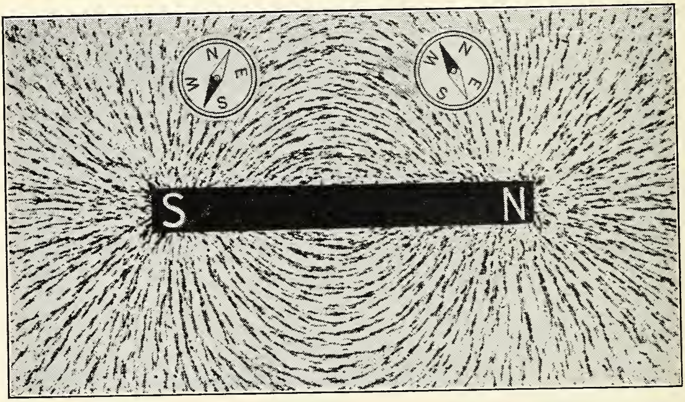

FIG. 3. Magnetic field near bar magnet. Iron filings orient themselves like minute compass needles.

340. The Law of Force. It becomes necessary to devise units of measure for magnetism as we have done for other quantities: force, time, heat, etc. The repulsion (or attraction) between two given magnet poles is reduced fourfold when the distance between them is doubled, ninefold when the distance is tripled. In other words, the magnetic force is inversely proportional to the square of the distance. (Compare gravitational attraction.)

Now consider four magnets, A, B, C, and D. First we measure the repulsive force between like poles of A and B at a given separation. Now suppose that when C is substituted for A the repulsion (now between poles of C and B) is doubled. We then say that C has twice the *pole strength* of A. Suppose that we now find in the same way that the poles of D are three times as strong as those of B. Then, if magnet C is substituted for A and D substituted for B, the force between C and D will be six times as great as that between A and B. Thus *the force between two poles is jointly proportional to their pole strengths and varies inversely as the square of the distance between them.* Representing the pole strengths by m_1 and m_2 and their separation by r:

$$F = c\frac{m_1 m_2}{r^2}.$$

So pole strengths can be compared. It remains to choose a unit: the **unit magnetic pole** has been taken as that pole which will repel an equal pole 1 centimeter away with a force of 1 dyne. With such a unit the constant in this equation becomes unity and the fundamental equation for magnetic force is

$$F = \frac{m_1 m_2}{r^2}. \qquad (1)$$

This is the force between two poles in the absence of other magnetic substances. Usually the poles are surrounded by air, which is itself slightly magnetic. The resultant force on the pole under these circumstances is slightly reduced — very much as the apparent weight of a submerged body is reduced by the buoyant force of the fluid. For poles submerged in a fluid the resultant force is

$$F = \frac{m_1 m_2}{\mu r^2}$$

where μ is a constant which for all ordinary fluids is very nearly equal to unity (1.0000004 for air). μ is called the *permeability* of the fluid.

341. Force between Magnets. There must be (at least) one pair of poles in every magnet, and hence between any two magnets there are (at least) four forces to be considered: two attractions and two repulsions. Suppose the two short magnets in Fig. 4 have poles of 3 and 12 units respectively. Then the force between N_1 and N_2 is $\frac{3 \times 12}{4} = 9$ dynes (repulsive); an equal force acts between S_1 and S_2; between N_1 and S_2 there is an attraction of $\frac{36}{1}$ and between N_2 and S_1 an attractive force of $\frac{36}{9}$ dynes. The total force is $36 + 4 - 9 - 9 = 22$ dynes, attractive. In more com-

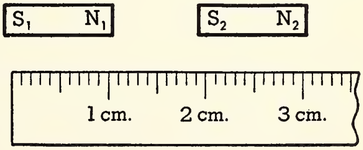

FIG. 4

plicated examples the forces will have different lines of action and the vectorial resultant must be found.

342. The Simple Theory of Magnetism. The fact that poles always exist in pairs prevents us from regarding a single magnet pole as a thing of independent existence. To explain the basic facts we suppose that each atom in iron is itself a small magnet with its north and south pole — we call it a *dipole.* In unmagnetized iron the dipoles are arranged at random. In this helter-skelter arrangement the opposite poles cancel in their effects. When the bar is magnetized there is a lining up of these elementary magnets. Even now, throughout the body of the magnet, the two kinds of poles are present in equal numbers and cancel in their effects. Not so at the ends. One end is preponderantly north, the other south, and this gives the magnet its two poles. This explains the creation of the opposite poles when the magnet is broken. (Suppose the bar in Fig. 5 is cut at the broken line.) It also leads us to expect that a magnet will lose its magnetism when heated, for as the atoms move more rapidly they must shake one another out of the definite arrangement.[1]

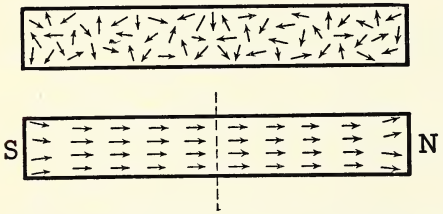

FIG. 5. Magnetization as alinement of dipoles.

Heat the magnetized knitting needle to a dull red and it loses its magnetism. Magnetize a piece of soft iron wire. Then twist it, and it is demagnetized. The twisting destroys the regular arrangement. But the phenomena of induced magnetism give perhaps the best illustration of the theory.

343. Induced Magnetism. Any piece of iron placed near a magnet pole becomes temporarily magnetized and a pole of opposite kind is created in the nearer portion of the iron. This is called induced magnetism. If the inducing pole is *S*, the *N* pole in each of the dipoles throughout the iron is attracted, the *S* poles are repelled; each elementary magnet is turned slightly, and there results a general magnetization of the material (Fig. 6*a*). In soft iron the lining up of the dipoles is temporary and is largely lost when the inducing pole is withdrawn.

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[1] It does not, however, explain the very sudden loss of magnetic properties which occurs at 770° C (§499). There is evidence to show that in strongly magnetic substances such as iron it is not in fact the individual atom which acts as an independent dipole, but a large group of atoms. These groups appear to be suddenly broken up at this temperature.

Place one end of an iron rod as close as you can to a compass needle without appreciably affecting it. Now bring a magnet near the further end of the iron and the needle is deviated (Fig. 7*a*). Why?

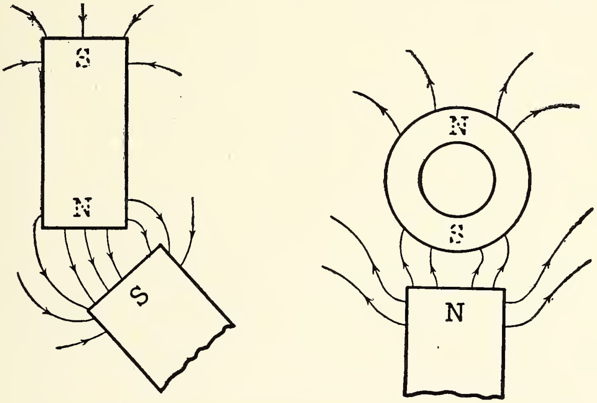
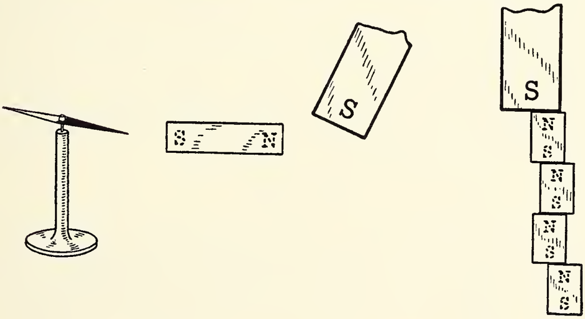


FIG. 6. Magnetic induction. (See also Fig. 29–8.)



(*a*) (*b*)

FIG. 7. Attraction by induction.

This magnetic induction explains why a magnet attracts iron. A magnet can attract only other magnets; the unmagnetized piece of iron becomes magnetized by induction, and the adjacent opposite poles attract each other (Fig. 7*b*).

344. Permanent Magnets. Induced magnetism is partially but not wholly lost when the inducing pole is removed. There is a certain tendency for the dipoles to stick in place. If, while it is being magnetized inductively by a neighboring pole, the iron bar (Fig. 7*a*) is hit with a hammer its magnetism will be increased. The disturbance enables the dipoles to turn further in response to the magnetic force than otherwise they would. And now, when the inducing field is removed, there is rather strong "residual magnetism." The same effect is obtained when we stroke a piece of spring steel or a knitting needle with a magnet pole. In this way we can make small permanent magnets; larger, stronger permanent magnets are made by induction in the field of an electromagnet (§482). Soft iron is demagnetized by a small reverse field and so is not suitable material for permanent magnets. Permanent magnets are made of hardened steel or of steel alloys which are more difficult to magnetize and to demagnetize and are also less susceptible to demagnetization by shocks. Permanent magnets may have a residual pole strength of more than 500 units for each square centimeter of cross-section (Table 34).

345. Magnetic Moment. If the magnets in Fig. 4 were shorter, the unlike forces upon the opposite ends would be more nearly equal and the force between the magnets less. Quite as important as the pole strength of the magnet is its length, and the product of the pole strength by the distance apart (L) of the two poles is called the **magnetic moment** (M) of the magnet:

$$M = mL. \qquad \text{(Def.)}$$

If the two magnets in Fig. 8 are placed side by side their total pole strength is doubled. Not so if placed end to end; then the adjacent poles cancel in effect and the result is a single long magnet with poles of the original strength at the ends. But in each case the magnetic moment of the magnet is doubled. Actually this strength of magnetic moment has a greater fundamental significance than the pole strength.

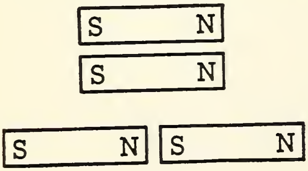


FIG. 8

Magnetic moment is a vector. Two similar magnets lying in opposite directions have together a zero moment. If they make an angle with one another the moment is obtained by vector addition. Place two similar magnets upon a board floating in water. Observe the different effects as the angles between the magnets are changed. The board will invariably turn until the resultant magnetic moment points north.

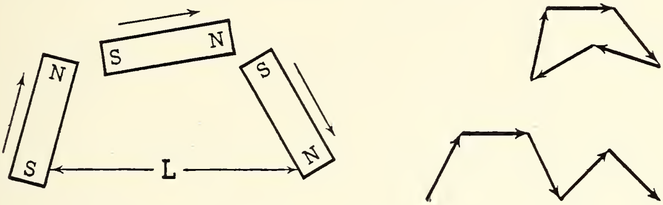


FIG. 9. Magnetic moment as a vector.

346. Intensity of Magnetization. A single large magnet can be considered as the aggregate of several smaller ones. Its magnetic moment is found as the resultant of the moments of these parts. The magnetic moment of a single cubic centimeter of the bar is called the **intensity of magnetization.** We can go to still smaller volumes; we can consider the magnetic moment of each minute crystal of the iron; or, going to smaller and smaller elements, ultimately we come to the magnetic moment of the individual elementary dipole. Each dipole has a definite moment, but in unmagnetized iron the dipoles are oriented at random so that their resultant is practically zero. When the bar is magnetized the magnetic moments become more or less alined. The resultant moment for all the dipoles in 1 cubic centimeter then gives the intensity of magnetization. The resultant for all the dipoles in the bar gives the total magnetic moment of the magnet.

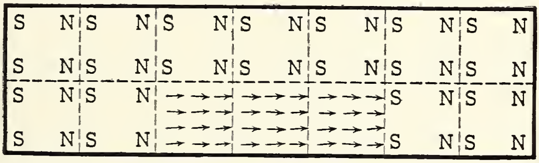


FIG. 10. Intensity of magnetization is the magnetic moment per cubic centimeter; it is the vector sum of all the dipole moments in a cubic centimeter; it is the pole strength per unit area of cross-section. (The figure represents an ideal case with no divergence of magnetization at ends.)

Consider a straight bar uniformly magnetized. Then

$$I = \frac{M}{V} \qquad \text{(Def.)}$$

or the magnetic moment (M) equals the intensity of magnetization (I) times the volume (V). Now $V = AL$ (cross-section × length) and $M = mL$. Hence

$$I\left(= \frac{mL}{AL}\right) = \frac{m}{A}.$$

The intensity of magnetization gives the pole strength per unit area of cross-section; this relation is often useful.

This is for a uniformly magnetized bar. Actually in a bar magnet the dipoles diverge near the ends; thus the intensity of magnetization is changed in direction and reduced in amount here; the poles are not located exactly at the ends. However, this effect may be relatively small in a long magnet and for simplicity it is neglected here. It is quite possible to magnetize only a small portion of a hard steel bar with poles close together. Or we can magnetize opposite ends in opposite directions so as to have a south pole at each end and a north pole of double strength in the middle, like two bar magnets joined with like poles together; the magnetic intensities in the two halves are opposite, and the magnetic moment of the whole bar may be zero. But such magnetization is rather unusual.

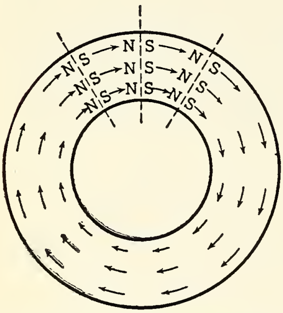


FIG. 11. Magnetized ring without free poles.

A very interesting and important example of magnetization is presented by an iron ring. When uniformly magnetized, a ring has no free poles. The intensity of magnetization in each cubic centimeter is the same in magnitude but not in direction, and the total magnetic moment is zero. If we saw the ring across the middle we have two ordinary magnets, with free poles at the ends.

347. Magnetic Substances. The maximum intensity of magnetization attainable in several magnetic substances is given in Table 34. This maximum intensity is the intensity when all the dipoles in the body are alined parallel. Several remarkable

TABLE 34

Intensity of Magnetization

	Maximum Intensity	Residual Intensity	Coercive Force (gauss or oersteds, §353)
Iron (pure, annealed).....	1700	850	0.4
Transformer steel........	1700	850	0.4
Hard steel..............		600	20.
Cobalt..................	1400	250	12.
Nickel..................	500	260	1.6
Cobalt steel............		800	250.
Tungsten steel..........		800	60.
Nickel-aluminum steel....		600	500.
Permalloy...............	900	450	0.05
Heussler alloy...........	300	200	7.3

alloys are included. Cobalt steel and tungsten steel with high residual intensity and high *coercive force* (resistance to demagnetization) are suitable for permanent magnets. Nickel-aluminum steel with less residual magnetism but very great coercive force is used for making remarkably strong short magnets. Permalloy is an alloy of iron and nickel with extremely low coercive force which has recently been developed by telephone engineers; the dipoles have an extraordinary freedom of motion, and, although the attainable intensity is not great, permalloy reaches full magnetization very readily. Heussler's alloy is remarkable as being a magnetic alloy of three non-magnetic elements (copper, manganese, and aluminum).

QUESTIONS

1. A magnet floating on a cork will turn to point north but will not move toward the north. Why?

2. Explain why a magnet attracts iron filings.

3. What is meant by magnetic moment and by intensity of magnetization?

4. Compare the magnetic moments of a bar magnet and a horseshoe magnet having equal magnetic intensities.

5. What is the force between two N poles, strength 10 units each and 2 cm apart?

6. What happens to the force between two poles if each is doubled in strength and the distance between them doubled?

7. What is the magnetic moment of a magnet with poles of 100 units 10 cm apart?

8. Why is annealed iron not suited for making permanent magnets?

9. Given two bars which attract each other. How can you determine whether only one or both are magnetized?

10. An old theory considered magnetism as the result of two kinds of weightless fluids. What objection is there to such a theory?

11. A bar magnet must have: (*a*) opposite poles near its two ends; (*b*) an equal number of *N* and *S* poles; (*c*) *N* polarity equal to *S* polarity in magnitude?

12. If a substance is magnetized it must certainly have: (*a*) free poles; (*b*) magnetic moment; (*c*) intensity of magnetization.

Vocabulary: Pole, magnetic field, pole strength, unit pole, permeability, dipole, magnetic induction, residual magnetism, magnetic moment, intensity of magnetization.

PROBLEMS

1. Compare the force between two magnet poles when 2 cm apart and when 10 cm apart.

2. Compare the force between two poles at a certain distance and two poles of twice the strength at half the distance.

3. Supposing the other poles so far away as to be negligible, find the force between an *N* pole of 250 units and an *S* pole of 50 units, 5 cm apart.

4. At what distance will the force between two 200-unit poles be approximately 10 gram-weights (*g* equals 1000, approximately)?

5. Assume each pole in Fig. 29–1 to be 200 units. Measure distances, and find the force on the left pole due to the three others.

6. Two magnetized knitting needles 20 cm long have 30-unit poles practically at the ends. The two needles are laid end to end, with unlike poles 10 cm apart. What force acts between these unlike poles? What force between the outside poles? What other forces act? Find the resultant force of attraction.

7. What is the attraction between two bar magnets, each with magnetic moment 10,000 and 10 cm long (between poles), placed in line with 50 cm between centers?

A typical bar magnet, quite commonly found in physical laboratories, is 15 cm in length and 1 sq. cm in cross-section, and weighs 120 grams. Its pole strength is about 300 units, and the poles approximately 10 cm apart. We shall use this magnet in problems, referring to it as our "typical magnet."

8. Find the force acting between two typical magnets placed in line with *N* poles 10 cm apart.

9. What is the force between two typical magnets in line, with 30 cm between centers?

10. Approximately, what is the attraction between two typical magnets placed side by side, 2 cm apart, opposite poles adjacent?

11. A typical magnet lies on the table; a like magnet, parallel, is held above it. Approximately how close must this approach to lift the magnet from the table?

12. What is the magnetic moment of a typical magnet? What is the intensity of magnetization (for the region between poles)?

13. What is the combined magnetic moment of the two knitting needles arranged as in problem 6? of the typical magnets in problem 8?

14. An iron bar 1 meter long and 2 cm square, magnetized by induction in a very strong field, has poles of 6000 units. What is its magnetic moment? (Consider poles at end.) What is its intensity of magnetization? Will it retain this magnetism when removed from the field? (Table 34.)

CHAPTER 29

MAGNETIC FIELDS

348. Lines of Force. A region in space where a magnetic pole is acted upon by a force is called a field of magnetic force. The neighborhood of our earth is such a force field (since anywhere about us a compass needle feels the effect of the earth's magnetism). A stronger field exists between the two poles of a horseshoe magnet and near the poles of a bar magnet. We shall define the **direction of the field** at any point of space as the direction of the force which would act on a north pole placed there. In consequence of this definition the force on a south pole at any point is exactly contrary to the field.

The direction of the field near a magnet can be found by exploring with a small compass. The compass needle (i.e., its north pole) points away from the north end of the magnet and we picture the field by vectors bristling outwards from this pole.

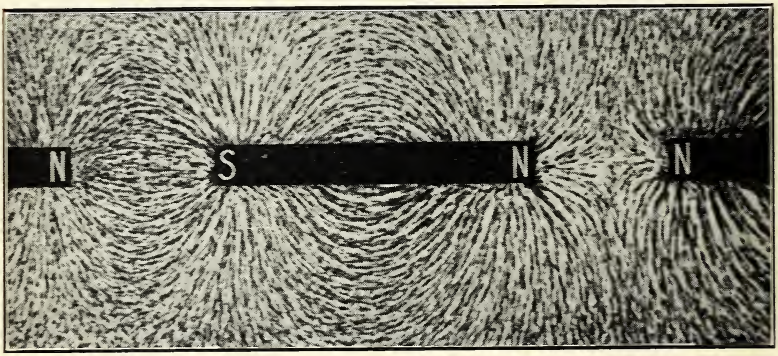


Fig. 1. Field between like and unlike poles.

We imagine the field as represented by **lines of force,** imaginary lines which go always in the direction of the field. With the help of the compass needle we can follow these lines around as they leave the north pole and enter the south. (Fig. 28–3.)

Iron filings sprinkled about in the neighborhood of the magnet set themselves along the lines of force and indicate the direction of the field. Arrange several bar magnets in various ways so that some like poles and some unlike poles face each other. Lay a sheet of paper over them and sprinkle with iron filings. Observe the lines of force passing between unlike poles but not between like poles (Fig. 1). Indicate with an arrowhead at several points in Fig. 1 the direction in which a north pole would tend to move. *Lines of force leave a north pole, enter a south.*

349. Theory of Elastic Lines of Force. According to one theory, the lines of force are supposed to have a real existence in an intangible fluid called the "ether," which pervades all space and all bodies; in these lines of force this theory finds the seat of magnetic attraction and repulsion. The lines may be thought to act like stretched rubber bands pulling opposite poles together. (See Fig. 1.) Neighboring lines are supposed to exert a sidewise thrust upon one another, and this forces like poles apart. This theory avoids the concept of action at a distance between poles.

Today the idea of an ether is discredited, but one can hardly avoid giving a certain amount of substantiality to the lines of force, since we shall find that it is possible for the field to persist after its poles are gone, and for lines which do not originate and end in poles to exist as closed curves! But for the present it is perhaps best to view the field as only a convenient method of representing the behavior of poles.

350. Field Strength. We have used a north pole as a test pole to determine the direction of the field. The magnitude of the force upon such a pole usually varies at different points in the field. This leads to consideration of the strength of the field. We define the **field strength** H at any point as *the force which would be exerted there upon a unit magnet pole.* Henceforth we shall consider the test pole to be a north pole of *unit* pole strength.

Supposing the field to have been explored and its direction and strength at each point determined, the force on a magnet pole (m) placed at any point can be determined at once from the relation (which follows immediately from the definition of H):

$$F = mH. \quad \text{(From Def.)} \tag{2}$$

Notice that m is the strength of the pole affected by the field; it does not represent the pole which is the cause of the field. Indeed, quite frequently the exact source of the field is not known. If the source of the field is known, the field strength can be computed as we shall now show.

351. Strength of Field Due to a Magnet. By eq. (1) of the last chapter the force acting on a *unit* north pole at a distance r from a pole m' is

$$\text{Force on unit pole} = H = \frac{m'}{r^2}. \tag{3}$$

By definition this is the field strength at this point due to the pole m'. It is directed away from a north pole, toward a south pole.

The field (H) due to a pair of poles is obtained by computing the effects of the separate poles and finding the resultant.

Examples. 1. Find the field strength at point a due to the upper magnet in Fig. 2. Due to 100 S:

$$H = \frac{100}{5^2} = 4 \text{ dynes per unit pole (to the left).}$$

Due to 100 N:

$$H = \frac{100}{10^2} = 1 \text{ dyne per unit pole (to the right).}$$

Total field is 3 dynes per unit pole toward the left.

2. The field at point b (due to the two north poles) is $\frac{100}{5^2} - \frac{100}{5^2} = 0.$

3. For the field at a point not directly in line with the two poles a vector construction is necessary. This is indicated in the figure. H is the resultant of a vector away from the N pole and a vector toward the S pole.

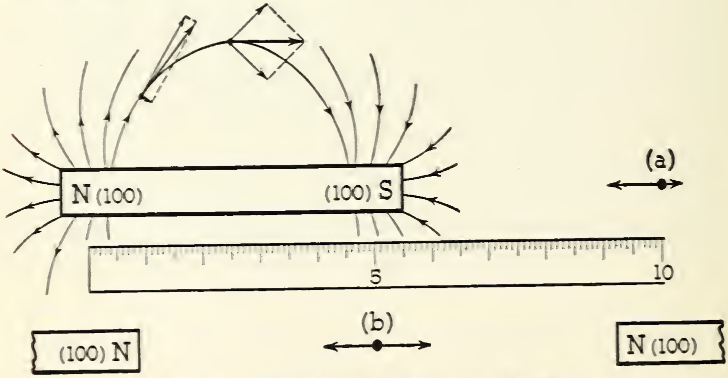


FIG. 2. Field strength near magnet poles.

352. Approximate Formula for Bar Magnet. It is frequently desired to find the strength of the field at a point along the extension of the magnet. Let the distance to this point, measured from the center of the magnet, be r, and call the length of the magnet (between poles) 2 a so that 2 ma is the magnetic moment M. Then the field due to the nearer pole (Fig. 3) is $\frac{m}{(r-a)^2}$, and that due to the further pole is $\frac{-m}{(r+a)^2}$. The resultant field is

$$H = \frac{m}{(r-a)^2} - \frac{m}{(r+a)^2} = \frac{4\,mra}{(r^2-a^2)^2} = \frac{2\,Mr}{(r^2-a^2)^2}.$$

If a is very much smaller than r, a^2 may be dropped in the denominator, giving the more convenient formula

$$H = \frac{2\,M}{r^3}. \quad \text{(Approximate.)} \qquad (3')$$

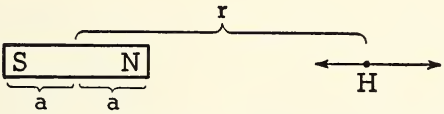


FIG. 3. For large distances: $H = 2M/r^3$.

353. Unit of Field Strength; "Number of Lines of Force." We have quite naturally expressed field strength as so many *dynes per unit pole.* This unit, however, has been given a special name: gauss.[1] The strongest fields usually available are about 20,000 dynes per unit pole or 20,000 gauss. In very small regions between the poles of the strongest electromagnets, a field of twice this intensity may be obtained. We shall find that the earth is about 0.6 gauss, varying with locality.

The strength of the field is sometimes expressed in a slightly different way. It has doubtlessly been observed that the force lines are thickest where the field is strongest. Hence the strength of the field can be represented as proportional to the number of lines of force passing through a unit area of a surface drawn perpendicular to the field. The electrical engineer has gone further and agreed (as a convention) to consider *the number of lines of force per unit area as equal to the strength of the field at the point.* We shall henceforth follow this practice. Near the *N* pole of a (rather weak) magnet the field may be a hundred gauss; we imagine a hundred lines of force bristling out from each square centimeter, one from each square millimeter; farther away the field is less and the lines, spreading out, become less closely packed. We shall frequently speak of a field of one line per square centimeter meaning a field of 1 gauss. Out from a strong magnet pole may come 20,000 lines per square centimeter.

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[1] Named for the German mathematician who was an early investigator of magnetism. Gauss was the first to propose a c.g.s. system of electromagnetic units. . . A distinction is sometimes made between the *acting* field, the field which acts on poles (eq. 2) and (as we shall see) on electric currents, and the *generated* field, the field directly produced by poles (eq. 3) or currents. When such a distinction is made the unit *gauss* is reserved for the former and the latter field is measured in *oersteds.* This distinction will be ignored in this text (App. 3).

354. How Many Lines per Unit Pole? It would be supposed that the total number of lines of force leaving (or entering)

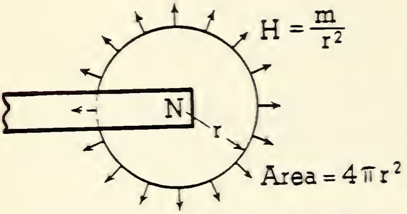

FIG. 4. $4\pi m$ lines leave a pole.

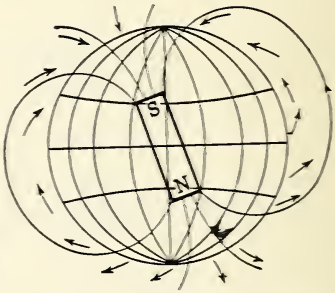

FIG. 5. Earth as a magnet.

a pole would depend upon the strength of the pole. This is true as we can see from a simple consideration. Consider a sphere of radius r drawn about the pole. Its area is $4\pi r^2$. The field strength at all points on this sphere is the same; by (2) it equals m/r^2 gauss, or, by our convention, there are m/r^2 lines per square centimeter. Hence the total number of lines emerging from this pole is $4\pi r^2 \times \frac{m}{r^2} = 4\pi m$. *$4\pi$ (approximately $12\frac{1}{2}$) lines issue from a unit pole.* This is " Gauss's Law."

355. The Field of the Earth. Gilbert explained the general behavior of terrestrial magnetism by considering the earth a magnet. Its poles are far below the surface, and its axis is inclined somewhat from the earth's axis of rotation. The southern pole is below a point south of Australia; the northern pole is below a point north of Hudson Bay. It is remembered that the southern pole is a pole of " north " magnetism. Lines of force leave it and enter the pole in our northern hemisphere; our compasses tend to set themselves along these lines of force. Modern investigators conclude that to a first approximation this earth of ours can be considered as a sphere with constant intensity of magnetization through much of its volume. The total magnetic moment of this earth magnet is 8×10^{25} units.

Magnetic north (the direction in which a compass needle points) usually differs from true north, and the difference is called the *angle of declination.* In the United States the declination varies from 20° west in Maine to 25° east in the state of Washington; in western Ohio the compass points due north. (See Fig. 7.) Columbus was probably the first to observe that the compass needle did not point to true north. In his first voyage across the Atlantic the changing angle of declination so disturbed his sailors that they nearly mutinied.

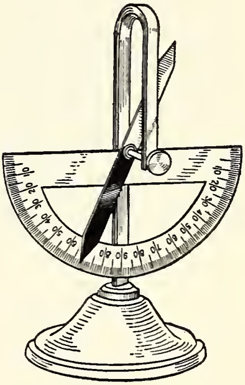

FIG. 6. Dip needle.

The ordinary compass needle which can rotate only in a horizontal plane shows only the *horizontal component* of the earth's magnetic field. If a compass needle were perfectly free to move it would dip downward, pointing more or less toward the earth's northern pole (or in the southern hemisphere with the south pole dipping toward the southern pole). This can be shown by means of the " dip needle," a magnetic needle free to rotate in a vertical plane. The angle with the horizontal is called the angle of dip (or angle of *inclination*). In the United States, the dip varies from 60° in Texas to 75° in Minnesota. A little south of the equator in America, a little north of the equator in the eastern hemisphere, the angle of inclination is zero, and in general in the southern hemisphere it is the south pole which dips below the horizontal.

In the United States the intensity of the earth's field is between 0.54 and 0.63 gauss. The *horizontal component* of this field varies from 0.175 gauss at the northern boundary to 0.275 gauss in southern Texas.

Despite the extensive study carried on in all parts of the world the fundamental cause of terrestrial magnetism is still unknown. The earth's field slowly changes; in the time of Gilbert the compass needle in London pointed north or perhaps a little east of north; today the declination there is 20° west. In 1800 the agonic (" no declination ") line passes through Washington, D. C.

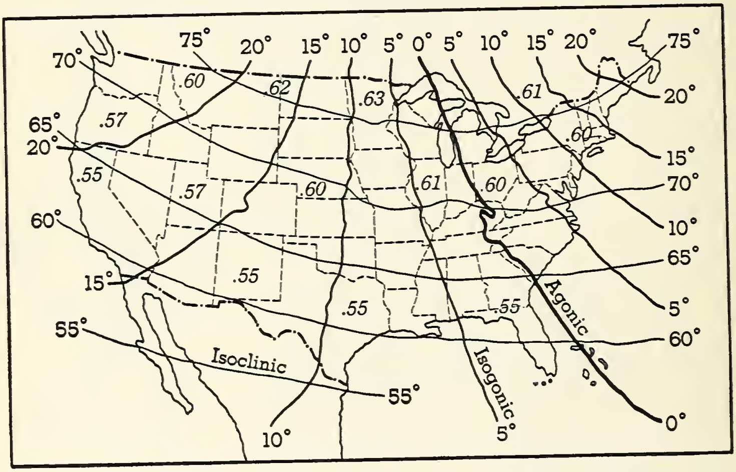

FIG. 7. Inclination, declination, and total field strength in the United States. Represented by isoclinic (" equal inclination ") lines and isogonic (" equal declination ") lines; the numbers within the states give the field strengths.

(Compare Fig. 7.) There are also smaller monthly changes and daily changes, and there are " magnetic storms ": disturbances of intensity, declination, and dip which are connected with showers of electrical charges given off from the sun.

356. Induction in the Earth's Field. A soft iron rod, held parallel to the earth's field, becomes magnetized by induction. The lower end becomes a north pole and repels the north pole of a compass. The intensity of magnetization induced is not great, but if a bar of permalloy is used the experiment becomes more striking because this alloy of nickel and iron is very susceptible to small fields. A bar of permalloy held parallel to the magnetic field of the earth will pick up small fragments of the same alloy; when the bar is held perpendicular to the lines of force it loses its magnetism. *The ratio of magnetic intensity to field strength is called* **susceptibility.** The susceptibility of permalloy is very great.

Railroad rails extending north and south and the vertical water pipes in our houses become magnetized; the northern ends and lower ends are north poles. Iron ore deposits are magnetized, and in iron districts large variations from the general trend are found in declination and in dip as well as in field intensity. In some localities the compass needle points south. These are all examples of magnetic induction.

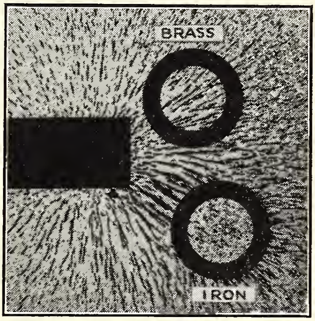

FIG. 8. Soft iron shields the magnetic field.

The mariner's compass is seriously affected by permanent as well as induced magnetism in iron parts of the vessel. The permanent magnetism of the ship is compensated by placing a bar magnet ("Flinders bar") some distance below the compass in such a direction as to produce an exactly opposite field. The induced magnetism (which changes as the vessel turns) was not serious in the day of wooden ships. It is a different matter today. The chief effect of induced magnetism in an iron shell is to weaken the field at points within it (see Fig. 8). In battleships and ocean liners there is little earth's field to control the compass. The gyroscopic compass (§186) has been generally substituted for the magnetic compass in large steel vessels.

357. Couple Acting in a Magnetic Field. A magnet floated on a large cork turns to the north as a compass needle; it will not drift toward the north. There are opposite forces acting on its two poles. A couple acts upon the magnet; this couple is greatest when the magnet is perpendicular to the field. In this position the couple is obviously HmL or HM. When the angle of deflection is θ the lines of action of the opposite forces are separated by a distance $L \sin \theta$ and the couple is $HmL \sin \theta$ or $HM \sin \theta$.

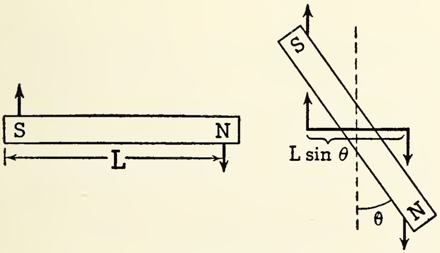

FIG. 9. Torque on a magnet.

$$\text{Couple} = MH \sin \theta. \tag{4}$$
$$\text{Maximum couple} = MH.$$

358.* Practical Measurements in Magnetism: 1. Deflection of Magnetometer. There are two methods by which the magnetic moment of a bar magnet (M) can be measured when the horizontal component of the earth's field (H_h) is known, or by which H_h can be found when M is known.

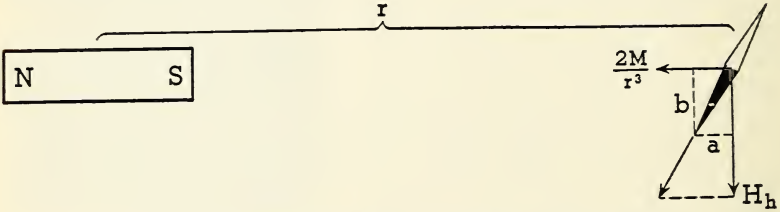

FIG. 10. Comparison of fields by magnetometer deflection.

(1) The magnet is placed some distance east or west of a small, delicately suspended compass needle. (The magnet itself lies in the east-west direction.) The magnet then produces a field perpendicular to that of the earth, and the compass needle sets itself along the resultant of the two fields. Suppose that the distance from the needle (r) has been adjusted so that the needle is deflected through exactly 45°. Then the cross field of the magnet evidently equals the horizontal component of the earth's field H_h. From (3′)

$$\frac{2\,M}{r^3} = H_h,$$

and M can be found if the horizontal component of the earth's field is known.

As the experiment is actually performed, instead of equalizing the two fields in this way the bar magnet is set at a fixed distance and the deflection of the needle carefully observed. Evidently (from the figure)

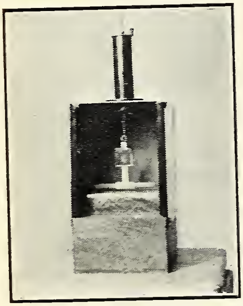

FIG. 11. Simple magnetometer.

$$\frac{2\,M}{r^3} : H_h = a : b. \qquad (a)$$

The angle (and hence the ratio of the sides $a : b$) being known, M can be found in terms of H_h and r as before.

Instead of an ordinary compass needle a *magnetometer* is used in this experiment. In this instrument a small bar magnet hangs suspended from a fine silk fiber and its deflection can be read with great accuracy by means of telescope and scale.

359.* Practical Measurements: 2. Period of Oscillation. A second method of determining M in terms of H_h (or *vice versa*) involves the measurement of the period of oscillation of the magnet. If the bar magnet under test is suspended by a fine fiber to swing horizontally in the earth's field it will oscillate with angular harmonic motion, since the torque varies approximately as the angle of deflection. By eq. (4) the torque in the horizontal plane is $MH_h \cdot \sin\theta$; and for small angles this is approximately $MH_h \cdot \theta$. Eq. (12''''), p. 185, gives the period of angular harmonic motion:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

or (in this case, where $\kappa = MH_h$),[1]

$$T = 2\pi\sqrt{\frac{I}{MH_h}}. \qquad (b)$$

Hence, by determining the moment of inertia of the magnet and the period, the product MH_h can be found, and either of the quantities M or H_h can be determined in terms of the other.

360.* This gives us two alternative methods for measuring the magnetic moments of magnets (or for measuring the horizontal intensity of the earth's field). It may be, however, that neither M nor H_h is accurately known. Then both methods must be used. The magnetometer method gives the ratio of M/H_h:

$$\frac{M}{H_h} = \frac{r^3}{2} \cdot \frac{a}{b} \text{ (from [}a\text{])},$$

and the oscillation method gives the product MH_h:

$$MH_h = \frac{4\pi^2 I}{T^2}.$$

The ratio and the product being known, M and H_h can be found separately (simultaneous equations). The student will find this an interesting experiment. He can make a rather accurate determination of the local field (horizontal component). The experiment was first performed by Gauss.

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[1] The torsion constant κ is the torque per unit angular displacement (§184) and hence equals $MH \sin_h \theta/\theta$ exactly or approximately MH_h since for small angles $\theta = \sin\theta$ approximately.

Magnetic observatories have been established in many parts of the world; vessels constructed without iron have been equipped for the mapping of the magnetic field over the oceans. Horizontal intensity is measured by the methods just described. The declination is also measured by the magnetometer, which is essentially only an extremely sensitive compass needle. (The magnetometers used in practice are much more elaborate than the simple instrument shown in Fig. 11.)

To find the angle of inclination the magnetic observatories use a very delicately balanced dip needle (inclinometer). The horizontal component and the angle of dip being known, the total intensity can be found.

361. Résumé. In the preceding chapter we found the law of action between poles; in this chapter we have studied the same phenomena from a different point of view.

The magnet field: a region in which magnet poles are acted on by a force. Its direction at any point: the direction of the force acting on a north pole placed there. Its strength (H) at any point: the magnitude of the force acting on a unit pole. Its unit: a dyne per unit pole (gauss). Its representation: by lines of force, always parallel to the field and in number considered as equal to H per square centimeter. Its computation: the field due to any pole is m/r^2; 4π lines of force issue from a unit pole. Its use: the force acting on any pole (m) is mH, and the torque on a magnet at right angles to the field is MH. Its inducing effect: a magnetic substance placed in the field is magnetized by induction. The intensity of magnetization so produced depends on the susceptibility of the material.

The earth's field has a strength approximately 0.6 gauss in most localities; the intensity and the declination and dip vary with the locality.

362. Magnet Field Compared with Gravitational Field. The student must not confuse magnet poles and magnet moments and intensity of magnetization in the magnet with the strength of the field which the magnet produces. Compare magnetism with gravitational attraction. All about the earth is a field of *gravitational* force; in treating gravitation we might have pictured imaginary gravitational force lines going into the earth like magnetic lines into a south pole, lines which would indicate for us the direction in which bodies fall. g dynes is the attractive force acting on 1 gram, so g is the measure of the strength of this gravitational field. We can imagine 980 lines drawn per square centimeter. The lines diverge and the field grows gradually weaker as we leave the earth just as the magnetic field does as the distance from a pole increases. But certainly this field of the earth and this g and these imaginary lines of gravitational force are not to be confused with the solid earth itself.

QUESTIONS

1. What is the value of the angle of declination, dip, field strength, and the horizontal component of the earth's field in your locality?

2. What is the force in magnitude and direction acting on a 50-unit south pole in your locality? What is the total force on a compass needle?

3. If a compass needle is 10 cm long and has poles of 100 units, what is its magnetic moment, and what couple is required to hold it transverse to the earth's field?

4. Approximately what would be the declination and dip 50 miles north of the northern magnetic pole?

5. Which pole of a compass will be attracted to the upper end of an iron water pipe?

6. Railroad rails which have lain in a N–S direction are found to retain their magnetism after they are removed. Why?

7. The field strength of a bar magnet varies with distance: (*a*) inversely as r^2; (*b*) inversely as r^3; (*c*) as (*a*) near one pole and as (*b*) at great distances.

8. Permalloy is notable for its high: (*a*) intensity of magnetization; (*b*) susceptibility; (*c*) coercive force.

Vocabulary: Lines of force, field strength, gauss, angle of declination, angle of inclination (dip), horizontal (vertical) component, susceptibility, magnetometer.

PROBLEMS

1. Find the strength of the field midway between the poles of a horseshoe magnet, each pole being 800 units strong and the distance between them 4 cm.

2. Find the strength of the field (*a*) 3 cm beyond the north pole of our typical magnet[1] (*b*) 10 cm beyond. How many lines of force leave one of its poles?

3. Approximately what is the strength of the field 50 cm from the middle of our typical magnet[1] along its extension?

4. In Fig. 2 find the field strength (due to upper magnet) at a point 3 cm to the left of a. Find the field strength 2 cm to the right of b (due to lower poles). Do the fields have the same direction in the two positions?

5. What are the direction and magnitude of the total force acting on a 300-unit S pole in your locality? What is the horizontal component of this force?

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[1] This is the magnet described on p. 342 (problem 8).

6. What force acts upon one pole of our typical magnet[1] when placed (*a*) in the earth's field; (*b*) at point *a* in Fig. 2?

7. What total couple acts on the typical magnet[1] when it is perpendicular to the earth's field in your locality?

8. In your locality what is the total couple acting on a magnet with magnetic moment of 2000 units oriented east and west? What couple acts upon it in a horizontal plane?

9. A sewing needle 5 cm long is magnetized (pole strength 2 units) and is floated on the surface of water. When the needle points northwest what turning couple acts upon it?

10. Find the field strength at a lateral point which is located 10 cm from each pole of our typical magnet.[1]

11. Assuming poles of 200 units for the bar magnet, measure distances and find the field strength at one of the small compasses in Fig. 28–3.

12. Approximately what are the inclination, declination, and the field strength in northern Montana, in southern Texas, in New York City?

13. From Fig. 7 compute the value of the horizontal component of the earth's magnetic field in central Texas.

14. Compute the moment of inertia and the period of oscillation of our typical magnet[1] swinging in the earth's field.

15. A bar magnet has a moment of inertia of 800 gram-cm^2. Its period of oscillation is 12.56 sec. Placed 20 cm east of a compass needle it deflects this 45°. Find: (*a*) MH_h; (*b*) M/H_h; (*c*) M and H_h. (*d*) Assuming an angle of dip of 67°, find the total intensity of the field.

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[1] Described on p. 342.

CHAPTER 30

ELECTRICITY AT REST

363. Rub a rubber rod with a piece of flannel. Bits of paper, of tinfoil, of sawdust, or other light objects will now cling to it. They will also cling to the flannel. We say that this is due to the " charges of electricity " which the rod and the flannel now hold. A similar effect is observed if sealing wax or amber is rubbed with the flannel or if glass is rubbed with silk. It is a common experience for a person to become charged in walking across a rug.

Any two different substances can be charged by rubbing them together. With metals it is necessary to hold the object by an insulating handle; a metal ball held by a handle of rubber or of glass can be charged by rubbing it across a varnished table.

364. Positive and Negative Charges. If we electrify two rubber rods in this way, suspend one of them horizontally by a thread as we did our magnets, and bring the other rod near it, we shall observe that the rods repel one another. These like charges repel. The same thing will happen if a piece of sealing wax or of amber is rubbed with flannel and brought near the suspended rod. But the charged glass rod acts differently. It will attract the electrified rubber rod. Suspend a charged glass rod, and the charge on another glass rod will repel it; that on rubber or amber or sealing wax attracts it. Evidently there are two kinds of electricity. The charges which we have produced on rubber, amber, and sealing wax are alike; the charge on glass is different. Like charges repel, unlike attract. The charge on the glass is called *positive*; that on the rubber, *negative.*

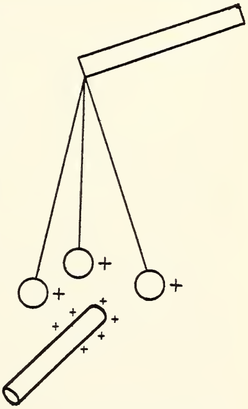

FIG. 1

Suspend several pith balls from silk threads. When the balls are touched with a charged glass rod some of the charge leaks to them and they repel one another. The glass rod now repels them, but they are attracted to a charged rubber rod.

A piece of gold foil held above a charged plate is attracted to it by induced electricity (§374). On contact with the plate it receives a charge and flies away. The like charges repel, and the foil may now be held floating in mid-air above the plate. Between two oppositely charged plates the foil will rush back and forth, carrying charges from one to the other, repelled by the like charge, attracted by the opposite charge, and continue shuttling back and forth until the two plates lose their charges and reach the same " potential " (Chapter 31).

365. Conductors and Insulators. In these experiments one can show that the rubber rod has the properties of electrification only upon those parts actually touched by the cloth. (Contrast with magnetism.) So it is with the glass rod. But when a metal rod is electrified as described above, the electrification is not thus localized. Early investigators considered positive and negative electricity as two kinds of fluids, and they distinguished between bodies in which these fluids can flow freely (conductors) and those in which they cannot (non-conductors or insulators). Metals are conductors; a charge received at one point flows over the whole outer surface of the metal. For this reason a piece of metal held in the hand cannot be charged; the charge flows from the metal through the body (which is a fairly good conductor) to the ground. (Contrast with magnetism.) Electrostatic experiments are difficult to perform in humid weather; a little moisture adheres to the surface of many insulators, notably glass, and renders them slightly conducting.

366. Opposite Charges by Contact. When rubber is rubbed with flannel, the flannel becomes charged as well as the rubber. Bring the flannel near the suspended charged rubber rod and we notice an attraction; the charges are opposite. Whenever charges are created by rubbing two substances together the one is charged positively and the other negatively. Moreover (as will be shown presently), the two opposite charges are of equal strength. We must suppose that the two opposite kinds of electricity are always present; in an uncharged (or " neutral ") body the two charges are present in equal amounts. When the flannel passes over the rubber rod a slight excess of the positive " fluid " clings to the flannel, of the negative, to the rod.

Although electricity generated in this fashion is sometimes called " frictional electricity " the only purpose of the rubbing is to bring the two surfaces into intimate contact. A piece of paraffin merely dipped into water becomes charged; a belt passing over the pulley of an engine becomes charged. When a water drop is broken in two, the parts usually become oppositely charged; this plays a part in the thunderstorm.

367. Electrons and Protons. It is already quite apparent that electricity behaves very differently from magnetism. Only a few substances can be noticeably magnetized; any substance can receive a charge. Charges can flow about in conductors. Charges do not, like poles, always exist on a body in pairs. As we pursue our study we shall find that individual magnet poles are little more than inventions of the physicist to describe magnetic effects; electric charges have a very substantial existence.

In the past there have been a number of different theories as to the nature of electricity. Today it is known that positive and negative electricity consists of two different kinds of particles: protons and electrons.[1] Protons are positive charges; electrons, negative. Both are material particles, but the mass of the proton is many times the greater. Its mass is very nearly equal to that of the hydrogen atom; the mass of the electron is 1840 times less. The hydrogen atom is merely the combination of a proton and an electron. In other atoms a number of protons (together with neutral particles, called neutrons) are united to form the atomic *nucleus*; then about this nucleus are bound an equal number of electrons, attracted by the positive charge. In any neutral substance the two kinds of charges — electrons and (buried in the nuclei) protons — are present in equal numbers; in a positively charged body there is a deficiency of electrons; in a negatively charged body there is an excess.

368. Force between Electric Charges. As has been found in other instances of action at a distance (gravitation and magnetism) the force between two electric charges varies inversely as the square of their distance apart, $F \propto 1/r^2$. It can also be shown that the force between them is jointly proportional to the charges (q_1, q_2), and the fundamental law of force is

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[1] These are the only charged particles with permanent existence in matter. A lighter positive charge (positron) and a heavier negative charge (meson) have been discovered in cosmic rays (Chapter 45), but these charges have only a transient existence.

$$F \propto \frac{q_1 q_2}{r^2} \cdot \quad \text{(Coulomb's Law.)} \qquad (5')$$

In the electrostatic system of units (e.s.u.), which we shall use for the present, the **unit of charge** is defined as the charge which will repel a like charge 1 centimeter away (in a vacuum) with a force of 1 dyne. This makes the constant of proportionality unity in vacuum (and practically so for air):

$$F = \frac{q_1 q_2}{r^2} \cdot \quad \text{(Coulomb's Law in e.s.u.)} \qquad (5'')$$

369. Electrical Units. Three different systems of units are in general use: the electrostatic system, which is the c.g.s. system that we are using; the electromagnetic system, which is another c.g.s. system; and a practical system in common use which we shall presently employ. In this last system each unit has its special name. The practical unit of charge is called the coulomb. Sometimes our electrostatic unit is called the statcoulomb, and we shall adopt this name.

How large is this unit? Ten statcoulombs on the average electroscope (§372) causes a marked deflection. When we charge a rubber rod, this may hold on its surface several times as much. (But billions of statcoulombs flow through a lamp filament every second!)

370. Force in Other Media. The force between electric charges is half as great if the charged bodies are immersed in petroleum oil; in paraffin oil it is one-fifth as much. Equation (5″) gives the observed force only for charges in vacuum and (very nearly) for charges in a gas. The more general relation is

$$F = \frac{q_1 q_2}{k r^2} \quad \text{(General Coulomb Law),} \qquad (5)$$

where k depends on the medium surrounding the charges. This medium through which the electric force acts (any insulating substance) is called the *dielectric*, and k is the *dielectric constant.*[1] Its values for several different substances are given in Table 35.

TABLE 35

Dielectric Constants

Petroleum....	2	Paraffin.....	2
Paraffin oil ..	5	Rubber	2
Alcohol	30	Mica.........	6
Water........	80	Glass	5–7

371. Electric Current. The electrons are more mobile than the positive charges in matter. In solid bodies the atomic nuclei (positively charged) are localized, merely vibrating with their thermal agitation about fixed positions. And in insulators the electrons are bound to them; but, in conductors, some of the electrons are free and can move through the conductor like a fluid. Or, perhaps more exactly, moving hither and thither, these free electrons move about among the atoms in a metal much as gas molecules do within a porous substance. It is these electrons rather than the protons which flow through the filament of the electric lamp when we close the switch. It is their motion which turns our electric motors.

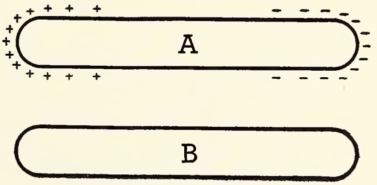

Fig. 2

Perhaps it is a little unfortunate that custom has called the protons rather than the electrons positive. But it must be observed that in most instances it is quite immaterial whether we consider the positive or negative charge as moving. If at a certain moment a metal rod has the charges distributed as in A (Fig. 2), electrical attraction will bring electrons and protons together and the rod becomes neutral (B). It is all one whether we say that the positive charge has moved to the right or that the negative charge has moved to the left. We shall speak freely of the motion of either positive or negative charge. When electricity moves there is an *electric current*, and by established usage the direction of the current is taken as the direction which positive charge would have to move to produce the effect. In Fig. 2 the " current " is to the right although investigations into the nature of metals have shown that it is actually the electrons which here move to the left.

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[1] The Greek prefix *dia* means *through.* Equation (5) applies to fluid dielectrics in which both charges can be immersed and the force measured directly. The effect of solid dielectrics will be considered in the next chapter. Equation (5″) may be regarded as the fundamental form of Coulomb's Law for the direct force between two charges, the factor k entering the denominator of (5) because of the electric force of the medium, which has an effect analogous to buoyancy. Recall the corresponding effect in magnetism (§340).

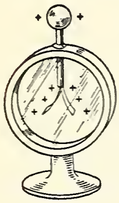

FIG. 3. Electroscope.

372. The Electroscope. For our further study we shall need a more convenient source of charge than frictional electricity and a more sensitive method of detecting electricity. Electric charges are conveniently produced by a " static machine," and the electricity is conveniently stored in a metal-lined glass jar (Leyden jar) which may be given a charge of many thousand statcoulombs. The explanation of this static generator and of the Leyden jar must be deferred to the next chapter.

The electroscope, used to detect electrical charges, is a very simple instrument indeed. Two strips of gold leaf or aluminum leaf are suspended from a metal stem. This is mounted as shown in Fig. 3. The stopper through which the stem passes should preferably be of sulfur, amber, or sealing wax for insulation. To shield the leaves from outer influences it is well to have a metal case. (However, a satisfactory electroscope can be made by the student with nothing more than a large nail for stem, two strips of foil, a rubber cork for insulator, and a bottle.) When a charge is placed on the knob of the electroscope, part of it flows down to the leaves. The two leaves then have like charges and repel.

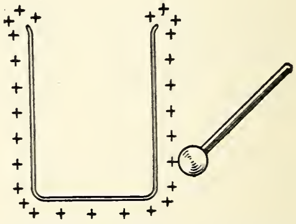

FIG. 4. Distribution of charges on metallic vessel.

373. Distribution of Charge on Conductors. Owing to the mutual repulsion between like charges, any charge which is placed on a conductor finds its place entirely on the surface. Moreover, in the case of a hollow conductor the charge is located only on the outer surface. To show this the metal vessel in Fig. 4

has been charged. If the small insulated disc (a " proof plane ") is touched to the exterior of the vessel, it receives a charge (as can be shown by carrying it to an electroscope). No charge is received when the plane is touched to the inner walls. There is no free charge on the inside of a hollow conductor.

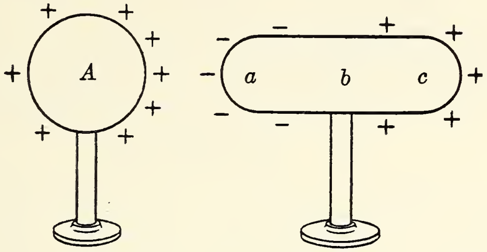

FIG. 5. Electrostatic induction.

In the same way it can be shown that the charge is strongest on points and sharp corners. The explanation is the same — mutual repulsion causes the charge to seek a place as remote as possible from the other charges.

374. Electrostatic Induction. When a positive charge (*A* in Fig. 5) is brought near an insulated conductor, the negative charges in the conductor are attracted and the positives repelled. The result is a separation of charges, and we have charging by induction. Unlike charge is on the nearer side (at *a*); like charge on the more remote parts (*c*).

Notice that, though the charges are now separated on the conductor, considered as a whole the body is still neutral. The charge attracted to one end equals that repelled to the other. And when the inducing charge is removed the induced charges flow together and all electrification disappears. If, however, while still under the influence of the inducing charge, the body is touched, momentarily, with the hand the situation becomes different. The repelled charge (in this case positive) flows off to the ground;[1] the negative charge is bound in place by the attraction of the neighboring positive charge. When now the inducing charge is removed, the conductor remains charged (negatively).

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[1] Actually electrons flow up from the ground to neutralize the positive charge but, as we have previously stated, this distinction can for the present be considered as immaterial. . . The same transfer occurs irrespective of *where* the conductor is touched, the bound charge (here negative) remaining and the unbound charge passing to the ground. This is explained more completely in the following chapter (§401).

Electric induction explains the attraction of metal foil by a charged rod. Unlike charge is induced on the nearer side of the foil, and these unlike charges then attract.

375. Charging the Electroscope by Induction. It is usually better to charge an electroscope by induction than by contact. To charge it positively a negative charge is used. The steps are:

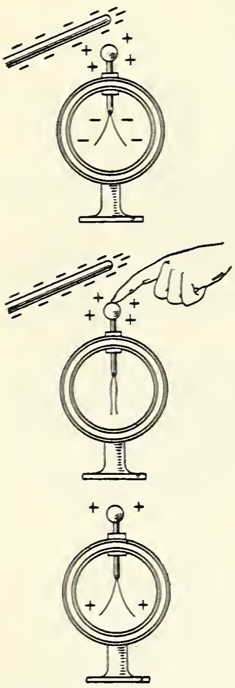

Fig. 6. Charging by induction.

(1) Bring near to the knob of the electroscope a negative charge. (Positive charge attracted to knob; negative repelled to leaves which diverge.)

(2) Ground the knob with finger. (Negative charge flows off from leaves, which collapse. Positive charge on knob is bound.)

(3) Stop grounding and then remove inducing charge. (Positive charge flows to leaves, which diverge, and electroscope is charged.)

It is often necessary to test the sign of the charge which is on the charged electroscope. We can readily do this by noting the effect when a negatively charged rubber rod approaches the knob. If the charge on the electroscope is positive the leaves will collapse because this charge is attracted up to the knob; if the charge is negative the leaves diverge further as the negative charge on the knob is pushed down to the leaves.

376. Total Induction. How much charge can a given charge induce? Michael Faraday asked himself this question, and he proceeded to place a charge within a hollow vessel. He had at hand a pail which was used for carrying ice, and the experiment is known as " Faraday's ice-pail experiment." It throws much light upon the meaning of electricity, and we should repeat the experiment. A hollow metal vessel, placed on an insulating stand, is connected to the electroscope. Suppose a positively charged metal ball to be brought within it. This will induce negative charge to the interior,[1] leaving the outside and the electroscope positive. If we now touch it to the vessel all the positive charge on the ball will flow off, neutralizing the induced charge, and any excess flows to the outside. But there is no such excess because, as we observe, the deflection of the electroscope is not changed. This proves that the total induced charge exactly equals the inducing charge.

Faraday then repeated the experiment in a slightly different way. Suppose that while the charges are being induced we touch the vessel with the hand. The negative charge remains bound; the positive charge flows off and the electroscope leaves fall. If now the ball is touched to the vessel the positive and negative charges exactly neutralize and the whole system remains neutral. Again there is no deflection of the leaves.

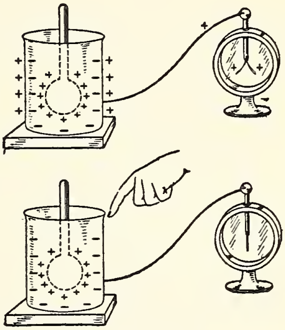
FIG. 7. Faraday's ice-pail experiment.

The experiment shows that a given charge induces an exactly equal opposite charge. True, the charge induced on a small conductor like the knob of an electroscope increases as the primary charge approaches it. This is because only a part of the total induction occurs on this particular conductor; much of the induction is usually on the table, or the walls and ceiling of the room. The *total* induction equals the inducing charge.

377. Equality of Frictional Charges. The equality of the opposite charges excited in frictional electricity (§366) can be shown by means of this same apparatus. If, after rubbing a rubber rod with a piece of flannel, both flannel and rod are placed in the "ice pail," no charge will be induced. The negative charge on the rubber evidently equals the positive charge on the cloth.

We have found three cases of equality of charge: (1) opposite frictional charges are equal; (2) charges induced on opposite ends of a neutral conductor are equal; (3) induced and inducing charges are equal.

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[1] This does not contradict the law of distribution found earlier (§373) because this charge on the inner surface is *bound*. Observe also that the total charge within the vessel is still zero since positive charge on the ball and negative charge on the inner surface are equal.

378. Strains in the "Ether." The ice-pail experiment led Faraday to his conception of lines of force. Out of each unit charge proceed 4π electric lines of force, just as magnetic lines proceed from magnet poles. They leave positive charges, enter negative ones. The lines tend to contract and pull opposite charges together. The sidewise thrusts between neighboring lines force like charges apart (Fig. **31**–1). By this theory the induced and inducing charges *must* be equal; every line of force that begins on the ball terminates somewhere, on an ice pail, or on a metal knob, or on the walls of the room.

379. History of Electrostatics. Six hundred years before Christ the Greek philosopher Thales found that amber had a "soul" which could be invoked by rubbing. Gilbert investigated this effect and coined the name "electric" (from the Greek word for amber). Von Guericke invented a frictional electric machine. But, showing how little was really known, it was not until 1733 that Du Fay, chief gardener for the king of France, discovered that there were two kinds of charge, which he named "vitreous" and "resinous." In Leyden the Leyden jar was invented. A few years later our distinguished American, Benjamin Franklin, drew electricity down his kite string from the clouds and showed that the thunderbolts of Jove were simply long electric sparks. He it was who first named the "vitreous" and "resinous" charges "positive" and "negative." That strange silent man, Cavendish, in England, investigated induction, measured forces between charges, and performed Faraday's ice-pail experiment before Faraday was born, but (as was his custom) he did not publish his results. Then came Faraday, and by 1833, a century after Du Fay's discovery of vitreous and resinous electricity, the fundamental facts about electricity at rest were fairly well established.

These earlier experiments on rubber rods attracting bits of paper and pith balls seemed rather unimportant at the time, not likely to play a role in human affairs. Near the end of the eighteenth century, Galvani observed a twitching in the leg of a dead frog (§473). And out of this came the study of electrical currents and an electrical age.

ELECTROSTATIC FORCES IN ATOMS AND MOLECULES*[1]

When men learned to harness electricity and make it turn fans and sewing machines and carpet sweepers and automobile starters and car wheels and drills and lathes and all the many things that electricity does turn today, it was not these forces of electrostatics which were used but the magnetic fields of electromagnets. It has never been possible to build an electrostatic motor larger than a toy. Nevertheless these forces of attraction and repulsion between charges are of supreme importance in making the world about us. The atoms themselves are simply small groups of electrons attracted to a positive nucleus. We can find in the world of the atom and molecule illustrations of all the laws of this chapter.

380. Atomic Nuclei. The fundamental building stones of nature appear to be the negatively charged *electron*, the much heavier, positively charged *proton*, and the uncharged *neutron*, which has about the same mass as the proton. Neutrons and protons combine — in about equal numbers except in the heavier atoms — to form the atomic nucleus.. In the lightest atom, hydrogen, the nucleus is simply the proton itself. In the next heavier element, helium, the nucleus consists of 2 protons and 2 neutrons.[2] It is 4 times as heavy as the hydrogen nucleus (1 proton) and has a *double* positive charge. In the next element (lithium) there are 3 protons (and either 3 or 4 neutrons). It is thus triply charged. And so on — nuclei have been found with positive charges numbering all the way from 1 (hydrogen) to 92 (uranium) (with only two omissions). The charge on the nucleus is equal to the atomic number. The composition of simple nuclei is shown in Fig. 8.

381. Atoms. Electrons are attracted to these nuclei. In number they are equal to the number of proton charges in the nucleus — 1 electron in hydrogen, 2 in helium, and so on. Each electron is attracted toward the positive charge at the center and repelled to a certain extent by the like charge of every other electron. The electrons do not fall into the nucleus because they are in motion and the electrical attraction furnishes the centripetal force which holds them in definite orbits about the center. They move like so many negatively charged pith balls whirling about a positive charge, tending to fly off at a tangent but restrained by electrical attraction. The attractive force varies inversely as the square of the distance, and so the electrons move about the nucleus as planets in the solar system move about the sun.[3]

There is, however, a certain regularity in the size of these orbits which we do not find in pith balls revolving about an attracting charge or in the solar system. The innermost electrons have an angular momentum of about 10^{-27} c.g.s. unit, and in larger orbits the angular momentum is always a multiple of this. These orbits and no others are possible. (This is a consequence of "Quantum Mechanics," Chapter 55.) So the electrons group themselves into definite "shells." There can be as many as 2 electrons in the first shell, 8 in the second, 18 in the next, and so on. This accounts for the periodic table, with periods of 2, 8, 8, 18, 18, 32 elements. (In the building up of elements only 8 electrons enter the third shell — completing a "sub-shell" — before the fourth shell is begun.)

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[1] The remainder of this chapter logically belongs here in electrostatics. Nevertheless it is suggested that these sections (and also the concluding sections of the next chapter) be omitted at a first reading. They may well be studied in a review of electrostatics, after the student has obtained some familiarity with electrodynamics.

[2] This is an extremely stable combination; it is called an "alpha (α) particle."

[3] This is the "Bohr model" of the atom, which, though today known to be only an approximate representation, remains the best *simple* model and explains, approximately at least, many atomic properties.

We are not at present concerned with this restriction to allowable orbits. Each electron pursues its path about the nucleus held in its orbit by electrostatic force. For the hydrogen atom, with a single electron (negative charge e) revolving about a single proton (positive charge e) the force is e^2/r^2, centripetal force holding the electron in its circular path (Coulomb's Law).

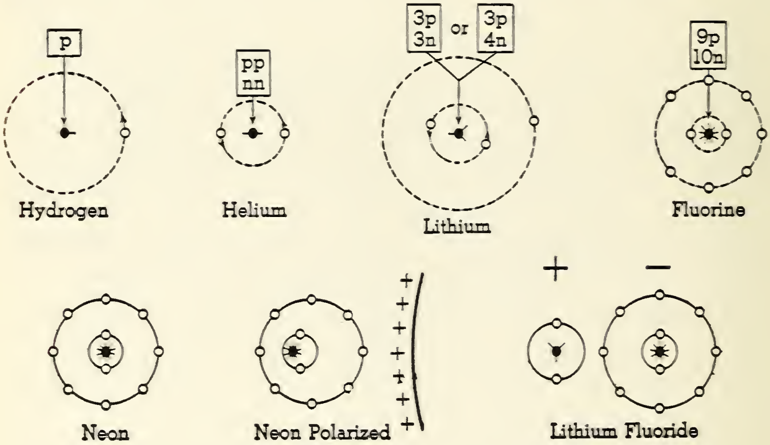

FIG. 8. (*Above*) Simple atomic models. (*Below*) (1) Two inner "shells" completed in neon. (2) Atom becomes a dipole in electric field. (3) LiF molecule formed by attraction of oppositely charged ions.

382. Ionic Attraction. Frictional electricity is due to the different "affinities" of different surfaces for electrons. When rubber touches wool a few of the electrons on the surface, at points of contact, leave the wool and adhere to the rubber, leaving the wool positive and making the rubber negative. A similar phenomenon occurs between individual atoms. Each atom has its own characteristic affinity for electrons, the metals least, the non-metals most. In a metal atom there is always at least one loosely bound electron in an outer shell (for example lithium in Fig. 8); such outer electrons are easily removed, and this leaves the atom with a positive charge. (A charged atom or molecule is called an *ion*.) The outer electrons in fluorine (Fig. 8) are closer to the nucleus and more tightly bound. Fluorine readily takes on another electron (completing its subshell of 8) and becomes a negative ion. If a free atom of lithium and a free atom of fluorine are brought together the fluorine steals an electron from the lithium, becomes negatively charged (F^-) and leaves the lithium positively charged (Li^+). These oppositely charged ions now attract each other and form the molecule Li^+F^- (lithium fluoride). Although it must not be supposed that all chemical forces can be explained thus, simply as due to electrical attraction, the attraction between oppositely charged ions does account for a wide variety of compounds between metals and non-metals which the chemist classifies as *salts.* (Common salt is sodium chloride, Na^+Cl^-.)

383. Atomic and Molecular Polarization. The phenomenon of electrical induction is found in the atom and molecule. The atom is a swarm of electrons about a nucleus. When the atom is in an electric field, for example near a positively charged body, the electrons are attracted slightly and the nucleus repelled, with the result that the negative charge is no longer quite symmetrically distributed about the positive. This is represented for neon in Fig. 8. One side of the atom becomes preponderantly negative, the other side positive; though there are here no free electrons, in effect the atom is now somewhat like a metal sphere with opposite charges induced on the two sides. As to external effects the swarm of electrons about the nucleus may be considered as if located at a single central point. Normally (in the free atom) this negative center coincides with the nucleus, and the opposite charges completely neutralize each other as regards external effects. But in an electric field the negative and positive centers are displaced. This positive-negative pair is called an *electric dipole.* (It might perhaps better be called a " dicharge " — i.e., double charge — to avoid confusion with the *N–S* dipoles of magnetism.) The amount of the displacement (the strength of the dipole) increases in proportion to the strength of the electric field.

This phenomenon occurs in any substance, solid, liquid or gas, in an electric field. Each atom in each molecule becomes a dipole, and all are alined in the direction of the field. The medium is then said to be polarized.[1] But certain kinds of molecules are already dipoles even in the absence of an electric field. For instance, we have seen that in salts (for example, Na^+Cl^-, or Li^+F^- in the figure) the atoms themselves are charged. One end of this molecule is positive, the other end negative. But, except in an electric field, such molecules in a body are oriented at random,[2] and the body is not polarized. The field more or less alines them and polarizes the medium. For such " polar molecules " these effects of polarization are considerably greater than for those in which the dipole must be created by the field.

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[1] It is important to distinguish between this electrical induction operating on the captive electrons in the atoms (polarization) and the electrical induction which causes a displacement of free electrons in metals. We shall in general use the term electrical induction only in this latter connection.

[2] In a crystal there is a regular arrangement of the atoms but it is ordinarily not such as to give a definite alinement of the dipoles. However, when a crystal (for example, quartz) is subjected to a strain there is such an alinement and the crystal becomes polarized, with positive and negative faces. This phenomenon (" piezoelectricity ") was discovered by Pierre Curie (co-discoverer of radium). It has a number of practical applications. When such a crystal vibrates (standing wave) there is a rapid fluctuation in the polariza-

Polarization accounts for the attraction of sawdust and paper by a charged body (§363). The negatively charged rubber rod either produces dipoles or orients polar molecules, brings the positive end of each molecule nearer, the negative end farther away; the nearer unlike charges then attract. Polarization also accounts for the properties of dielectrics as will be shown in the next chapter.

384. Ionic Dissociation. Because of the extremely high dielectric constant of water (Table 35) the forces between charges are much less in water than in air. This is rather difficult to show directly because even the purest water conducts slightly. But when a salt, with opposite ions bound together by electrical attraction, is dissolved in water, the forces between the ions are so greatly reduced that a certain proportion of the molecules break up (" dissociate "). When common salt (Na^+Cl^-) is dissolved in water, one finds free sodium ions (+) and free chlorine ions (−) in the solution. As we shall find in Chapter 36, the presence of the ions makes the solution a conductor.

385. Metallic Conductors. The diagrams in Fig. 8 represent the individual free atoms or molecules. When metallic atoms unite to form the solid state, it is as positive ions that they assume their regular array in the metallic crystal; their outer, loosely bound, electrons are freed to wander about at random among them. These free electrons act like molecules of a very light gas. Their speed depends on the temperature. They are about 3600 times lighter than hydrogen *molecules*, hence (§218) move about 60 times faster or at ordinary temperatures about 60 miles per sec. (This is according to the usual mechanics; the New Quantum Mechanics shows that the actual average speed is about ten times greater.) But this speed has little or nothing to do with the speed of propagation of electric currents. When additional electrons enter one end of a long wire they repel other electrons; these others and all the electrons in the wire begin to drift, and almost instantly other electrons leave the other end. The repulsive impulse which sets the electrons in motion passes along the wire with the speed of light (3×10^{10} cm per sec. or 186,000 miles per sec.).

The laws of electrostatics go far toward explaining many atomic phenomena. There has been in recent years a vast amount of research in these fields, and some of these rather too simple concepts of electrons and atoms have been modified. What we have given are the primitive models — of electrons in atoms, molecules, metals — out of which the newer theories have developed. These simple models will suffice for our purposes.

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tion. This alternating surface electrification is used for frequency control in radio transmission (§545). Crystals are also used in high-fidelity microphones (telephone transmitters).

The converse effect is the production of vibration by polarization, and this is used as the source for generating supersonic waves (§307).

QUESTIONS

1. Mention several respects in which a magnetic pole and an electric charge differ in behavior.

2. Explain the steps in charging an electroscope positively by induction.

3. Explain why a charged rubber rod attracts a piece of foil.

4. If a charged metal ball is held above a piece of metal foil on a metal plate, the foil is attracted to the ball, touches it and flies away to the plate, then back to the ball, and so on. Why? Will the process continue indefinitely?

5. If a charged proof plane is touched to the inside of a metal can, it will give up all its charge; if to the outside, only part of it. Why?

6. A charged body is brought near an insulated metal sphere and attracts it. Why will the attraction be greater if the sphere is touched with the hand?

7. Two oppositely charged spheres are some distance apart. Why will the attraction between them be increased if a third sphere, either conductor or insulator, is placed between them?

8. Explain in terms of motions of electrons what happens in the induction experiments illustrated in Fig. 7.

9. A positive charge is brought near a small body. This body now has an induced negative charge over its whole outer surface to greater or less degree, if this body is: (*a*) an insulator; (*b*) a conductor held in the hand; (*c*) a conductor only if it is supported by an insulator.

10. Faraday's ice-pail experiment shows that there is an equality between: (*a*) the charges induced on the rubber and wool by contact; (*b*) the induced and inducing charges; (*c*) the charges on outer and inner surfaces.

Vocabulary: Positive (negative) charge, electron, statcoulomb, conductor, insulator, dielectric, dielectric constant, current, electroscope, electrostatic induction.

PROBLEMS

1. What happens to the force between two charges if one is doubled, the other halved, and their separation doubled?

2. Compare the forces between two charges placed 2 cm apart and charges each three times as great placed 1 cm apart.

3. What force acts between two charges of 1000 statcoulombs 10 cm apart?

4. What is the attraction between two charges of 40 and −100 statcoulombs 10 cm apart? 20 cm apart?

5. Two balls, with charges of 60 and − 20 units, respectively, are placed 4 cm apart in paraffin oil; what is the attraction between them?

6. A piece of tinfoil weighing a milligram holds a charge of 4 units; 10 cm above it is a metal ball. Approximately how much charge must be placed on the ball for it to lift the foil?

7. The spheres in Fig. 31–8 were given opposite charges of 3,000,000 statcoulombs; find the force of attraction in pounds if the spheres are 15 meters apart (between centers).

8. Three charges, +10, +10, and −10, are placed in line, in this order, and 5 cm apart. Find the force on each of the charges due to the two others.

9. The three charges in problem 8 are placed at the corners of a 5-cm equilateral triangle. Find the direction and magnitude of the force on each charge.

10. An electron has a charge of about 5×10^{-10} statcoulomb. How many electrons does it take to deflect a typical electroscope (§369)?

11. A piece of rubber is lifted from a glass plate and, in separating, steals 2 billion electrons (problem 10). What is the force between the two when they are 5 cm apart?

12. Two pith balls, 2 cm apart, have charges of 4 and −10 electrostatic units respectively. What force acts between them? Suppose that 4 billion electrons (problem 10) are added to each; what will the force now be?

13. What is the force between two electrons (problem 10) 1 angstrom (10^{-8} cm) apart? The radius of the electron orbit in hydrogen is about 5×10^{-9} cm; find the force between proton and electron.

CHAPTER 31

ELECTRICAL POTENTIAL ENERGY

The energy of electricity is more important to us than the electricity itself. We purchase not electricity but energy from the Light and Power Company. In this chapter we shall study the energy of electric charges. Our discussion must be introduced with a description of the electric force field.

THE ELECTRIC FIELD

386. General Properties of the Electric Field. Around a charge is an *electric field*, a region in which another charge experiences a force. A *positive charge* is now used (instead of a *north pole* as for the magnetic field) to plot the direction of the field. We draw lines of force marking the direction in which the force acts on the + charge; we may consider the lines (as Faraday did) as stretched bands (strains in the ether) pulling unlike charges together, or merely as convenient representations of electrical force. Bits of paper, or thread, or tinfoil, can be used to demonstrate the lines of force, as iron filings were for the magnetic field. The electric field resembles the magnetic field in several properties; these we can summarize briefly:

(1) Definition: *Strength of field* (E)*:* force which will act on a unit positive charge placed at the point.

(2) Law: Field strength in air or vacuum near a charged particle:[1]

$$E = \frac{q'}{r^2}. \tag{6}$$

(3) Lines of force: leave positive charges, terminate on negative charges. The number of lines of force per unit area represents the strength of field E. Number of lines of force from one statcoulomb: 4π.

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[1] To distinguish between the charge which is the cause of the field and that which is acted upon by the field, we shall for the present use the primed letter to represent the former.

(4) The force on a charge (q) in the field: $F = qE$ (from definition of E).

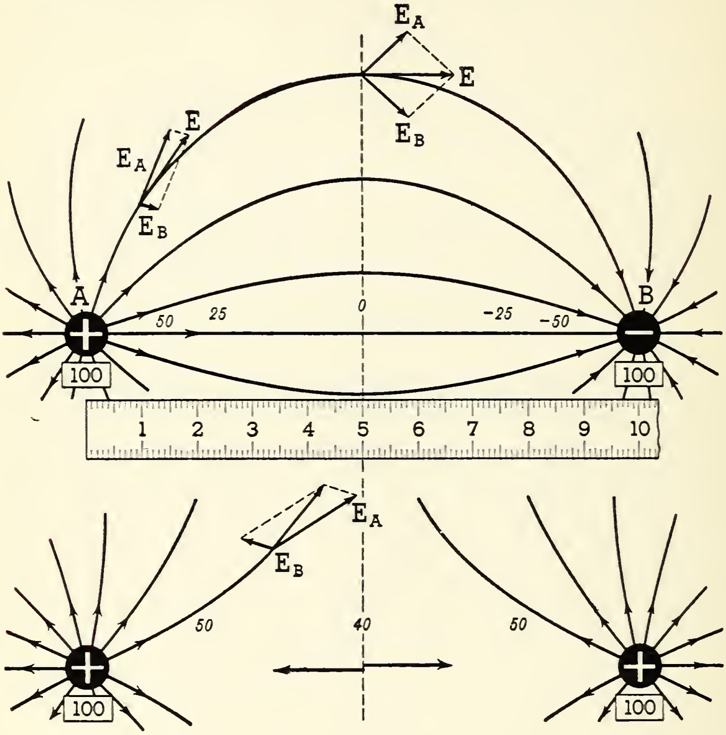

FIG. 1. Electric fields near charges. (Numbers in fields represent potentials.)

Examples. 1. The field strength at the midpoint between two opposite charges 10 cm apart (Fig. 1, above scale) can be computed. The field here due to the positive charge (E_A) is $\frac{100}{25} = 4 \rightarrow$; that due to the negative is the same, and the total field is $8 \rightarrow$. How much force will be required to hold a charge of $+10$ at this point?

2. In general a vector addition is required. Consider the point near the top of the figure, 7 cm from each charge. E_A is $\frac{100}{7^2} = 2 \nearrow$, E_B is $2 \searrow$, and the total field is $2.8 \rightarrow$. In what direction will an *electron* move at this point?

3. The field midway between two like charges is zero because the two repulsions act here in opposite directions (lower figure).

387. Electric Field Near Conductors. To these we must add several other characteristics of electrostatic fields which do not apply to a magnetic field (there are no conductors in magnetism):

(5) There is no field (lines of force) within a conducting medium. (For in the conductor charges are free to move and any such electrical strain would be at once relieved.)

(6) There are no lines of force inside a hollow conductor. (For lines of force terminate on charges, and as was shown in the last chapter there are no charges on the inner surface.)

(7) Lines of force leave a conductor perpendicular to the surface.

(8) The field is most intense near points. (The charge is strongest here, and 4π lines leave each unit charge.) Sparks occur most readily between points. (Whenever the electric field in air at atmospheric pressure exceeds 100 dynes per statcoulomb a spark occurs.)

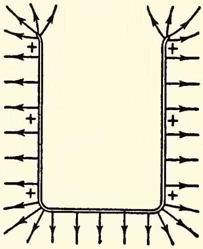

FIG. 2

(9) Between two neighboring parallel plates which are oppositely charged the field is of uniform strength except near the edges. A negatively charged body in the field represented in Fig. 3 would experience an upward force of qE dynes between the plates. If the force equaled the weight, the body would be suspended in mid-air. Upon this fact is based one of the fundamental experiments of modern physics.

388. The Charge on the Electron. Probably many people have wondered how scientists can weigh the individual atom, measure its size, measure the charge on the electron and proton. The atomic world is a world of exceedingly small magnitudes; these atomic magnitudes are interrelated so that many of the others are found rather easily after one or two quantities are found. In 1909 Millikan (at the University of Chicago) measured the charge on the electron. Above the upper plate in Fig. 3, Millikan produced a fine mist of oil droplets. At first the plates were uncharged. The mist settled slowly and some of the particles fell through the hole down toward the lower plate. Here Millikan observed them

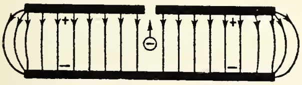

FIG. 3. Millikan's oil-drop experiment.

with his microscope and measured their rate of fall. The rate at which such a particle settles down through the air depends on its weight, and so, from its rate of fall, Millikan could find the weight of any individual oil particle. (The law of fall follows directly from Stokes' Law for motion in a viscous medium [§155].) This constituted the first part of the experiment, and an extensive investigation was carried out validating this result.

Most of the oil droplets were charged. When the larger drops were broken up in the spray to form the mist the parts became charged, a kind of frictional electricity (§366). Millikan charged his plates, and by adjusting the strength of the field between them he was able to arrest the fall of any given drop and hold it suspended in mid-air, the upward attraction just compensating for the weight. The electrical force in the field is qE; hence

$$qE = \text{weight in dynes};$$

and, the weight of the droplet and the strength of the field being known, the charge (q) on the droplet can be found. For example, Millikan found that if he used a field of (say) 10 units this field would in many instances just balance a droplet weighing 4.8×10^{-9} dyne. Hence $q = 4.8 \times 10^{-10}$ unit. Sometimes this field would support droplets twice as heavy or three times as heavy (which Millikan naturally assumed had double or triple charges); some droplets were uncharged; but never was a charged droplet found carrying a charge smaller than 4.8×10^{-10} e.s. unit. Therefore Millikan took this as the charge of the individual electron.

Thus the element of charge (usually represented by e) is 4.80×10^{-10} (in round numbers 5×10^{-10}) statcoulomb. Or (taking the reciprocal) there are about 2 billion electrons in a statcoulomb.

In its actual performance, the famous "oil-drop experiment" was somewhat different from that described. We have given its general principle — it was really the weighing of minute oil drops first under the influence of gravity alone and then in an electric field. We have given the presently accepted value of e which is about 1 per cent higher than Millikan's original value.

The charge on the electrons being known, the masses of the atoms could be found; the method is described in a later chapter. Some years later Millikan was awarded the Nobel Prize.

PROBLEMS

1. What is the field strength 1, 2, 3, and 4 cm from a charge of −12 units?

2. What is the force on a charge of −6 units placed at each of these points (problem 1)?

3. What is the field strength midway between a charge of +10 and −10 statcoulombs placed 4 cm apart?

4. Three charges, +8, +8, and −8 units, are placed in this order at intervals of 2 cm along a straight line. What is the strength of the field at each of the charges due to the two others?

5. What are the magnitude and direction of the force on each charge in problem 4?

6. If one statcoulomb is placed on a small sphere 1 mm in radius, how many lines of force per square centimeter leave the surface, and what is the strength of the field at the surface? What is the field strength 1 cm from the center of the sphere?

7. A charge of +10 units is placed on an insulated ball inside a metal cylinder and +5 units of charge are placed on the cylinder. (*a*) How many lines of force will leave the ball? (*b*) How many lines will leave or enter the inside surface of the cylinder? (*c*) The outside? (*d*) If the cylinder is touched with the finger how much charge will leave it?

POTENTIAL

From this brief summary, from the study of the diagrams and from the analogy with magnetic fields, it is to be hoped that the student has now at least a passing acquaintance with electric fields and with the lines of force about positive and negative charges about charged conductors. All this has to do with the *force* on an electric charge. We now turn to consideration of electrical energy—the *work* done in carrying a charge from point to point in the field.

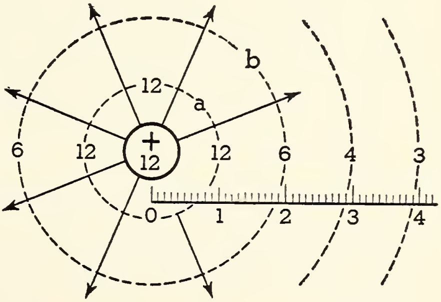

FIG. 4. Potential near charge of 12 statcoulombs.

389. Potential Energy of Electric Charges Two positive charges, near together, can do work as they are pushed apart by the repulsive forces. When near together they have potential energy just as a compressed spring has. The same is true for two negative charges. But unlike charges attract each other; they are more like a stretched spring; their potential energy is greatest when they are farthest apart.

A positive charge has greater potential energy at a than at b in Fig. 4, and at either place the energy is higher than "at infinity." The opposite is true of a negative charge.

390. Electric Potential. For definiteness we shall consider the electrical work done upon a *unit positive* charge. And we shall call its potential energy zero when it is at a great distance from other charges (at infinity). (For definiteness some one definite value must be chosen; for gravitation we called the potential energy zero on the ground or on the floor.) The potential energy of this unit positive charge at any point in the field is called the potential at the point. In other words, the **potential** at a point in space or on a conductor is the work required to carry a unit positive charge from infinity to the point. The numbers in Figs. 1 and 4 represent such potentials. In Fig. 4 the potential at a (and at all points on the "equipotential" circle) is 12 units; this means that 12 ergs of work are required to carry a statcoulomb up to a from infinity. The *difference in potential* between a and b is 6 units, which means that 6 ergs of work are required to carry a unit positive charge from b to a.

As we pass along a line of force the potential increases or decreases. If the test charge is carried along a path which is exactly perpendicular to the lines of force no work is done and all points along the path are at the same potential. So the broken lines in Fig. 4 pass through points of equal potential. In Fig. 1 a charge can be carried from infinity to the midpoint between the unlike charges by such a path always perpendicular to the field; no work need be done, and this point is at zero potential.

391. Negative Potential. High potentials are caused by the proximity of positive charges. On the other hand, the potential energy of the test charge (positive unit) is less near the negative charge in Fig. 1 than at infinity. One must distinguish between the charge on the body and the potential which it produces. For instance, points near or on a negatively charged body might well be at a high potential if a larger positive charge were in the neighborhood. Though the two ends are oppositely charged all parts of the cylinder in Fig. **30**–5 are at the same positive potential.

392. Computation of Potential Difference in a Uniform Field. The strength of the field is the *force* per unit charge. When a charge is carried along a line of force in a uniform field, the field strength (E) multiplied by the distance (s) is the *work* done per unit charge or the potential difference.

$$\text{Potential difference} = E \times s \quad \text{(for uniform field).} \tag{7}$$

Examples. I. In our description of the oil-drop experiment (Fig. 3), the field strength was considered to be 10 units. If the plates were 2 cm apart the difference in potential between lower and upper plates was 20 units (statvolts).

II. Suppose the parallel plates in Fig. 3 have an area A, are d centimeters apart, and hold charges of $+q'$ and $-q'$. Find the field between them and their difference of potential.

(1) The total number of lines of force leaving one plate and entering the other is $4\pi q'$ (§386).

(2) These lines of force may be considered as practically all between the two plates. Hence the number of lines per square centimeter is $\frac{4\pi q'}{A}$. This is the strength of the field E (§386).

(3) The difference of potential between the plates is

$$E \times d = \frac{4\pi q' d}{A}.$$

When the lines of force diverge and the field varies as we go from point to point, the computation is less direct. Let us see how the potential may be computed in one important case.

393. Computation of Potential Near a Charged Particle. The potential in the neighborhood of a charged particle varies inversely as the distance. The exact relation is

$$\text{Potential} = \frac{q'}{r} \quad \text{(about charge located at single point).} \tag{7'}$$

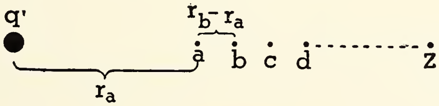

Fig. 5. Potential $= q'/r$.

Proof: We cannot now apply our "force times distance" relation directly since the force is not constant; it increases as we approach the charge. Consider first the work done as the unit positive charge is carried between two neighboring points, say from b to a. At b the force on unit charge is $\frac{q'}{r_b^2}$;

at a, $\frac{q'}{r_a^2}$. As a mean value, suitable for this computation, make a compromise in the denominator and take the mean force[1] throughout this interval as $\frac{q}{r_a r_b}$. The distance is $r_b - r_a$, and the work is

$$W = \frac{q'}{r_a r_b}(r_b - r_a) = \frac{q'}{r_a} - \frac{q'}{r_b}.$$

This is the difference in potential between a and b.

Similarly, going from r_c to r_b the work is $\frac{q'}{r_b} - \frac{q'}{r_c}$, and so on. The work in going to a from some distant point z is

$$\frac{q'}{r_a} - \frac{q'}{r_b} + \frac{q'}{r_b} - \frac{q'}{r_c} + \frac{q'}{r_c} - \dots + \dots - \frac{q'}{r_z} = \frac{q'}{r_a} - \frac{q'}{r_z}.$$

If now we suppose r_z to be infinite we get the work in carrying our unit charge from infinity to r_a (potential at a); its value is

$$\text{Potential at point } a = \frac{q'}{r_a} - \frac{q'}{\infty} = \frac{q'}{r_a}.$$

Compare this relation with the expression (q'/r_a^2) for the *force* on a unit charge at distance r_a. Potential varies inversely with the distance. One centimeter from a charge of 100 statcoulombs the potential is 100 units. The potential energy of a unit + charge placed here is 100 ergs. Two centimeters away the potential is 50 units; 10 centimeters away it is 10 units; etc. What is the difference in potential between a point 10 centimeters and 2 centimeters away?

This relation between potential and distance is shown clearly in Fig. 4.

394. Unit of Potential. Potential (or the difference in potential) is measured in *ergs per statcoulomb.* In the practical system of units another unit will be used, the volt. In our present electrostatic system of units the unit is called the statvolt. One statvolt means a potential of 1 erg per statcoulomb.[2]

Potential, like work and energy, is a scalar quantity. This makes the computation of potential simpler than that of field strength. At the midpoint between the *like* charges in Fig. 1 (lower) the potential due to charge A is Pot. $_{(A)} = \frac{100}{5} = 20$ ergs per statcoulomb (or statvolts); that due to B is the same; total potential 40 statvolts. Forty ergs of work are required to carry a unit charge up to this point. . . Midway between the *unlike* charges (Fig. 1, upper) the potential due to A is 20 statvolts; that due to B is -20 statvolts; total potential zero. A charge can be brought to this point without doing work.

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[1] This is the "geometric mean"; its use is justified when the computation is made exactly, using calculus.

[2] One statvolt equals 300 volts.

395. Work and Difference of Potential. It must be clearly understood that the electric field strength and electric potential have for definiteness been consistently described in terms of the effect on a unit *positive* charge. A + charge tends to move along the lines of force, a − charge in the opposite direction. Potential is a measure of the potential energy of a unit positive charge. The *potential energy* of positive charges is greatest at points of high potential; the opposite is true of a negative charge. (Work would be done not by us but for us if a negative charge were brought up from infinity to point a in Fig. 4; we must *do* work in carrying the negative charge *away*.) Charges always tend to move from points of higher to lower potential energy (just as balls tend to roll downhill): *positive* charges tend to move down the lines of force *from high potential to low;* negative charges tend to move up from low potential to high. When in Fig. 4 we carry a + charge in from the right edge of the figure (low potential to high) we do work against the field. When the + charge moves (as it tends to do) in the contrary direction the electrical field does work on the charge (electrical work).

The amount of work done depends upon the charge carried and the potential difference between the two points. The electrical work done on a unit positive charge when it moves from a potential of 12 statvolts (as at a) to 3 statvolts (as at the edge of the figure) is 9 ergs. The work on any charge q carried between two points in the field is equal to

$$W = q \times \text{Potential difference.}$$

It is customary to represent the potential difference by the letter V:

$$W = qV. \quad \text{(From Def.)} \qquad (8)$$

396. Electrostatic Potentials in Conductors. The following laws apply only to electricity at rest (electrostatics).

(1) The potential is the same at all parts of a conductor. (For there are no lines of force in a conductor and no work

is required to carry a charge from one part of the conductor to another.) This law is reminiscent of the laws of hydrostatics — water seeks the same level; electricity seeks the same potential.

(2) The potential is the same at all points inside a hollow conducting vessel. (This is for the same reason as [1]; the field strength within the vessel is zero.)

(3) A conductor connected to the ground is at zero potential. (For the ground is a conductor and, extending to great distances, must be at zero potential.)

In other words, no work is required to carry a charge from one point to another on or within a charged conductor; it takes the same work to carry a charge from infinity to any point on or within the conductor; if the conductor is grounded this work is zero.

Since the potential at all parts of a conductor is the same, it is customary to speak of the potential " of the conductor," meaning this common value for all points on it or within it.

397. Potential and Pressure. Electrical potential is analogous to pressure in a liquid. Potential is potential energy per unit positive charge; pressure is potential energy per unit volume (§147). Positive electricity (or a fluid) flows from points of high potential (or pressure) to low. The flow of a liquid in a pipe depends on the *difference in pressure* between the two ends of the pipe. We shall find that the flow of electricity in a conductor is proportional to the *difference in potential* between its two ends. The hydraulic work is: Volume $\times \Delta P$ (§147); the electrical work is: Charge $\times\ \Delta$ Potential (or QV).

398. Electric Sparks. Too great a difference in the hydrostatic pressures on two sides of a wall may break the wall; similarly, when great potential differences exist across small air gaps, a spark will occur. In air the breakdown occurs for a field strength of 100 statvolts per centimeter (or 30,000 volts per centimeter).[1] If the gap is filled with some other dielectric, such as glass, mica, or oil, a much stronger field is required to produce a spark through the insulator. If the dielectric is a solid it is punctured by the spark.

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[1] From eq. (7), $E = V/s$; hence the unit of field strength (E) can be expressed as a *statvolt per centimeter* as well as a *dyne per statcoulomb*, the two expressions having identical meanings.

Lightning is an instance of a spark in air. There normally exists in the atmosphere a field of several volts per centimeter (downward). (We do not usually observe this since our laboratory rooms act as hollow conductors.) When the charged clouds of the thunderhead come overhead there is an enormous increase in the field strength, and a spark may occur.

Electricity leaks away from points, from sharp edges, or from fine wires, at comparatively low potentials. This is because of the strong field which exists about a point or an edge. (To avoid this leakage the conductors in a static machine terminate in heavy knobs.) The lightning rod utilizes this property. The rods leading up from the ground terminate above the house in sharp points. They are erected with the expectation that they will dissipate the electricity from the vicinity before the lightning strikes. The leaves of trees and the sharp roofs of houses in the city have the same effect and give a certain amount of protection to their neighborhoods. Lightning is much less likely to strike a house in the city than a house in the open, treeless country.

399. Static Machines. 1. *The Electrophorus.* Charge a hard-rubber plate. Bring near a large metal disc held by an insulating handle. Ground the disc momentarily. Negative charge flows off; positive charge is bound. The disc has been charged by induction. Carry this positive charge to charge a metal vessel or a Leyden jar. The negative charge remains on the rubber plate and the process can be repeated as often as desired. This is the *electrophorus.* It was invented by the Italian physicist, Volta; it was used for obtaining charges in much of the pioneer work in electrostatics and illustrates the principle of the static machine.

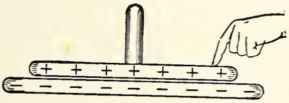


FIG. 6. Electrophorus.

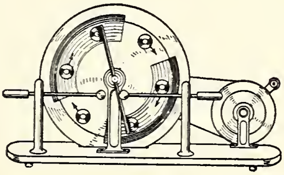


FIG. 7. Static machine.

2. *Static Machines.* Figure 7 shows a commonly used static machine. The static machine is really an automatic electro-

phorus. Two pieces of paper shellacked to the surface of a stationary glass plate take the place of the rubber plate. Small metal discs on a revolving glass plate transport the induced charges. These induced charges are bound on the discs as they pass near the charged paper; free charges flow off to the knobs.

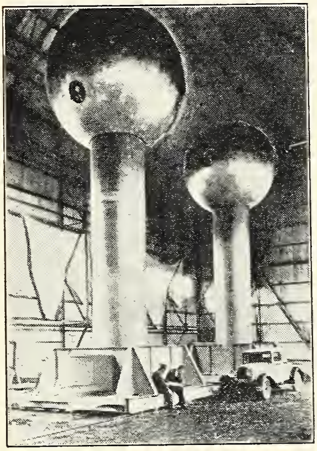


FIG. 8. Large electrostatic machine for nuclear investigations (Round Hill, Mass.)

A huge static machine has been built which gives a difference of potential of several thousand statvolts (about 10,000,000 volts) and produces electric sparks several meters long. Supported on heavy trucks are two large metal spheres, the " knobs " of the static machine. The spheres are supported by large insulating columns; within each column is an endless belt, motor driven, which carries charges up to the sphere. There the charge flows from the belt to the outside of the hollow conductor. One sphere is 5,000,000 volts above ground; the other 5,000,000 volts below. An intense electric field is excited throughout the neighborhood except of course within the balls themselves. These interiors are fitted as laboratories, and here the investigators stay to make their observations. This static machine was designed to aid in the study of the nucleus of the atom. (A number of other machines, similar but not quite so large, are in operation in laboratories in this country.) The apparatus is quite as remarkable for the large quantity of electricity which it transports on its fast-driven belts as for the potential to which it raises this charge. With a single belt running at full speed it carries 1 millicoulomb (3,000,000 statcoulombs) of charge per second.

400. Electrostatic Voltmeters. An instrument to determine difference of potential is called a voltmeter. The electroscope is in reality a voltmeter. Its deflection depends on the potential of any conducting body to which it is connected. (It obviously does not measure all the *charge* which is on the body any more than a water-level gauge measures all the water in a boiler.)

The usual electrostatic voltmeter is a more rugged form of electroscope in which a needle takes the place of the metal foil. A scale is provided from which the potential can be read directly in volts. The instrument serves to measure potentials up to perhaps 6000 volts (20 statvolts).

A much more sensitive instrument is the quadrant electrometer. Here a light vane, suspended from a fine quartz fiber, hangs within a pillbox-shaped conductor. The "pillbox" is divided into four quarters, opposite quadrants connected together. A charge is put on the vane, and whenever a difference of potential exists between the neighboring quadrants the vane is deflected in the field created between them. One can easily measure a millionth part of a statvolt on this instrument; electrometers have been built so sensitive that they will indicate the arrival of electrons at the rate of only one or two per second.

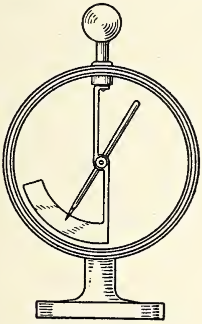


FIG. 9. Electrostatic voltmeter.

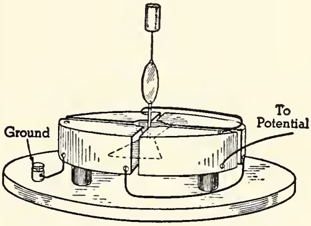


FIG. 10. Quadrants of electrometer.

401. Electrostatic Induction. With our knowledge of electric potentials we are now able to appreciate more clearly the meaning of induced charge. When the charged body A, in Fig. 30–5, is brought near the neutral conductor, momentarily — before conduction occurs — the potential at a (nearer A) is greater than at c. (The student should always analyze the meaning of potential — would it take work to carry a positive charge to the point in question?) Hence positive charge flows from a to c. This lowers the high potential, raises the lower, until both ends are at the same potential. It then takes the same work to carry a + charge from infinity to a as to b or to c. (The negative charge on a partially neutralizes the strong repulsion of A when we bring a + charge to the left end of the cylinder.)

a, b, and c are now all at a rather high potential. Hence if they are connected to the ground positive charge flows off (or, more accurately, negative charge flows on) until the potential of all points is reduced to zero. a now has an increased negative charge, b a negative charge, and the charge at c is reduced to zero or slightly negative. The conductor as a whole has been charged negatively by induction.

CONDENSERS

402. Capacitance. In electrostatic experiments we desire conductors on which we can store considerable quantities of electricity. We are limited by the fact that the potential of the conductor rises as charge flows onto it, very much as the level of a lake is raised by the inflow of water or the pressure in a bottle is increased when more gas is forced into it. The quantity of charge (Q) which a conductor will hold for unit rise in potential is called its **capacity** or, to use the more recent nomenclature, its **capacitance** (C).

$$C = \frac{Q}{V}. \qquad \text{(Def.)}$$

Capacitance is measured in terms of stat*farads*. A metal ball 1 centimeter in radius has a capacitance of 1 statfarad; 1 statcoulomb of charge raises its potential by 1 statvolt. The capacitance of a conductor increases with its size. But much greater capacitances are obtained by making use of the property of induction.

403. The Condenser. When a + charge is placed upon a plate (A) its potential is raised. However, in Fig. 11 another grounded plate (B) is placed close by, and on this an almost equal − charge is induced. This charge lowers the potential of A until it is only a little above zero (the potential of B). Because of the very close proximity of the grounded plate B a much greater charge can be placed on A before its potential rises to a high value.

Such a device is called a *condenser*. The Leyden jar is one form of condenser. Both the inner and the outer surface of the glass jar are coated with metal foil; when the outer coating is grounded a large amount of electricity can be placed on the inner surface before the potential becomes so high that sparking takes place. An ordinary Leyden jar may have a capacitance of 1000 statfarads. This is as much as a sphere with 10-meter radius. Such a jar holds 1000 statcoulombs for every statvolt of potential. It may easily be charged to a potential of 100 statvolts, and then it stores some 100,000 statcoulombs.

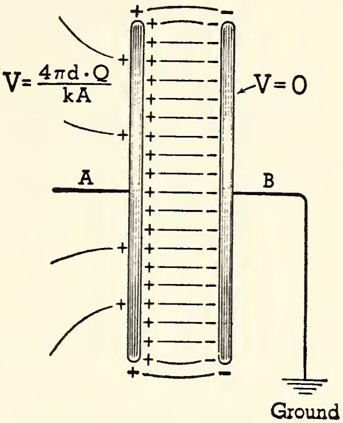


FIG. 11. Plate condenser.

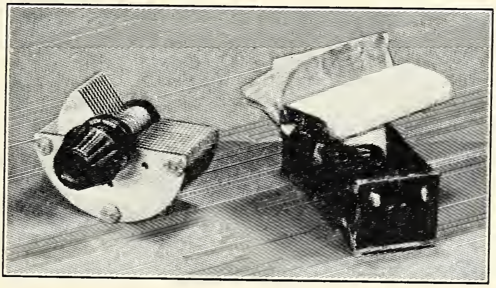


FIG. 12. Condensers: variable condenser for radio and fixed condenser, opened to show tinfoil-paper construction.

Condensers are made in a variety of other forms. Variable condensers consist of two sets of interpenetrating plates; one set is fixed, the other (grounded) set is movable, and as these grounded plates are turned to penetrate farther and farther between the stationary plates, the capacitance of the latter increases. Such variable condensers are used in radio sets and usually have a maximum capacitance of several hundred statfarads. . . Fixed condensers of large capacitance are made by using for grounded and insulated plates two long strips of tinfoil separated only by the thickness of a piece of thin paraffined paper. The foils and insulating papers are rolled up into a small cylinder, and yet, because of the large area of the foils and small separation, such a condenser, not much larger than a spool of thread, may have a capacitance of one or two million statfarads. . . The electrolytic condenser (§463) has a still greater capacity. Here the insulator is only a film of oxide on the surface of aluminum (or certain other metals). The film may be only 0.0001 mm thick and the capacitance then is about a million statfarads (about 1 microfarad) for each square inch of surface.

404.* Capacitance of Conducting Sphere. The capacitance of a spherical conductor (in electrostatic units) is equal to its radius in centimeters. *Proof:* When charge Q is placed on the sphere it is uniformly distributed and acts for all exterior points as if it were concentrated at the center. Hence the potential at any exterior point is (by eq. 7′) Q/r, where r is the distance from the center; thus the potential at the surface itself is

$$\text{Potential} = \frac{Q}{\text{Radius of sphere}},$$

and the capacitance is

$$\text{Capacitance (statfarads)} = \frac{Q}{\text{Potential}} = \text{Radius of sphere (centimeters).}$$

405.* Capacitance of a Plate Condenser. It has been shown (Example, §392) that the potential difference of two parallel plates is

$$\text{Potential difference} = \frac{4\pi d \times Q}{A},$$

where A is the area and d the distance apart (in air). Thus a charge Q on the insulated plate in Fig. 11 raises its potential to $\frac{4\pi d \times Q}{A}$. Hence the capacitance of the parallel plate condenser *with air between the plates*, is

$$C = \frac{Q}{\text{Potential}} = \frac{A}{4\pi d}\ \text{statfarads.}$$

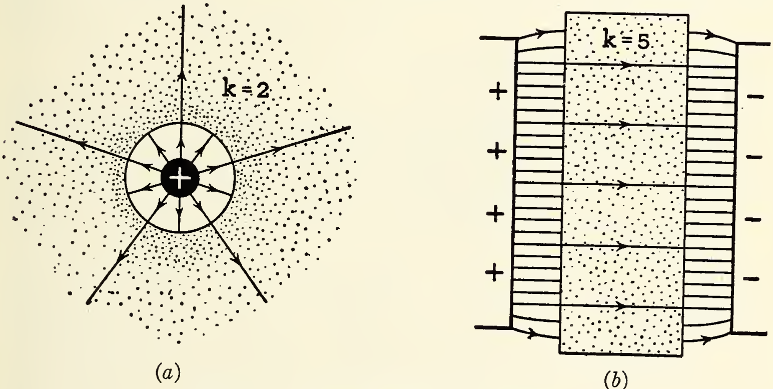
406.* Potential Energy of the Charged Condenser. Suppose that we gradually charge a condenser. While the charge rises from 0 to Q its potential difference rises from 0 to V; the average potential difference during the charging process is $\frac{1}{2}V$. Therefore, the total potential energy of the charged condenser is $\frac{1}{2}QV$. What is the energy (in joules) of a Leyden jar charged to 100 statvolts with 100,000 statcoulombs? What is its capacitance?

Condensers are used for the storage of electricity — but this must not be misunderstood. As compared with the quantities of electricity passing every second through our motors and electric lights, the quantity which can be stored in a condenser is very small. The paraffin paper condenser mentioned above had perhaps a capacitance of 2,000,000 statfarads. With a potential of 1 statvolt (300 volts) it holds 2,000,000 statcoulombs. The energy stored in it ($\frac{1}{2}QV$) is then 1,000,000 ergs. But this is only a tenth of a joule, and a 40-watt lamp requires an expenditure of 40 joules of energy per second. The electricity stored in a Leyden jar is just enough to furnish one good, strong spark. Measured in the units of electrostatics the charge stored in a condenser may not seem extremely small but in comparison with the practical units in common use the statcoulomb is a very small unit.

THE FIELD IN A DIELECTRIC

407.* Field Strength and Potential in Dielectric. The relations which have been developed are for charges in air. In §370 we found that the force between charges was reduced when they were surrounded by an insulating medium (dielectric). Thus when a small charged sphere is surrounded by a dielectric the electric field in the medium is not q/r^2 but q/kr^2. In Fig. 13*a*, a positive charge (q) is placed in a small spherical cavity in a block of paraffin ($k = 2$). From the charge itself we must imagine $4\pi q$ lines of force diverging; in the paraffin there are only half as many lines; the field strength is half as great. The presence of this insulating substance has reduced the intensity of the electrical forces.

Potential differences are also reduced in the dielectric. The work done in carrying a unit charge from point to point in the paraffin ($k = 2$) is half as great as it would be between the same points in air.[1] If we conceive of the medium as extending to infinity the potential at any point (the work done in carrying unit positive charge from infinity) is reduced to half value. In general the potential near a point charge immersed in a large dielectric is



(*a*) (*b*)

FIG. 13. Electric field in a dielectric.

$$\text{Potential} = \frac{q}{kr}.$$

Figure 13*b* represents two condenser plates separated by glass ($k = 5$).

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[1] The reader naturally finds it difficult to conceive carrying a charge from point to point in a solid dielectric. He can imagine a small needle-shaped cavity made parallel to the lines of force and consider a small charged body carried through this. The field in such a cavity is the same as in the dielectric.

Within the dielectric the field is reduced fivefold. Five times less work is required to carry a charge from plate to plate, and the potential difference is reduced fivefold. This means that the capacitance is increased fivefold. Thus the capacitance of a plate condenser, with plates separated by a medium of dielectric constant k, is

$$C = \frac{kA}{4\pi d}.$$

408.* Theory of the Dielectric. Lines of force start from positive charges, end in negative charges. When a dielectric is present there perhaps seems to be an exception to this rule. In Fig. 13 lines of force terminate on the surface of the dielectric upon which, at first thought, there would seem to be no charge. But let us consider what happens when a piece of glass or paraffin is placed in an electric field. We have seen (§383) how the positive and negative charges in each atom are displaced by the field, making of each a "dipole," polarizing the medium. This polarization has introduced no electric charge within the medium; in any small volume the positive and negative charges are equal. The situation is different on the surface. On one surface (Fig. 14) the positive ends of the dipoles protrude, upon the other the negative ends. Thus because of its polarization the dielectric has a surface charge (though it has no free charge), and it is upon this that the lines of force which are lost from the medium begin and end.

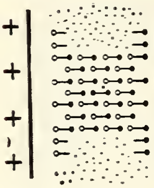

FIG. 14. Polarization of the dielectric.

(Charges:
● positive,
○ negative.)

When glass ($k = 5$) is placed between our charged plates 4 units of opposite surface charge appear for every 5 units of charge on the plates and only one-fifth of the lines enter the medium. In paraffin ($k = 2$) the polarization is less; 1 unit of surface charge appears for 2 units on the plates and the field in the medium is half as strong as the field in air. Observe that in a conductor the induced surface charge *equals* the inducing charge (Faraday's ice-pail experiment) and there is *no* field within the material; this is because the electrons in the conductor are free. In a dielectric, with bound electrons, the "induced charge" due to polarization differs in magnitude from the inducing charge; this charge on the surface is less than the inducing charge by one part out of k, and the field in the dielectric is reduced, not to zero (as in a metal), but to $1/k$ of its original value.

It is convenient and customary to neglect these surface charges due to polarization and to introduce the dielectric constant to take account of this effect. For this charge is bound in the atom; it cannot be removed and tested on a proof plane. Polarization reminds us of magnetization. But it disappears when the electric field is withdrawn — there is no phenomenon of permanent polarization. Sometimes, however, the polarization lingers an appreciable time and, when high-frequency alternating fields are applied, produces "dielectric power loss."

409. Résumé. We purchase energy, not electricity, from the Light and Power Company, and indeed if the company should ask us to accept any appreciable quantity of electricity we should certainly not know what to do with it. The power company sends us electrons with high potential energy. We use these to light our lamps or to turn our motors and return them to be once more given potential energy, to be pushed into the low-potential wire (where these *negative* charges have high energy) and returned to do more work. And we pay, not for statcoulombs, but for joules or for kilowatt-hours of work. Hence the importance of electrical potential.

Potential at a point is the work which must be done pushing a unit of positive charge from infinity up to the point. Its value is high near positive charges, low near negative charges (assuming that these are the only charges in the neighborhood). More important than the absolute value of the potential is the difference of potential (V). The work done when any charge (Q) moves from point to point is QV.

Except as pertains to conductors the properties of the electric field about charges are in many respects like those of the magnetic field about poles. The field strength is zero and the potential is constant within a conductor and within a hollow conducting vessel. About a single charged particle the field strength equals q/r^2 and the potential q/r. Within a dielectric these values are divided by k.

The condenser holds electricity. Its capacitance is the charge it holds per unit rise in potential: $C = Q/V$. The capacitance of a sphere in air equals its radius; of a plate condenser, $\frac{kA}{4\pi d}$ (both in electrostatic units).

QUESTIONS

1. What is the force on 2 statcoulombs in an electric field of 3 units? How much work is required to carry this charge 2 cm against the field in this region?

2. In Fig. 30–5, the left end of the cylinder has a negative charge, yet the potential of all parts of the cylinder is positive. Explain.

3. What is the potential 3 cm from a point charge of 6 units? What does this mean?

4. Why is the potential constant inside a hollow vessel? How strong is the field there?

5. Given two charged metal cylinders of different sizes, connected by a wire. Why will their total capacitance be decreased if placed one inside the

other? Will either the total charge or their potential be changed? Will the capacitance be changed if they are merely set close together? Will the capacitance of a solid ball be greater than that of a hollow one?

6. Consider a roller window shade, insulated and made of conducting material. Does its capacitance change when it is rolled up? Suppose that it has a negative charge on it; will its voltage change?

7. What is the potential on the surface of a metal ball, 3 cm in radius, holding 6 units of charge? What is the potential at the center?

8. How does polarization of a dielectric resemble and differ from the magnetization of a piece of iron?

9. A body connected to the ground necessarily has: (*a*) zero charge on it; (*b*) the same charge as the ground; (*c*) zero potential.

10. When two conductors are brought into contact the electric charge upon them is shared: (*a*) equally; (*b*) in proportion to their capacitances; (*c*) inversely as their capacitances.

Vocabulary: Electric field, field strength, (positive, negative) potential, potential difference, statvolt, condenser, capacitance.

PROBLEMS

1. Find the field strength at points 2, 3, 4 cm distant from a charge of 6 units.

2. Find the potential at the different points in problem 1. How much electrical work is done when 10 statcoulombs (positive) pass from the 2-cm to the 4-cm point?

3. If a 0.1-gram pith ball with 6 statcoulombs of charge starting from the 2-cm point of problem 1 is pushed away by the electric field, what will be its kinetic energy and velocity at the point 4 cm from the charged body? What will be its kinetic energy and its speed after it has completely escaped from the influence of the charge?

4. Find the field strength and potential (*a*) midway between two 100-statcoulomb charges, 4 cm apart; (*b*) between opposite charges $+100$ and -100.

5. The spheres in Fig. 8 were given opposite charges of 3,000,000 statcoulombs; they were 15 meters apart. What was the field strength midway between them? What was the potential here?

6. What is the potential one centimeter above the 3-cm division of Fig. 1? above the 6-cm division? What is the difference in potential between these two points?

7. Ten charges, each 50 units, are distributed on the circumference of a circle whose radius is 10 cm. What is the potential at the center?

8. Assuming that the field strength between the plates in Fig. 2 is 5 electrostatic units, what would be the force here on a droplet holding 10 electrons? If the plates are 3 cm apart what is their potential difference?

9. The capacitance of a telephone condenser is 1,000,000 statfarads. How much electricity flows into the condenser when it is connected to a dry cell which gives a potential difference of $\frac{1}{200}$ statvolt?

10. We have three Leyden jars each with capacitance of 10,000 statfarads. How much electricity do they hold when the plates are charged by a static machine to a potential difference of 10 statvolts? Make a sketch showing how the three jars can be charged simultaneously ("in parallel").

11. What is the capacitance of a metal sphere 10 cm in diameter? If 20 statcoulombs are placed on the sphere, what is its potential? How much work is required to add 2 more statcoulombs to the sphere?

12. What is the capacitance of a plate condenser consisting of two plates 10 by 10 cm separated by 1 mm of mica?

13. If one plate of the condenser in problem 12 is connected to ground and 1000 units of negative charge is placed on the other, what is the potential of this insulated plate? How large a sphere would be required to hold this same charge at this same potential?

14. A Leyden jar is 10 cm in diameter, and the metal coatings extend over the bottom and 20 cm high on the walls. The glass is 3 mm thick. Find its capacitance.

15. By comparison with the other objects in the figure estimate the capacitance of the sphere in Fig. 8.

16. What is the capacitance of the earth?

CHAPTER 32

FLOW OF ELECTRICITY

410. The following experiment is illuminating. A metal sphere is charged until its potential is several ergs per unit charge (i.e., statvolts). This potential can be read on an electrostatic voltmeter. If a meter stick is now placed joining the sphere with the ground, the charge will gradually leak off. (It will be well to give the sphere a larger supply of charge by connecting it to a Leyden jar.) A very simple modification of the electroscope allows us to measure the charge that passes along the stick: a small metal plate (connected to the ground) is placed near the leaf; the leaf receives a charge, deflects until it touches the plate, is thereby grounded, and falls back to be charged again.[1] With a typical electroscope it took 10 units of charge to deflect the leaf to the plate. So by watching the leaf as it hands on charge after charge to the grounded plate we can measure the rate of flow of electricity.

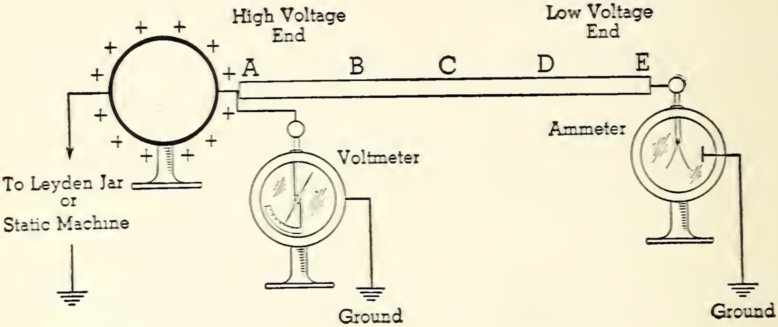

Fig. 1. First experiment in electrodynamics: current through a stick.

411. Electric Current. The rate of flow of electricity is called the *current* (I). The current is the charge (Q) leaving the sphere per second.

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[1] It will be found that an electroscope of the needle type (Fig. 31–9) works most satisfactorily; the gold leaf tends to stick to the plate.

$$I = \frac{Q}{t}. \qquad \text{(Def.)}$$

A special name has been given to the current unit — a statcoulomb per second is called a stat*ampere.*

In a typical experiment the sphere was charged to a potential of 20 units and the current-measuring electroscope mentioned above deflected once (transferring 10 statcoulombs) in 2 seconds. Clearly the current passing through the stick was then 5 statamperes.

412. Ohm's Law. Positive charge moves from high potential to low. We can show in this experiment that the rate of flow (I) is proportional to the difference in potential (V) between A and E (or ground):

$$V = RI. \quad \text{(Ohm's Law.)} \qquad (9)$$

We charge the sphere until its potential is 40 statvolts above ground and the electroscope makes one deflection per second; reduce its potential to 10 units and the electroscope makes one deflection in 4 seconds. (What is the current in each instance?)

The proportionality constant R equals V/I. This ratio depends on the nature of the stick used; it is called the **resistance.** In this experiment the resistance was 2 statvolts per statampere. If the stick contains much moisture its resistance will be relatively low; in dry weather its resistance may be quite high. Resistance is the ratio of difference of potential to current.

413. Resistances in Series and Parallel. The longer the stick the greater its resistance. If the current must pass through two similar sticks placed end to end, the resistance will be doubled. If, on the other hand, two sticks are placed side by side, giving a double path for leakage to the ground, there will be twice the flow. That is, when two similar resistances are placed " in parallel " the resistance is halved. (Compare Fig. 14–14.)

414. Potential Drop along a Conductor. These experiments bring out again the close resemblance between the concepts of potential in electricity and pressure in fluids. Electrical potential is sometimes referred to as " electrical pressure." The current of water which flows through a pipe depends on the pressure difference existing between the two ends. The water current also varies with the resistance offered by the pipe; a long pipe offers more resistance to flow than a short one; two pipes in parallel carry away water twice as fast as one. Water flows from high pressure to low. For water currents and electric currents the reason for the flow is the same — the tendency of both water and electricity to move from places of high potential energy to low. Pressure is the potential energy per unit volume; electrical potential is the potential energy per unit charge.

One further observation upon our meter stick will emphasize the similarity between potential and pressure. In a pipe carrying a current of water the pressure falls gradually from one end to the other (Fig. 14–8). We shall find a similar thing along the meter stick. In the first experiment described above A was 20 statvolts above ground. Change the voltmeter now to point B; we find that the potential of this point is 15 statvolts. The potential of C is 10 statvolts, of D 5 statvolts. What difference of potential will be read if, by connecting the outer case of the voltmeter to B and the needle to A, we measure the difference in potential between A and B? What is the potential difference between B and E? From left to right the potential gradually falls. The charge gradually loses its potential energy as it passes down the stick.

What becomes of this energy? The electrons passing through the stick, striking the molecules, increase the molecular motion and generate heat. In this experiment, where 5 units of charge passed per second from a potential of 20 ergs per unit charge (statvolts) to zero, 100 ergs of potential energy is lost per second and 100 ergs of heat energy added to the stick. (This is only 100 ÷ 42,000,000 calories per second and cannot be detected. If the stick had lower resistance, and more current flowed, the generation of heat per second would be greater.) In any case

$$\text{Heat (in ergs) per second} = \frac{Q}{t}V \quad (\text{or } IV),$$

where Q/t is the current and V the potential difference.

415. Source of Electromotive Force. In this experiment the potential of the charged sphere gradually decreases as more and more charge leaves it, and the current diminishes. To avoid this, a static machine can be connected to the sphere and can be operated continuously during the experiment, raising charge to the high-potential sphere as rapidly as charge leaks away. (This is represented in conventional manner in Fig. 2.) Any device which raises the potential in this fashion is called a *source of electromotive force* (e.m.f.). (The name is perhaps a misnomer; it is actually a source of electromotive potential.)

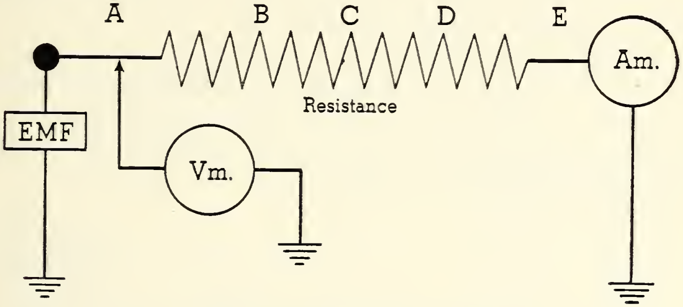

FIG. 2. Conventional diagram.

416. Larger Currents. To demonstrate certain other characteristics of electricity in motion we shall need currents billions of times greater than these. We could substitute a wire of low resistance for the meter stick, but this would almost instantly discharge the Leyden jar. Nor could the static machine be operated fast enough to maintain the potential. Something is needed which will transfer charge much more rapidly than a static machine does — it need not furnish such high potentials. When (1800) the Italian physicist Volta found that two different metals dipped in an acid would give a potential difference and would furnish electricity millions of times faster than any static machine he laid the foundations of a new world. This was the *voltaic cell;* today Volta's cells are in wide use in the form of dry cells and storage-battery cells. Later, the electric generator was discovered — a much more powerful source of electromotive force. These sources of electrical energy will be described in later chapters.

So novel were the phenomena found with these large currents that at first it was not certain that voltaic electricity and static electricity were the same thing. For one thing, the potential difference produced by these cells was exceptionally small. Two hundred dry cells connected " in series " (so that their electromotive forces add up) give only 1 electrostatic unit of potential difference — enough to deflect an electroscope slightly. It would take several times as many cells to make a spark jump even a very small gap. The difference in potential used on our electric lamps (although enough to give one a shock) will hardly affect an electroscope. However, metal wires have small resistances, and these comparatively small electrical pressures are enough to furnish the currents which we need.

The experiments described above can easily be repeated with a dry cell for electromotive force, a wire instead of a stick for resistance, and with other kinds of measuring instruments — voltmeters and ammeters — appropriate to the smaller potential differences and the larger currents.

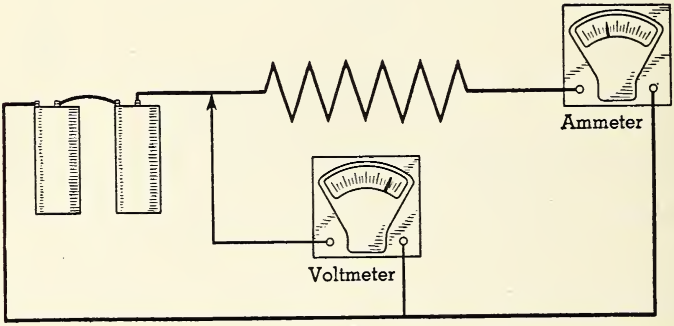

Fig. 3. Second experiment in electrodynamics: current through a wire resistance. In each cell the center terminal is at higher potential than the case.

Ohm's law will be studied in greater detail in the next chapter. We must now mention (casually at present) some of the other remarkable phenomena of current electricity.

417. Heating Effect. When electricity passes through a wire it heats it. A rather fine iron wire fastened between the terminals of a dry cell becomes red hot. The shorter the wire, the less its resistance and the more current flows through it and the more it is heated. Our houses are lighted by very fine wires (in incandescent lamps) heated white hot by the flow of current. Enormous numbers of electrons flowing through resistance wires in toasters and coffee pots and electric ranges play their role in cooking our food. This heat energy is released when electricity loses its potential energy, positive charge passing from high potential to low. The heat generated is

$$\text{Heat in calories} = QV \div 42{,}000{,}000.$$

418. Chemical Effects. If two copper plates are suspended in a solution of copper sulfate, electricity can be made to flow through the solution from one plate to the other. It will be found that one of the plates, where the current leaves to enter the solution, is wasting away. An equal amount of copper is deposited upon the other plate where the current passes out of the solution. Using other solutions, other metals can be deposited. This is called electroplating. We can demonstrate the effect in a few minutes with the current furnished by several dry cells; it would take several hundred centuries to deposit a noticeable amount of copper with the current of a few statunits which passed through the meter stick.[1] Solutions containing silver or gold can be used for silver plating or gold plating; these and other applications of electroplating are familiar to all.

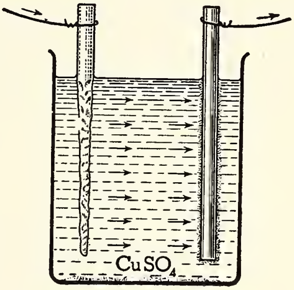

FIG. 4. Electroplating.

419. Magnetic Effects. We must now describe what is certainly one of the most remarkable and most important properties of the electrical current. If a wire is placed between the poles of a magnet so that the current flows across the lines of force as shown in the figure, the wire will experience a force in a direction at right angles both to the field and to the current! The wire in Fig. 5 will be thrust toward the reader (out of the page). There is a convenient rule to indicate the direction of the moving force: the thumb and first two fingers of the *left* hand are to be extended in mutually perpendicular directions; then,

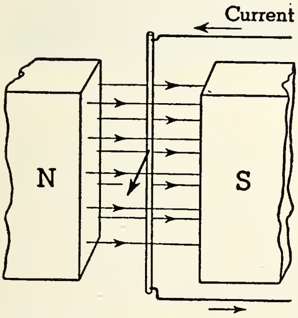

FIG. 5. Force on current in a magnetic field.

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[1] This in spite of the fact that 2 billion electrons and 1 billion atoms are deposited by a statcoulomb in a second. The atom is a very small thing.

if the first finger points in the direction of the magnetic field and the center finger points in the direction of the current, the thumb will indicate the direction of the force.[1] Use this rule to determine the direction of the force on the right side of the coil in Fig. 6.

This effect was discovered by the Danish physicist, Oersted, in 1819. Half a century later Rowland, distinguished American physicist, showed that this same magnetic field existed about rapidly moving electrostatic charges and so completed the proof of the identity of " static " and " voltaic " electricity.

When first discovered by Oersted, this cross motion induced by the magnetic field seemed very strange indeed, as it still must seem to the student when he first meets the effect. To be sure, it seemed at first that this force was too small ever to be of practical importance. When a current of 3×10^{10} statamperes flows in the wire across a magnetic field of 1 line per square centimeter (or 1 gauss) the force on the wire is only 1 dyne on each centimeter of its length. By using more wires, stronger fields, still larger currents, this force grows. A century later this force had grown until it was doing the work of 20,000,000 horses in our factories. Out of this discovery of Oersted's came the electric motor.

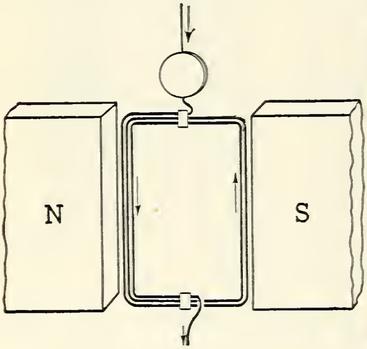

FIG. 6. Principle of galvanometer.

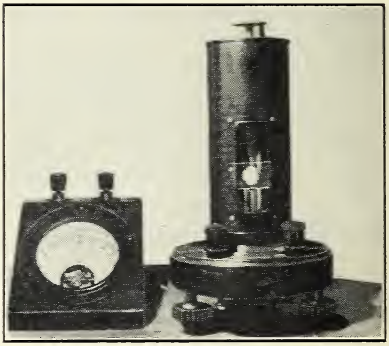

FIG. 7. Galvanometers.

420. The Galvanometer. The galvanometer employs this transverse force to measure currents. The coil shown in Fig. 6

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[1] To remember this rule: The *Th*umb gives the *Th*rust; the *F*irst finger gives the *F*ield; the *C*enter finger gives the *C*urrent . . . The first finger must be extended as in pointing.

either is suspended by a fine wire, or (in portable galvanometers) is pivoted in delicate bearings and held in position by a hair spring. When current flows down one side and up the other across the magnetic field a couple is exerted on the coil. The deflection is sometimes indicated by a delicate pointer; in more sensitive instruments a small mirror is mounted on the coil and the deflection is observed by the motion of a beam of light reflected to a scale. The deflection of the coil is a measure of the current flowing.

421. Electromagnetic Units. As measures for these much greater currents another system of units has been adopted. These are called electromagnetic units (e.m.u.). *A current which will, in a field of 1 gauss, give a transverse force of 1 dyne per centimeter of length of wire* is taken as the unit. This electromagnetic unit is called the **abampere.** It is 3×10^{10} times greater than the units which we have been using. In terms of this unit of current we define a new unit of charge. This is (naturally) the amount of charge passing a given point in a second when a current of 1 abampere flows. It is called the abcoulomb.

The unit of potential is the erg per abcoulomb (or the abvolt). It is an extremely small unit $\left(\frac{1}{3 \times 10^{10}} \text{ statvolt}\right)$.

422. Practical Units. This electromagnetic system of units is (like the electrostatic system) based on the centimeter and gram and second (c.g.s.), but it is not the system most commonly used in practice. The units of charge and current are a little too large for convenience. A practical system was adopted at an international conference 60 years ago. The **ampere** and **coulomb** are one-tenth as large as these fundamental " ab " units. The ampere is of convenient size. About half an ampere flows in an average incandescent lamp, perhaps 5 amperes in a flatiron, a few hundredths of an ampere through the telephone. A new dry cell, with a low-resistance wire connected directly between its terminals (" short-circuited "), will furnish 20 or 30 amperes for a short time.

When an ampere flows *for* a second 1 coulomb has passed. The total charge delivered is therefore

$$Q = It. \qquad \text{(From Def.)}$$

$$\text{Coulombs} = \text{Amperes} \times \text{Seconds}.$$

Expressing the relation another way: an ampere is a coulomb *per* second.

$$I = \frac{Q}{t}. \qquad \text{(Def.)}$$

How many coulombs pass through our "average incandescent lamp" in a minute?

In this practical system of units the joule (rather than the erg) is the unit of work (§129). It follows that the volt is a potential difference of 1 joule per coulomb (instead of 1 erg per statcoulomb). More explicitly: if the potential at two points is such that 1 *joule* of work is done when a *coulomb* passes from the one point to the other, their difference of potential is by definition 1 **volt.** Since a joule is 10^7 ergs and a coulomb is one-tenth of an electromagnetic unit, the volt (or joule per coulomb) is 10^8 times as large as the abvolt. (Hence our statvolt is 300 volts.)

Unit of charge:	coulomb	= one-tenth e.m.u.
Unit of current:	ampere	= one-tenth e.m.u.
Unit of potential:	volt	= 10^8 e.m.u.

423. Motion of Electrons in Electric Currents. It might appear that the magnetic effect mentioned above would always be extremely small in comparison with electrostatic forces. But not so. Consider what is happening in the wire when a current is flowing. There are billions of billions of free electrons in a cubic millimeter of wire. When there is no potential difference they are moving at random. But when there is a difference in potential and an electric field in the wire there is a more or less rapid drift of all these electrons in a single direction. This is the electric current. But all the time the moving electrons are equaled in number by the stationary positive charges; there are current, motion of charge, heat generation, and magnetic effects; but the wire is not charged, and we can neglect the electrostatic effects. When we charge a conductor we add or subtract a few electrons on the *surface.* Never more than a minute fraction of a coulomb is collected, even on the most highly charged body. Currents of many coulombs per second are readily created, the wires remaining practically uncharged. There are in the wire several coulombs of moving negative charge balanced by the same amount of positive charge.

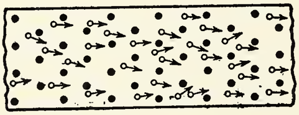

FIG. 8. Current in wire.

Suggested Reading. Pilley's "Electricity" (Oxford) and Bragg's "Electricity" (Macmillan) give very readable accounts of fundamental experiments in electricity. Both contain interesting illustrations. For a simple treatment of the practical use of electricity in the home, see Avery's "Household Physics" (Macmillan).

QUESTIONS

1. What heat is generated in a resistance when a statcoulomb passes between two points 1 statvolt different in potential? When an abcoulomb passes between two points with potential difference of 1 abvolt?

2. Explain what happens in Fig. 1 in terms of the motions of electrons.

3. How does the current passing through a given resistance depend on the potential difference across its ends? How will the current be affected if the resistance is doubled?

4. Would the current through the meter stick in Fig. 1 be changed if the potential of both ends were increased the same amount?

5. How does the potential vary as we pass from point to point along the stick? Compare the difference in potential between A and D with that between B and C. Suppose that the right end of the stick is disconnected from the ground so that no current flows; what will the potentials then be along the stick?

6. What is the direction of the force acting on a wire carrying a current to the east in a magnetic field directed northwards?

7. There is an ammeter ("amperemeter") on the instrument panel of the automobile. How much current flows to charge the battery? How much to light the lights (read with the generator at rest)? If the headlights are on for an hour how much charge flows from the battery?

8. When two like resistances are connected in series the potential difference (V) across either one is: (*a*) equal to, (*b*) half of, the total potential difference; and the current I in either one is: (*a*) equal to, (*b*) half of, the total current flow.

9. When a current flows downward in a magnetic field directed northward the force on the wire is: (*a*) N; (*b*) E; (*c*) W.

Vocabulary: Resistance, potential drop, Ohm's Law, resistances in series (parallel), electromotive force, cell, generator, electrolysis, galvanometer, ammeter, voltmeter, coulomb, ampere, volt.

CHAPTER 33

RESISTANCE

In this chapter we shall consider the flow of currents through resistances. We now consider only parts of circuits containing no electromotive forces. To them Ohm's Law applies directly. In Chapter 34 we shall generalize the law to apply to partial circuits containing electromotive forces and to the complete circuit.

424. Resistance and Conductance. Resistance, defined in §412, is a measure of the opposition which a body offers to the flow of current. It plays much the same role in electricity that friction does in mechanics. Were it not for resistance, a current, once started in a wire, would flow forever in undiminished intensity without any further application of electromotive force. The measure of resistance is the potential difference required to send unit current (ampere, abampere, or statampere) through the conductor.

The practical unit of resistance is the *ohm*. The resistance in ohms is the number of volts required to produce 1 ampere of current.

$$\text{Ohms} = \frac{\text{Volts}}{\text{Amperes}} \cdot$$

In other words: the *ohm* is the resistance of a conductor of such dimensions that 1 volt potential difference is required to send 1 ampere of current through it. For wires all 1 millimeter in diameter, 1 ohm is the resistance of 50 meters of copper, or 7 meters of iron, or 3 meters of German silver, or $1\frac{2}{3}$ meters of constantin, or 65 centimeters of nichrome. Wire of this size (" 18-gauge ") is commonly used in the laboratory, and much of the flexible lamp cord wire, made of stranded copper, has the same effective cross-section.[1]

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[1] In the electrical industry wires are usually measured by the " American Wire Gauge." According to this system an increase of 3 in gauge number corresponds to a halving of the cross-section, a doubling of the resistance. For city power distribution 4-gauge copper wire with a resistance of about an ohm per mile is commonly used. On the other hand small electromagnets (for example in telephone receivers) are often wound with 40-gauge wire, with a resistance of about an ohm per foot.

The reciprocal of resistance is called the *conductance* (K). $K = 1/R$ (or I/V). For example, the resistance of a meter of 18-gauge copper wire is $\frac{1}{50}$ ohm — its conductance is 50 units; the conductance of iron wire is less. What are the resistance and conductance of 1 meter of 18-gauge nichrome wire?

425. Specific Resistance. The resistance of wires of other sizes can be determined by proportion. The resistance varies inversely as the cross-section (or inversely as the square of the diameter). For example, the resistance of copper wire $\frac{1}{2}$ millimeter in diameter (" 24-gauge ") is four times that given above. Of course the resistance is also proportional to the length of the conductor. Hence

$$R = \mathbf{r}\frac{L}{A}. \tag{10}$$

The proportionality constant **r** is called the specific resistance or the *resistivity*. (**r** can be described as the resistance of a centimeter cube of the material).

TABLE 36

Resistance of a Centimeter Cube (Resistivity)

Silver	1.6×10^{-6} ohm
Copper	1.7×10^{-6} ohm
Aluminum	2.8×10^{-6} ohm
Iron	9.8×10^{-6} ohm
Pure water	2×10^{7} ohms
Sulfur	2×10^{17} ohms

Table 36 shows the wide range of values found for resistivity. Pure water has a resistivity 10^{13} times greater than copper. (Ordinary water has a much smaller resistivity.) A copper bar 1 square centimeter in cross-section wrapped around the earth's equator 25 times would have the same resistance as a cubic centimeter of pure water. And the resistance of sulfur (the best insulator of all) is 10^{10} times greater still.

The resistance of a conductor of any size can be computed from the resistivity (eq. 10). The resistance of the wires used in the laboratory can be determined more quickly by proportion, from the values for 18-gauge (1-millimeter) wires given above.

426. Direct Method of Resistance Measurement. By Ohm's Law $R = V/I$. To measure the resistance of a conductor a suitable potential difference is applied to its two ends, and this

" voltage " is measured with a voltmeter; the current is measured with an ammeter. The ratio gives the resistance.

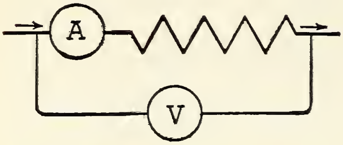


FIG. 1. Measurement of resistance.

Experiments. Measure the diameter of constantin wire, compute the necessary length per ohm, cut in lengths to give four resistances of slightly more than 1, 2, 3, and 6 ohms, respectively, and wind on wooden spools. Using a dry cell, voltmeter, and ammeter, measure resistance of each and adjust exactly to the above values.

Let the current from a dry cell pass through your body and then through a galvanometer. Measure the current and compute the resistance of your body. . . . Body resistance depends mostly upon the contact. It is the current through the body, not the voltage itself, which produces a shock. When good contacts are made, as by the hands immersed in water, comparatively low potential differences are sometimes fatal.

427. Interesting Facts about Resistance. (1) Resistance of pure metals increases with temperature, almost in direct proportion to the absolute temperature. For instance, at 0° C the resistance of silver increases by 1 part in 260 (instead of 1 part in 273), and that of copper by 1 part in 250 for each degree of temperature rise (Fig. 2). In incandescent lamps the resistance of the tungsten filament is about 15 times greater at the operating temperature than at room temperature.

This resistance change is explained by modern theory as due to the increased disorder among the atoms in the metallic crystals. When the atoms are at rest (as at absolute zero) they are arranged in almost perfect regularity, and through this regular array the electrons can travel great distances without collision. At higher temperatures the vibration of the atoms destroys this simple regularity and reduces this uninterrupted flight of the electron.

(2) The resistance of alloys is in general greater than that of elemental metals. Their resistance increases with temperature, but the change is usually much less than for the pure elements. Measure by the voltmeter-ammeter method the resistance of a piece of nichrome wire, in an oil bath, at room temperature and at 100° C. For this range the increase in resistance is only about 1 per cent.

(3) Suddenly, at a temperature a few degrees above absolute

zero, the resistance of certain metals seems to vanish completely. (Note the mercury and lead curves in Fig. 2.) A current may be started in a conductor and it continues to flow for many hours in undiminished intensity, without the aid of a battery or other electromotive force! This is called superconductivity. The electrons no longer seem to know the atoms are there; they no longer make collisions with them. The complete explanation has not as yet been given.

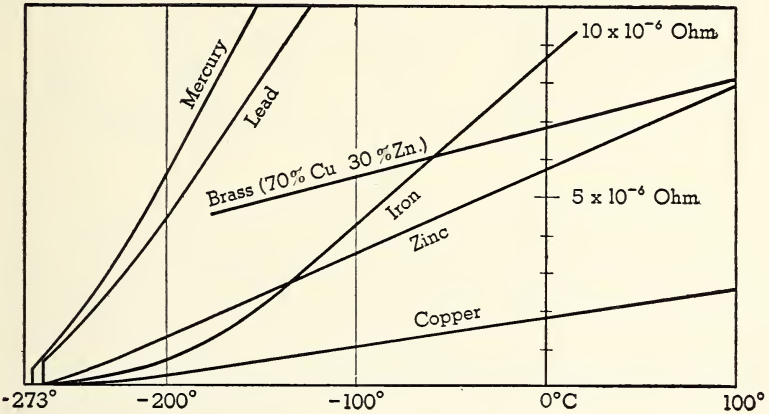


FIG. 2. Resistivity and temperature.

(4) The conductivities of metals for electricity and for heat are closely proportional. Silver and copper are the best conductors of both. The explanation is that it is not the slow-moving atoms but the faster-moving electrons which play the chief role in transferring heat. Substances without free electrons (non-metals) are always poor conductors of heat.

428. Resistors and Rheostats. Resistances are widely employed for the control of currents; when used for this purpose they are called *resistors*. The resistor may consist simply of an open coil of high-resistance wire, or the wire may be wound on an insulating core and then is often covered with vitreous enamel. Nichrome, an alloy of nickel and chromium, is the wire quite generally used in resistors because of its high resistivity and its resistance to oxidation at high temperatures.

Provision may be made for varying the control resistance; this variable resistor is called a rheostat. In the type most commonly used in the laboratory (Fig. 3), a sliding member makes contact with the resistor, allowing any portion of the resistance to be used.

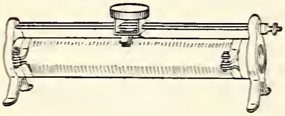


FIG. 3. Common type of rheostat.

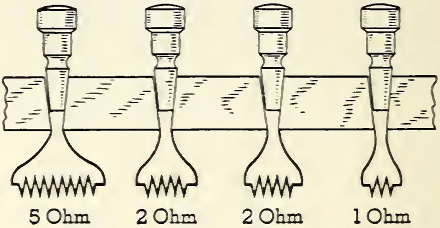


FIG. 4. Plug type resistance box.

429. Resistance Boxes. Standardized resistances are needed in electrical measurements. Fine manganin wire is used; this alloy is notable for the constancy of its resistance over a wide range of temperatures. In a common type of resistance box the resistance coils are connected between heavy brass blocks (Fig. 4) and any or all of the coils can be shunted out by means of plugs: with the plugs out the current passes through all the resistances; any plug inserted offers a by-pass for the current, eliminating some one resistance. The boxes in common use in the laboratory have a range from 1 to 1000 or from 1 to 10,000 ohms, and, in 1-ohm steps, any resistance between these limits can be obtained.

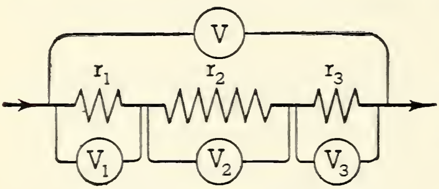


FIG. 5. Resistances in series.

430. Resistances in Series. If several resistances (r_1, r_2, r_3) are connected in series, the total resistance (R) is equal to the sum of r_1, r_2, r_3:

$$R = r_1 + r_2 + r_3. \tag{10'}$$

Proof: The potential difference across the combined resistances (V, measured by the upper voltmeter in the figure) is equal to $V_1 + V_2 + V_3$, the sum of voltages across the individual resistances. The same current (I) flows through each resistance. Then since $R = V/I$ (by definition) and $r_1 = \frac{V_1}{I}$, etc.,

$$R = \frac{V}{I} = \frac{V_1}{I} + \frac{V_2}{I} + \frac{V_3}{I} = r_1 + r_2 + r_3,$$

giving equation (10′).

Observe: 1. The current (I) can be computed from a knowledge of the potential difference across any part of the circuit (say V_1) and the resistance of that part; or it can be computed from the total potential difference and the total resistance.

$$I = \frac{V_1}{r_1} = \frac{V_2}{r_2} = \frac{V_3}{r_3} = \frac{V}{R}.$$

2. Since $V_1/r_1 = V_2/r_2$ (or $= V_3/r_3$), potential differences (V_1, V_2, etc.) across the various resistances are proportional to the resistances. This may be written

$$\frac{V_1}{V_2} = \frac{r_1}{r_2}.$$

A greater voltage is required to send the current through a greater resistance. Similarly $V_1 : V = r_1 : R$; *the voltage across any resistance is to the total voltage as that resistance is to the total resistance.* In this connection read again the discussion in §414.

For example, r_1, r_2, r_3 may represent lamps of 100, 200, 100 ohms resistance; if the total potential drop is 100 volts, there will be a 25-volt drop across the first lamp, a 50-volt drop across the second, and a 25-volt drop across the third. The current can be computed as 25/100 = ¼ ampere, considering the first lamp, or as 100/400, using the total voltage and the total resistance.

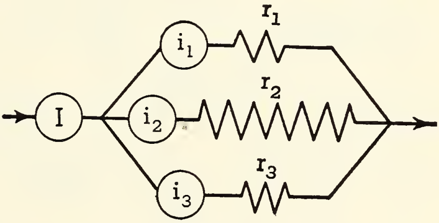


FIG. 6. Resistances in parallel.

431. Resistances in Parallel. It is perhaps obvious that two or more wires in parallel are equivalent to a single wire of increased cross-section. Indeed the conductance equals the sum of the conductances:

$$K = k_1 + k_2 + k_3,$$

or, as it is usually written,

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}. \qquad (10'')$$

Proof: The total current I, measured by the ammeter in the main line in Fig. 6, is equal to $i_1 + i_2 + i_3$, the sum of the divided currents. There is the same potential difference (V) across each resistance. Then since $K = 1/R = I/V$ (by definition)

$$\frac{1}{R} = \frac{I}{V} = \frac{i_1}{V} + \frac{i_2}{V} + \frac{i_3}{V} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3},$$

giving eq. (10″).

Observe: 1. The potential difference across the parallel resistances can be computed from a knowledge of the current in any branch and its resistance ($V = i_1r_1$), or it can be computed from the total current and the total resistances ($V = IR$)

$$V = i_1r_1 = i_2r_2 = i_3r_3 = IR.$$

2. Since $i_1r_1 = i_2r_2$ (or i_3r_3), the currents in the different branches are inversely proportional to the resistances

$$i_1 : i_2 = r_2 : r_1.$$

Similarly $$i_1 : I = R : r_1.$$

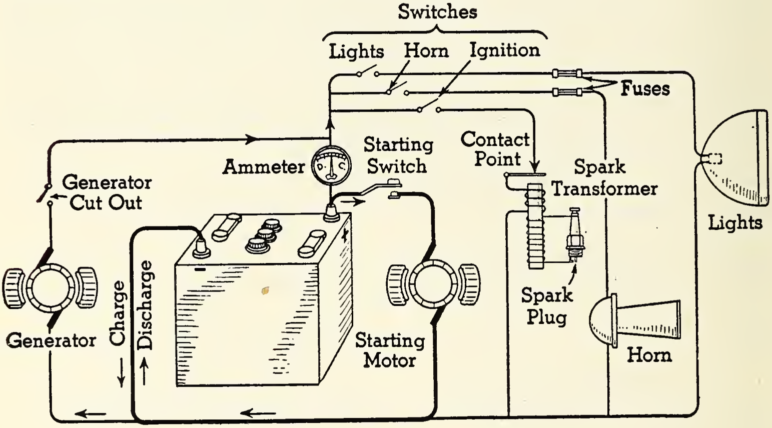


FIG. 7. Parallel circuits: wiring of automobile.

For example, consider the same 100-, 200-, 100-ohm lamps, now connected in parallel. To find the resistance of the combination:

$$\frac{1}{R} = \frac{1}{100} + \frac{1}{200} + \frac{1}{100} = \frac{5}{200} = \frac{1}{40}, \text{ and } R = 40 \text{ ohms.}$$

Suppose that 100 volts is applied to these lamps; then

$i_1 = 100/100 = 1$ amp. ($r_1 = 100$ ohms.)
$i_2 = 100/200 = \frac{1}{2}$ amp. ($r_2 = 200$ ohms.)
$i_3 = 100/100 = 1$ amp. ($r_3 = 100$ ohms.)
Total current $= 2\frac{1}{2}$ amp. ($R = 40$ ohms.)

Check: $I = V/R = 100/40 = 2\frac{1}{2}$ amperes. Observe that the currents divide inversely as the resistances.

This relation (10′′) becomes very simple when all the resistances are alike. When a number (n) of like resistances (r) are connected in parallel the combined resistance is equal to r/n. Three 100-ohm lamps connected in parallel carry three times the current of one, and the total resistance is $33\frac{1}{3}$ ohms.

Example. The resistance of our 3-ohm coil and 6-ohm coil (§426) connected in parallel is 2 ohms; for $1/R = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ and $R = 2$. What is the resistance of the 1-, 2-, 3-ohm coils in parallel?

Experiment: Connect these three resistances in series and in parallel as in Figs. 5 and 6 and measure their resistance in each combination; compare with the computed values.

432. Application of Ohm's Law: Current through a Network. The current through any part of a circuit (not including within the part an electromotive force) is given by Ohm's Law

$$I = \frac{V}{R}.$$

Let us illustrate this by a special problem. Suppose that we connect our resistances as in Fig. 8 and apply a potential difference of 15 volts.

1. What current will flow? We first find the resistance. The resistance of the 6-3 combination is 2 ohms (example above). Therefore the total resistance is $1 + 2 + 2 = 5$ ohms. The current is given by Ohm's Law: $I = V/R = \frac{15}{5} = 3$ amperes.

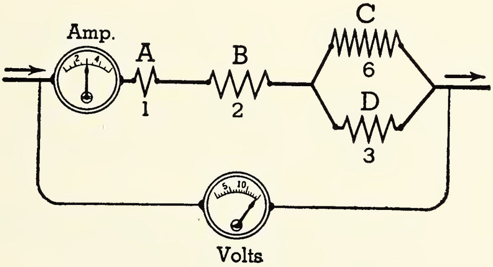


FIG. 8. Current through network.

2. What is the potential drop across each resistance? Again use Ohm's Law (now knowing the current). Across B the resistance is 2 ohms. Hence

$$V_B = IR = 3 \times 2 = 6 \text{ volts}.$$

The potential difference across *C-D* is also 6 volts (same resistance); that across *A* is 3 volts ($IR = 3 \times 1$).

Check: $3 + 6 + 6 = 15$ volts total.

3. How does the current divide in the 6-3 combination? We now know the potential difference. Applying Ohm's Law to the lower branch ($V = 6$ volts, $R = 3$ ohms), we get $I = V/R = \frac{6}{3} = 2$ amperes. Similarly in the upper branch $I = \frac{6}{6} = 1$ ampere.

Check: 2 amperes + 1 ampere = 3 amperes total.

433. Distribution of Voltage and Current by Proportion. Instead of steps 2 and 3 above it is often easier to solve such problems using the proportional relations given (in fine print) in §§430, 431. The 15 volts divides up in proportion to the resistances (1 : 2 : 2). Or, since the total resistance is 5 ohms, we have for the potential difference across *A*:

$$V_A : 15 = 1 : 5 \quad \text{and} \quad V_A = 3 \text{ volts.}$$

Again observe that the 3 amperes is distributed among the 6-ohm and 3-ohm coils inversely as the resistances (as 1 : 2). Or (what amounts to the same thing) the current in one branch is to the combined current as the combined resistance is to the resistance of the branch. Here the combined resistance is 2 ohms:

$$i_C : I = R : r_C$$
$$i_C : 3 = 2 : 6,$$

giving $i_C = 1$ ampere as before. If a 10-ohm and a 1-ohm resistance are connected in parallel, how will 11 amperes flowing through the combination divide?

Example. Since this is a very important problem the student should work the example of §432 through with different values. Suppose that the potential difference is 110 volts, that the resistances *A*, *B*, *C*, and *D* are 10, 30, 60, and 20 ohms.

1. Find $R_{C\text{-}D}$ and R_{total} and the current I through the ammeter.
2. Find the potential difference across each resistance.
3. Find the current through each branch of *C-D*.
4. Check 2 and 3 by proportion.

434. The Ammeter. The ammeter is an instrument for measuring fairly large currents. It is a portable galvanometer with a low parallel resistance (a " shunt "), through which most of the current by-passes, only a definite small proportion going through the galvanometer itself. The range of the ammeter depends on the resistance of the shunt. For instance, suppose that the galvanometer gives the full deflection when 0.01

ampere flows through it. Then with a shunt resistance $\frac{1}{999}$ as great as that of the galvanometer, we have an ammeter with 10-ampere range, for only 1 part in 1000 (0.01 ampere) will flow through the galvanometer and 999 parts through the shunt.

The ammeter has a low resistance. It is placed *in* the line (Fig. 8). If placed across the line like a voltmeter it will probably be burned out.

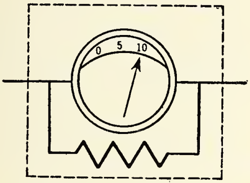


FIG. 9. Ammeter circuit.

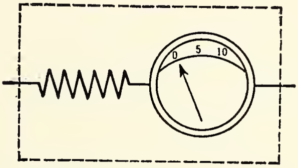


FIG. 10. Voltmeter circuit.

435. The Voltmeter. If, instead of a shunt, a *series* resistance is added to this same galvanometer so that the total resistance is exactly 100 ohms, we shall have a voltmeter with 1-volt range (for 1 volt through 100 ohms will furnish 0.01 ampere and so produce full-scale deflection). With 10,000 ohms the range would be 100 volts. (Why?) A voltmeter is a galvanometer with a high series resistance. The voltmeter is placed in parallel with the line (Fig. 8), but because of its high resistance little current flows through it.

The shunting resistance of the ammeter and the series resistance of the voltmeter are usually included within the case of the instrument. Ammeters and voltmeters sometimes have several different shunts or series resistances, with corresponding binding posts allowing the instrument to be used for several current or voltage ranges. In the laboratory the student should be careful to connect the instruments for the proper range.

Very low resistance in an ammeter and very high resistance in a voltmeter are properties much to be desired. The instruments placed in a circuit to a certain extent modify the current and voltage distribution. In Fig. 1 the ammeter introduces additional resistance in the line; the voltmeter is in parallel with the line, by-passes a small amount of current, and decreases slightly the circuit resistance. To reduce these effects to a minimum the ammeter should have as little resistance and the voltmeter as much resistance as possible.

436. Rheostat with Variable Pickoff. When current flows there is a gradual fall in potential along a conductor (§414).

Figure 11 shows how a resistance can be used to furnish any desired potential difference up to the full voltage of the power line. It is placed across the line so that current flows through it and gives a potential drop along the resistance. By adjusting the sliding contact the potential between *b* and *c* can be varied at will. The leads from *b* and *c* go to the "load" — for example, a lamp or motor — and this output voltage can be made any fraction of the whole.

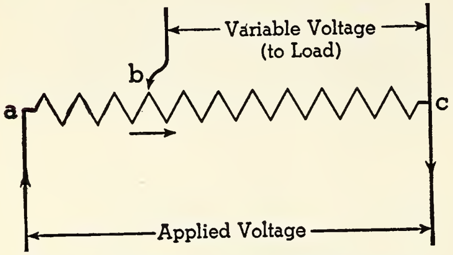


FIG. 11. Rheostat with variable pickoff.

This same circuit (but with standard resistances) is used in the *potentiometer*, an instrument for exact measurement of electromotive forces by comparison with the electromotive force of a standard cell (§442).

437. Wheatstone Bridge. In Fig. 12 the current from the battery flows through two branches (*abd* and *acd*) in parallel. As we pass along either branch the potential falls in proportion to the resistance passed over. Hence if the resistances are adjusted so that $r_1 : r_2 = r_3 : x$ the potentials at points *b* and *c* are exactly the same. (You can show this using for r_1, r_2, r_3, and x your 1-, 2-, 3-, and 6-ohm coils and substituting a voltmeter for the galvanometer *G*.)

This arrangement (called the Wheatstone bridge) is used for the exact measurement of resistance. A sensitive galvanometer bridges the points *b* and *c*. The resistance to be measured (x) is inserted as one arm of the circuit; r_1 and r_2 are set in some simple ratio and r_3 adjusted until no current flows in the galvanometer. Then points *b* and *c* must be at the same potential; hence

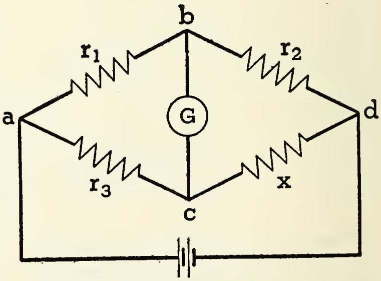


FIG. 12. Wheatstone bridge.

$$\frac{r_1}{r_2} = \frac{r_3}{x},$$

and from this relation the unknown (x) is determined. By this method resistances can be measured with a high degree of accuracy. Since the measurement of the unknown resistance is made by comparison with a known resistance (r_3), it avoids the errors necessarily involved in the reading of the instruments in the voltmeter-ammeter method.

438. The "*IR* Drop." According to Ohm's Law, when current flows through a conductor there is a drop in potential equal to *IR*. The engineer refers to the "*IR* drop." We have measured the *IR* drop across each resistance in Fig. 8; in the variable pickoff on the rheostat the *IR* drop was used to obtain any desired potential; in the Wheatstone bridge there was an *IR* drop in each branch of the bridge, and the *IR* drops between *ab* and *ac* were balanced. The *IR* drop occurs wherever electrical energy is being transformed into heat. In the lamp or the electric heater this generated heat serves its useful purpose; but an *IR* drop also occurs in power lines and in motors and generators and cells, and here it represents wasted power.

Figure 13 shows a power line supplying current for 4 lamps each taking 2.5 amperes, or 10 amperes in all. The two power wires are supposed to have together a total resistance of 0.2 ohm. There is therefore an *IR* drop of 2 volts in the line (1 volt is lost in each side). A potential difference of 112 volts is required at the power plant to supply the 110 volts at the lamps.

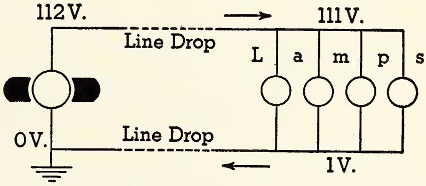


FIG. 13. Power line with 2 volt *IR* drop.

The *IR* drop increases with the current. Probably all have noticed the dimming of the house lamps when an electric heater or electric motor is started, drawing heavy current. This is due to the increased *IR* drop in the power line, decreasing the voltage at the house.

QUESTIONS

1. Given three 30-ohm resistances, how can they give a total resistance of 90 ohms? 10 ohms? 45 ohms?

2. Compare the resistivity of copper, iron, German silver, and nichrome. Compare the conductivity of copper and pure water.

3. How much current flows when a 10-ohm heating coil is connected to a 110-volt power line?

4. What is the resistance of a lamp which carries a half ampere (110-volt power line)?

5. How will a current of 10 amperes divide through resistances of 2 ohms and 3 ohms in parallel?

6. How much current flows through ten 220-ohm lamps connected in parallel across a 110-volt line?

7. Twelve volts are applied to resistances of 1, 2, and 3 ohms in series; by proportion what will be the potential difference across each?

8. How can a potential difference of $\frac{1}{2}$ volt be obtained from a dry cell and variable rheostat?

9. Why does a bird perched on a power line get no shock? Does any current *at all* flow through its body? Would the amount be different if the resistance of the wire were increased?

10. How many statunits in an ampere; how many volts in a statvolt; hence how many statohms in an ohm?

11. Resistance increases approximately as the absolute temperature: (*a*) for all metals; (*b*) for all substances; (*c*) for elemental metals.

12. With a 1-ohm and 3-ohm resistor in parallel: (*a*) one-third, (*b*) one-quarter, (*c*) two-thirds of the total current flows through the larger resistor.

Vocabulary: Conductance, resistivity, superconductivity, resistor, rheostat, Wheatstone bridge, *IR* drop.

PROBLEMS

In each problem the student should draw the circuit.

1. Find the resistance of (*a*) a copper wire $\frac{1}{2}$ mm in diameter and 10 meters long; (*b*) a copper bar 1 by 2 cm by 1 meter.

2. Work out on paper questions 3 to 7 above.

3. Eight Christmas-tree lights are to be connected in series and are to take 2 amperes when connected to a 110-volt line. What must the resistance of each lamp be? How many dry cells would be required to operate a single lamp?

4. A 20-ohm resistance coil and a 30-ohm coil are connected in parallel, and a potential difference of 6 volts is applied. (*a*) What is the combined resistance? (*b*) What is the total current? (*c*) What current flows through each coil?

5. Find the current if the coils of problem 4 are connected in series.

6. Compute the resistance of our 1-, 2-, 3-, and 6-ohm resistance coils (§426): (*a*) in series; and (*b*) in parallel.

7. If 12 volts potential difference is applied to each of these combinations (problem 6) how much current will flow in each instance?

8. What will be the potential difference across each resistance (problem 7) when the coils are connected in series? What current in each when in parallel?

9. In Fig. 8 take 20, 25, 30, and 15 for the resistances of *A*, *B*, *C*, and *D*. Suppose 110 volts to be applied to the circuit. Find the current through each resistance and the potential difference across each.

10. To a 200-ohm lamp and a 20-ohm flatiron connected in *series* 110 volts are applied. How much current flows through each? If a person touches the two terminals of the lamp he gets a shock; not so for the flatiron. Why? What is the potential difference across each?

11. Two 200-ohm lamps and two 40-ohm heating coils are connected in series (in this order) to form a complete circuit (for example, in a circle). What current will flow from a 120-volt power line, if the two sides of the line are connected, one side between the two lamps and the other between the two coils? What current if the power line is connected directly across one of the coils in the circle?

12. I connect a 3-ohm auto lamp and a 100-ohm house lamp in parallel; in series with this combination I connect another lamp of each kind. The operating voltage for the auto lamp is 8 volts, for the house lamp 120 volts. If 200 volts is applied to the circuit which lamp will burn out?

13. In Fig. 5 suppose that r_1 is 20 ohms, r_2 60 ohms, and r_3 40 ohms. If the voltmeter V reads 150 volts, what current will flow? What will the instruments V_1, V_2, V_3 read?

14. In Fig. 6 suppose the resistances are respectively 20, 60, and 40 ohms. If ammeter I reads 11 amperes how much will the instruments i_1, i_2, i_3 read?

15. In Fig. 6 suppose that resistances are respectively 20, 60, and 40 ohms. If the ammeter i_1 reads 3 amperes, (*a*) what is the potential difference across the upper (20-ohm) resistance? (*b*) What is the potential difference across each other resistance? (*c*) What do the ammeters i_2, i_3, I read?

16. Suppose four lamps, arranged as in Fig. 13, have resistances of 30, 40, 60, and 50 ohms (from left to right). Suppose that in the line leading to the first lamp there is 10 ohms resistance and that there is also 10 ohms between each lamp. If 100 volts is applied to the line what current will flow through each lamp? (Suggestion: first compute the resistance of the last two lamps and interconnecting line, then the last three, etc.)

17. A galvanometer gives full-scale deflection for 1 milliampere (0.001 ampere). Its resistance is 5 ohms. With 0.001-ohm shunt it is used as an ammeter. Approximately what is the full-scale reading of the instrument?

18. In a voltmeter the galvanometer described in problem 17 is used with a 10,000-ohm series resistance. Approximately what is the voltage range of the instrument?

19. The Wheatstone bridge (Fig. 12) is used to measure a high resistance (x). The bridge balances with r_1 10 ohms, r_2 100 ohms, r_3 1346 ohms. Find the unknown resistance.

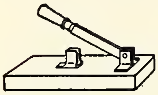
20. Approximately how much does the house voltage fall when a 22-ohm flatiron is turned on? Assume the line resistance to be $\frac{1}{2}$ ohm (110-volt power line).

21. Suppose that the lamps in Fig. 13 have resistances of 100, 200, 400, and 400 ohms; voltage as indicated in the figure. (*a*) How much current flows through each and through the main line? (*b*) What is the resistance of the main line?

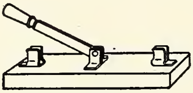
22. A 30-volt farm generating plant, located in the barn, furnishes power for ten 28-ohm lamps in the house. The power wires to the house have a resistance of 0.1 ohm each. Find (*a*) the resistance of lamps and line; (*b*) the total current; (*c*) the *IR* drop in the power line; (*d*) the voltage at the house.

WIRING PROBLEMS

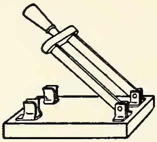
In the following problems switches of different types are used. In their simplest form these are illustrated below.



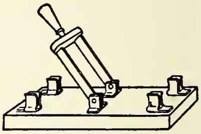
Single Pole Single Throw



Single Pole Double Throw

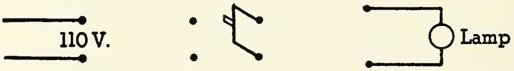


Double Pole Single Throw

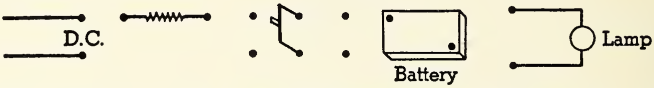


Double Pole Double Throw

1. Ordinary house switch: Connect the double-pole single-throw switch to turn the light on and off.

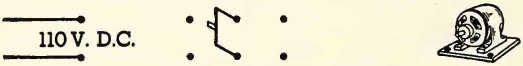


2. Connect the double-pole double-throw switch so that the battery can be connected either to the line (through a resistance) for charging or directly to the lamp.

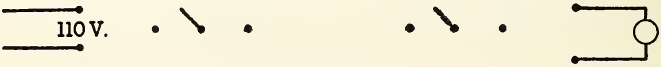


3. Work problem 1 with a single-pole single-throw and problem 2 with single-pole double-throw switch.

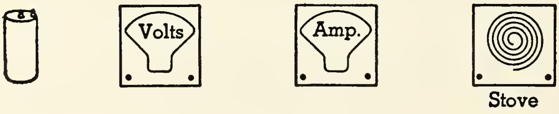
4. Reversing switch: Connect the double-pole double-throw switch so that the direction of current in the motor can be reversed by throwing the switch.



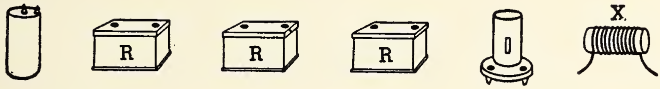
5. A common problem in hall lighting: Connect the following apparatus so that the light can be turned either on or off at either switch.



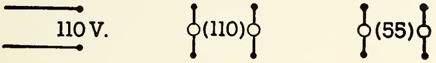
6. Connect the following apparatus to measure the resistance of the electric stove:



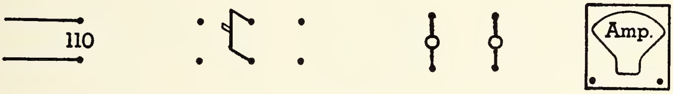
7. Connect the following as a Wheatstone bridge to measure the resistance of the resistor, X.



8. Given two 110-volt lamps and two 55-volt lamps. Connect all four to a 110-volt circuit.



9. Connect the following so that both lamps are lighted and the ammeter measures the current in one or the other depending on which way the switch is closed.



CHAPTER 34

OHM'S LAW WITH ELECTROMOTIVE FORCES

439. Electromotive Force. A source of electromotive force is any agency by which some other form of energy (mechanical, chemical, magnetic) can be transformed into electrical energy. Examples are static machines, dry cells and storage batteries, electric generators. Electromotive force is analogous to hydromotive force; a cell or a generator is analogous to a water pump. A dry cell does not create electricity but simply transports positive charge to a high potential (or, actually in most instances, negative charge to low potential) and so increases its potential energy. (A pump does not create the water but raises its height or increases its pressure.) By the magnitude of the electromotive force (e.m.f.)[1] we mean *the difference in potential which the cell or generator or other device maintains when no current is flowing.*

For example, refer again to Figs. **32**–2 and 3. The electromotive force is a static machine in the first figure and two dry cells in the other; positive charge is raised from low potential to high, by mechanical work in the static machine, by the expenditure of chemical energy in the cells. Imagine the circuits opened at the resistance so that no current can flow; the voltmeter across the static machine will read some thousands of volts; across the two dry cells it will read 3 volts. This potential difference, *with no current flowing*, is the e.m.f.

TABLE 37

TYPICAL ELECTROMOTIVE FORCES

Static machine	10,000–100,000 volts
Dry cell	1.5 volts
Storage cell	2.0 volts
Automobile generator	6–8 volts
Power generator	110–22,000 volts
(Transformed to 110–120 volts for consumption)	

Small electromotive forces may also be produced directly by heat (§458) and by light (§548).

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[1] This quantity is commonly referred to by its initials: the " ee-em-ef."

440. Internal Resistance. A pump raises the level of water; but its piping also offers a certain amount of resistance to flow, and there is a loss of pressure when a large current flows. Similarly the cells in Fig. **32**–3 offer more or less resistance to the flow of current (internal resistance). Therefore when current flows there is not only an electromotive force (raising the potential) but also an IR potential drop within the cells; hence when current flows the rise in potential in the battery is less than its electromotive force ($\mathcal{E}$)

$$V_{\text{rise}} = \mathcal{E} - IR. \tag{9'}$$

For example, when a current of 2 amperes flowed in the experiment illustrated in Fig. **32**–3 a student found that the potential difference read on the voltmeter was only 2.6 volts (instead of 3 volts). Here the IR drop in the cells was evidently equal to 0.4 volt, and (since I was 2 amperes) the internal resistance of the two cells in series was 0.2 ohm.

Old cells may have much greater resistance than this. The resistance of a storage cell is much lower, usually less than 0.01 ohm. Any source of electromotive force has a certain amount of internal resistance. In the generator this is the resistance of the wire through which the electricity must flow as it is boosted from low to high potential. Usually the windings are thick bars of copper and the resistance is quite small, but in the small hand-driven generator (magneto, used in rural telephone sets) there are thousands of turns of very fine wire with very high internal resistance. A static machine has a high e.m.f. but it has an effective internal resistance so extremely high that it cannot deliver any appreciable amount of current.

441. Cells in Series and in Parallel. Figure 1 represents a battery of four dry cells, connected in series and in parallel. (This shows the conventional representation of a cell. We let the shorter line represent the low potential, the longer line the high potential terminal.) The electromotive force of the cells in series is the sum of their individual electromotive forces. Unfortunately the resistance of the combination is likewise equal to the sum of the individual internal resistances; for a number of cells the internal resistance may become of serious magnitude. . . It is sometimes advantageous (when the external circuit has a very low resistance) to connect the cells in parallel. This does not increase the electromotive force, but it reduces the internal resistance. If r is the internal resistance of each cell, the resistance of n cells in series is nr ohms; for n cells in parallel the internal resistance is r/n ohms.

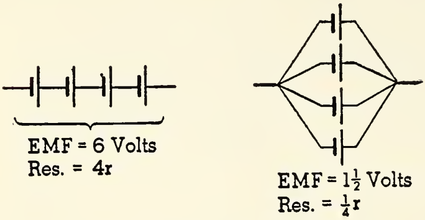

FIG. 1. Dry cells in series and parallel.

In the majority of cases cells are connected in series. It can be shown that, to obtain the greatest current through a given circuit from a battery of several cells, the cells should be arranged in such a way that their internal resistance matches most nearly the external resistance. With high external resistance this means the series connection — in a high-resistance circuit the increased internal resistance is unimportant. But when the external resistance is a small fraction of an ohm, dry cells should be connected in parallel.

442. The Potentiometer. Electromotive forces can often be measured with sufficient precision by measuring the open-circuit potential difference with a voltmeter, but for high-resistance cells and for thermoelectromotive forces (§458) the ordinary voltmeter introduces an error because of the current which it draws. For greatest precision, electromotive forces are measured by comparison with standard cells of known electromotive force (§472).

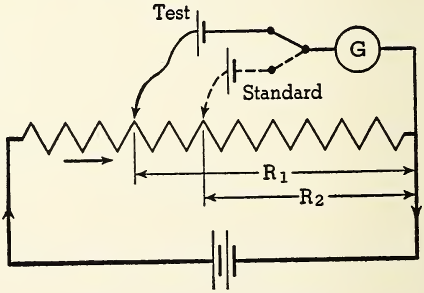

FIG. 2. The potentiometer.

The comparison is made with a *potentiometer*. In Fig. 2 a steady current from a storage battery flows through the resistance R, and there is a gradual IR drop in potential along the resistance. (Compare Fig. **33**–11.) The cell (or other e.m.f.) under test and a galvanometer are in a "pickoff" branch circuit, with sliding contact. The cell is connected so as to oppose the flow of current. Considered as a simple divided circuit (without e.m.f.) the current would flow toward the right through the galvanometer; the cell tends to drive the current in the opposite direction (a "counter electromotive force" [§444]). If the IR drop is greater than the e.m.f. of the cell, current will flow against this counter electromotive force (from left to right through the galvanometer); if less, the cell will force current in the opposite direction. By adjustment the contact can be placed so that no current flows in this part of the circuit, the electromotive force being exactly balanced by the IR drop. A standard cell is then substituted for the test cell and the adjustment again made. Then, since the voltage drop along the resistance is proportional to the resistances,

$$\frac{\text{E.m.f.}_{\text{test}}}{\text{E.m.f.}_{\text{standard}}} = \frac{R_1}{R_2}.$$

Since when actually balanced no current is flowing through the cells, this gives the ratio of the true electromotive forces.

443. Ohm's Law for the Complete Circuit. *In any complete circuit the potential rise in electromotive forces must exactly equal the potential drop in resistances.* For example, if two dry cells are connected in series with several resistances making a complete closed circuit, the combined IR drops in these resistances *plus the IR drop within the cells* equals the electromotive force (3 volts). Thus when a current flows through a circuit containing one or more electromotive forces and several resistances in series:

$$\begin{aligned}\text{Total e.m.f.} &= IR_1 + IR_2 + IR_3 + \text{etc.}\\ &= I \times \text{Total resistance,}\end{aligned}$$

or, as the equation is usually given, the current flowing in the circuit is

$$I = \frac{\text{Total e.m.f.}}{\text{Total resistance (external and internal)}}. \qquad (9'')$$

This is one of the most useful forms of Ohm's Law, applying as it does to the whole circuit.

Example. The 18-volt battery in Fig. 3 has a resistance of 1 ohm; find the current flowing in the circuit and the potential difference across its terminals.

$$R = 1 + 2 + 2 \text{ (external)} + 1 \text{ (internal)} = 6 \text{ ohms.}$$

$$I = \frac{\mathcal{E}}{R} = \frac{18}{6} = 3 \text{ amperes of current flowing.}$$

$$V_{\text{rise}} = \mathcal{E} - IR_{\text{int.}} = 18 - 3 \times 1 = 15 \text{ volts.}$$

The voltage across the external resistances is thus 15 volts, and 3 amperes is flowing; given these conditions the example in §432 shows how the voltages and currents for individual parts of the circuit can be found.

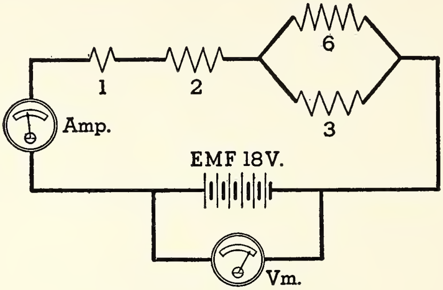

FIG. 3. Network with e.m.f.

444. Counter Electromotive Force. Imagine a circuit with a number of dry cells connected in series (for example, Fig. 3). Tracing around the circuit in the direction of current flow we find at each cell the potential raised by $1\frac{1}{2}$ volts (or a little less because of internal *IR* drop). Now we reverse one of the cells, to oppose the current flow. In this cell the potential now *falls* by $1\frac{1}{2}$ volts (or a little more because of the *IR* drop). Such an electromotive force, opposing the flow of current, is called a counter electromotive force.

A forward electromotive force furnishes electrical energy; a counter electromotive force consumes electrical energy — here the reversed cell is being charged (very inefficiently if a dry cell) — electric energy converted into chemical energy. An electric motor has a counter electromotive force. (The motor is, in fact, simply an electric generator with current reversed, forced to flow against the electromotive force, furnishing mechanical work and consuming electrical energy.)

With open circuit the potential drop in the reversed cell is equal to the counter electromotive force; when current flows there is in addition an *IR* drop:

$$V_{\text{drop}} = \mathcal{E} + IR.$$

In Fig. 4 two cells are connected "bucking each other." A 1-ampere current from an external power source is sent through the circuit. The *IR* drop in each cell is $\frac{1}{2}$ volt. The electromotive force of the left cell is in the direction of the current (forward electromotive force); the potential is *raised* by $(1\frac{1}{2} - \frac{1}{2})$ volt. The right cell has a counter electromotive force; in this cell the potential *falls* by $(1\frac{1}{2} + \frac{1}{2})$ volts.

The combined electromotive force of the two cells is zero; the resistance of the circuit shown is 3 ohms; hence there must be a potential difference of 3 volts at the terminals to produce the 1-ampere current.

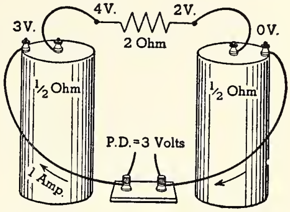

FIG. 4. Direct and counter electromotive forces. Current from an external source (power line) is being sent from left to right through the cells.

445. Generalization and Summary of Ohm's Law. Ohm's Law, in its original form ($V = IR$), deals with the potential drop as current flows through resistance. It says nothing of the potential rise (or fall) produced by electromotive forces. It tells only half the story. As we have seen, there is a potential rise in a voltaic cell (as it is generally used, driving the electrical current). The potential rise across its terminals is

$$V_{\text{rise}} = \mathcal{E} - IR. \quad \text{(Forward e.m.f.)} \qquad (9')$$

This may be considered a more general form of Ohm's Law applicable to a part of the circuit containing a battery, a generator, or other source of electromotive force which is directed so as to drive the current.

Equation (9′) also applies to an electromotive force opposing the current (counter electromotive force), but now the electromotive force is to be reckoned as negative:

$$V_{\text{rise}} = -\mathcal{E} - IR,$$

or as we can more conveniently use it,

$$V_{\text{drop}} = \mathcal{E} + IR. \quad \text{(Counter e.m.f.)}$$

Thus we may summarize our general discussion of Ohm's Law:

I. Ohm's Law (partial circuit, no e.m.f.):

$$V_{\text{drop}} = IR. \qquad (9)$$

II. Ohm's Law (partial circuit, e.m.f.):

$$V_{\text{rise}} = \mathcal{E} - IR. \qquad (9')$$

(A counter e.m.f. is negative, giving V_{drop}.)

III. Ohm's Law (complete circuit):

$$\text{E.m.f.}_{\text{total}} = I \times R_{\text{total}}$$

or as generally used

$$I = \frac{\text{E.m.f.}_{\text{total}}}{R_{\text{total}}}. \qquad (9'')$$

Example. Find the current which flows through a circuit which contains a 110-volt generator and a motor with 90-volt counter electromotive force, supposing that the internal resistance of each is 2 ohms and the resistance of the line is 1 ohm. Here the net electromotive force ($\mathcal{E}_1 + \mathcal{E}_2$) is 20 volts. The total resistance is 5 ohms. Hence (from 9″)

$$I = \frac{20}{5} = 4 \text{ amperes.}$$

What is the potential difference across the terminals of (*a*) the generator and (*b*) the motor? From eq. (9′)

(*a*) Generator: $V_{\text{rise}} = \text{E.m.f.} - IR = 110 - 2.4 = 102$ volts.
(*b*) Motor: $V_{\text{drop}} = \text{E.m.f.} + IR = 90 + 2.4 = 98$ volts.

The IR drop in the line is 4 volts (from 9).

QUESTIONS

1. How much current flows if a dry cell with resistance 0.1 ohm is short-circuited? How much if connected through a 0.1-ohm resistance?

2. The e.m.f. of a storage cell is 2 volts. Under what circumstances is the potential difference across it smaller than this? When greater?

3. Under what circumstances may the center "positive" terminal of a dry cell be at lower potential than the outer "negative" terminal?

4. Assume the left end of the series cells in Fig. 1 to be grounded (zero potential). Label the potential of the positive terminal of each cell.

5. Repeat question 4 with the cells grounded (*a*) at the center, (*b*) at the right.

6. What forward and counter e.m.f.'s are shown in Fig. **35–1**? What is the total e.m.f. of the circuit?

7. What is the voltage of power distribution in your community?

8. Electrical energy increases as (positive) charge goes from low potential to high in a cell; it then decreases as the charge flows again to the lower potential through a resistance. Where does this energy come from, and where does it go?

9. A dry cell is connected into a power circuit. The potential difference across the cell is greater than 1.5 volts: (*a*) when the external resistance is very great; (*b*) when the current passes through the cell from zinc case to carbon center; (*c*) when current flows from carbon to zinc.

10. Through a given external circuit a number of cells give the greatest current when they are so connected that: (*a*) their internal resistance equals the external resistance; (*b*) their internal resistance is the least possible; (*c*) their e.m.f. is a maximum.

Vocabulary: Internal resistance, potentiometer, standard cell, counter e.m.f.

PROBLEMS

Assume the e.m.f. of dry cells as 1.5 volts. Draw circuit for each problem.

1. What are the e.m.f., the IR drop in the cell, and the potential difference across the two binding posts of a 0.1-ohm dry cell through which 3 amperes are flowing (cell furnishing energy)?

2. A storage battery has 0.01 ohm internal resistance. Assuming an e.m.f. of 6 volts, find the potential difference across its terminals when it is furnishing a 50-ampere current.

3. Four 0.1-ohm dry cells are connected in series, with a voltmeter across the group. What is the voltmeter reading (*a*) when no current flows; (*b*) when the cells furnish a 15-ampere current?

4. A length of iron wire with $\frac{1}{2}$-ohm resistance is connected across the terminals at a dry cell, internal resistance $\frac{1}{4}$ ohm. What current flows? What is the voltage drop in the wire?

5. What current will flow if the cell in problem 4 is short-circuited (zero external resistance)?

6. In a flashlight three small dry cells are in series with a $1\frac{1}{2}$-ohm lamp. If each cell has an internal resistance of $\frac{1}{4}$ ohm, how much current will flow? What is the potential difference across the lamp?

7. A doorbell has 6 ohms resistance; the line is 50 meters of 18-gauge copper wire. What is the current with two dry cells ($\frac{1}{4}$ ohm each) in series? What is the potential difference across the bell?

8. In Fig. 3 assume the resistors (1, 2, 3, and 6) to have resistances of 20, 30, 60, and 180 ohms, respectively; assume the e.m.f. to be a 200-volt generator with 5 ohms internal resistance. What will the instruments read?

9. Three resistors, 2-ohm, 3-ohm, and 6-ohm, are connected in parallel. In series with this group are connected a 1-ohm resistor and two $\frac{1}{4}$-ohm dry cells. How much current will the 1-ohm resistor carry? The 2-ohm resistor?

10. In Fig. 33–12 assume that r_1 is 10, r_2 is 2, r_3 is 5, and x is 1 ohm; that the e.m.f. is 3 volts and internal resistance $\frac{1}{2}$ ohm. Find the potential difference across x.

11. Two dry cells, each with $\frac{1}{4}$-ohm internal resistance, are connected (*a*) in series, (*b*) in parallel; the external resistance is $\frac{1}{8}$ ohm. Find the e.m.f., the internal resistance, and the current in (*a*) and (*b*).

12. Four $\frac{1}{4}$-ohm dry cells are connected in " series-parallel " (two chains in parallel of two cells each). The external resistance is $\frac{3}{4}$ ohm. Find the e.m.f., internal resistance, and the current.

13. Can three $\frac{1}{4}$-ohm dry cells be connected in such a way as to produce through a $\frac{1}{4}$-ohm external resistance a current greater than 4.5 amperes?

14. A student connects four $\frac{1}{4}$-ohm dry cells in series, but one cell with terminals reversed. The external resistance is 1 ohm. What current flows?

15. What is the potential difference across the four cells in problem 14? What is the potential difference across the reversed cell?

16. An electric generator drives a motor. The e.m.f. of the generator is 120 volts; the counter e.m.f. of the motor is 118 volts. Assuming an internal resistance of 0.1 ohm in each machine and a 0.2-ohm line resistance, what current flows?

17. A 10-volt storage battery, resistance 0.1 ohm, is being charged by a 50-volt generator which has a 0.2-ohm internal resistance. A 4.7-ohm resistance is in series. When charging, the battery acts as a counter e.m.f. (*a*) What are the total e.m.f., total resistance, and the current? (*b*) What is the potential difference across the battery? across the resistance?

18. The battery in problem 17 is removed from the power line and discharged through the 4.7-ohm resistance. Find the current and the potential difference across the cell.

19. In an automobile, two headlights (resistance 3 ohms each), the tail light (12 ohms), and the dash light (12 ohms) are in parallel. Find the combined resistance. Assuming a 6-volt battery with negligible internal resistance, what will be the total current and the current through each lamp?

20. In Fig. 33–7 suppose that a current of 5 amperes flows in the ignition circuit, 10 amperes in the light circuit, and 20 amperes in the generator circuit; which way does current flow through the battery? how if the horn circuit is closed, carrying 10 amperes? If the battery has 6 volts e.m.f. and 0.01 ohm resistance, what is the voltage across it in each case?

21. The internal resistance of the automobile battery is 0.01 ohm, and the resistance of the starter is the same; the e.m.f. of the battery is 6 volts; the counter e.m.f. of the starter, 5 volts. Find the current when the starting switch is closed.

22. Suppose the generator in Fig. 33–7 to be charging the battery (all circuits at the right of the figure open). Assume the e.m.f. of the generator to be 7 volts, its internal resistance 0.03 ohm; the battery e.m.f. 6 volts and resistance 0.01; the wire resistance 0.01 ohm. Find the charging current and the potential difference across the battery.

CHAPTER 35

ELECTRICAL ENERGY AND HEAT

446. Energy in the Electric Circuit. Electrical energy is obtained when positive charge is raised from low potential to high. There are several ways of doing this. We have seen the slow, inefficient static machine, and we shall later study the electric generator. Mechanical work is done by the operator as he turns the static machine, or by the steam engine as it turns the generator; electrical energy is obtained. In the voltaic cell chemical energy disappears and electrical energy is created. These are examples of electromotive forces. In every case as charge (Q) passes:

$$\text{Electrical energy created} = \text{E.m.f.} \times Q. \qquad (11)$$

There are several ways of using electrical energy. Mechanical work may be done, as in ringing a doorbell or in turning an electrical motor. Or the electricity may do chemical work, as in charging a battery. In each of these cases there is a *counter* electromotive force, " counter " because the electromotive force lowers rather than raises the potential energy of the passing charge.

$$\text{Work done} = \text{Counter e.m.f.} \times Q. \qquad (11')$$

Finally electrical energy may be used up as heat in the conductors — and this gives the IR drop

$$\text{Heat generated (in joules)} = IR \text{ drop} \times Q. \qquad (11'')$$

Figure 1 illustrates the creation and use of electrical energy. It shows the electromotive force of the generator and battery, cooperating to send current about the circuit. It shows the motor and the electroplating cell with their back electromotive forces, and it shows the IR drop as current flows through and heats the lamps. And there is always, unfortunately, also an IR drop in the line and within the cells and in the generator and the motor, as the current generates heat. The potential rise in the forward electromotive forces is not as great as the electromotive force, and the potential drop in the counter electromotive forces is greater than it should be, because of these internal IR drops. Energy is wasted as heat, entropy increases, electrical energy is degraded into heat energy (Second Law of Thermodynamics).

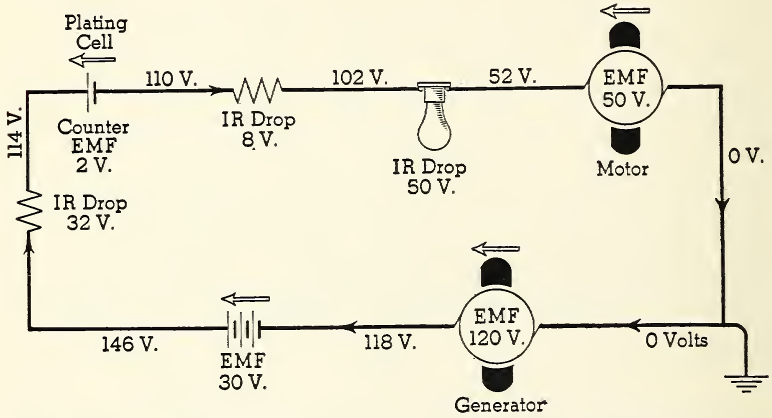

FIG. 1. Direct and counter e.m.f.'s. (E.m.f. shown by open arrows. A 2-volt IR drop is assumed in each e.m.f.)

Ohm's Law, as applied to a complete circuit containing e.m.f.'s, was given in the last chapter. Figure 1 is another illustration of this general law (9'').

$$\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3, \text{ etc.} = IR_1 + IR_2 + IR_3, \text{ etc.}$$

The sum of the e.m.f.'s equals the IR drops. The counter e.m.f.'s must be taken into consideration as well as the forward ones and are of course reckoned as negative.

447. Work and Power. When Q coulombs of electricity flow from higher potential to lower, the work done (or heat generated) is

$$\text{Work (or heat)} = QV \text{ joules.}$$

If V is an IR drop, the result is heat (11''); if the fall in potential is due to a counter electromotive force, the result is mechanical work, chemical energy, etc. (11').

Example. How much work is done and how much heat generated each minute when $\frac{1}{2}$ ampere flows through a motor from a 100-volt line? (Motor 80 per cent efficient.) Q is 30 coulombs and $QV = 3000$ joules. Twenty per cent of this is heat in the motor, 80 per cent is work. The electrical power input is $3000/60 = 50$ joules per sec. (watts).

The rate of doing this electrical work (or generating heat) is

$$\text{Power} = \frac{Q}{t}V; \quad \text{or} \quad \text{Power} = IV. \tag{12}$$

This is the power in joules per second or *watts*.

The power can be measured with a voltmeter and ammeter. For example, from the readings on the instruments in Fig. **33**–8 the power expenditure in the four resistors is 45 watts (45 joules [or 10.7 calories] of heat are developed per second).

Observe that when the power is 1 watt 3600 joules of work[1] is done in an hour. This quantity of work is called a watt-hour; it is a unit of energy frequently used in engineering practice. The kilowatt-hour (3,600,000 joules) is the unit in terms of which electrical energy is sold to the consumer.

448. The Wattmeter and the Watt-Hour Meter. Electrical power can be measured directly with a wattmeter. This instrument well illustrates the joint dependence of power on current and potential difference. However, to explain its operation it is necessary to anticipate our later discussion of electromagnets (Chapter 37). Compare Fig. 2 with Fig. 32–6. In both figures there is a central coil suspended in a magnetic field but in the galvanometer this field was furnished by the two poles of a permanent magnet, while here an exactly similar field is furnished by two coils and its strength depends on the current in these "field coils." The wattmeter has three binding posts and these are connected with the circuit in such a way as to make the instrument a cross between an ammeter and a voltmeter. The field coils are connected through a high resistance directly across the line. This is like a voltmeter; the current in these coils (and hence the magnetic field) is proportional to the potential difference (V). The current of the circuit (I) flows through the suspended coil. Thus the deflecting torque acting on the moving coil is jointly proportional to the current in the circuit and to the voltage across it; the deflection depends on IV and thus measures the power in the circuit.

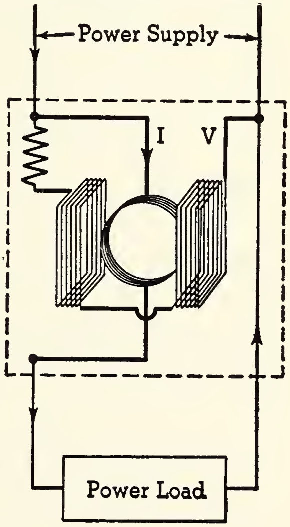

FIG. 2. The wattmeter.

The student has seen the watt-hour meter which the power company places in the home for measuring electrical energy. This operates on the same general principle as the wattmeter. But here the moving coil, instead of giving a steady deflection, rotates continuously; it operates as a small, slow-moving electric motor rather than as a galvanometer. This rotating coil is geared to the recording dials which count the revolutions.

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[1] A joule per second is a watt; conversely a watt-second (the work done with 1 watt power in a second) is a joule.

449. Electrical Work and Heat. We must distinguish between the power expended against a counter electromotive force and the power expended in sending current through a resistance. From §447 the total power input is

$$\text{Power} = IV.$$

1. If the potential drop (V) is due to a counter electromotive force, not to an IR drop, mechanical or chemical energy is generated.

$$\text{Useful power output} = I \times \text{Counter e.m.f.} \quad \text{(Mech., Chem., etc.)}$$

2. When current flows through a resistance, heat is generated, $V = IR$, and

$$\text{Power output (as heat)} = I^2R \text{ watts.} \quad \text{(Joule's Law.)} \quad (12')$$

This is the heat in joules generated per second. In calories

$$\text{Heat per second} = \frac{I^2R}{4.2} \text{ cal. per sec.}$$

In later chapters we shall study the energy of cells, motors, and generators; then the heat generated will represent wasted power, the "I^2R loss." We now study the devices in which this heat serves a useful purpose — electric lamps and furnaces.

Examples. 1. In the motor of the preceding example (§447) the counter e.m.f. was actually 80 volts, the IR drop 20 volts. The electrical power input was

$$\text{Power} = IV = \tfrac{1}{2} \times 100 = 50 \text{ watts.}$$

The mechanical power output was

$$\text{Useful power} = I \times \text{E.m.f.} = \tfrac{1}{2} \times 80 = 40 \text{ watts.}$$

2. What is the resistance of a lamp marked: 55 watts, 110 volts? $P = IV$; $55 = I \times 110$; $I = \frac{1}{2}$ ampere. But $R = V/I$ and $R = 220$ ohms.

3. How much heat is generated per second when a 22-ohm resistance is placed on a 110-volt line? The current is 5 amperes. Heat generated per second equals $5 \times 110 = 550$ joules per second or 130 cal. per sec.

4. How much heat is generated when 10 amperes flows through the above resistance? Heat per second $= I^2R = 100 \times 22 = 2200$ joules per sec. or 520 cal. per sec. How many times as much as for 5 amperes (in the preceding example)?

TABLE 38

Power (Watts) of Electrical Appliances

Typical Values

Fan, sewing machine	50	Toaster, flatiron	500
Refrigerator	150	Radiation heater	1000
Washing machine	600	Stove (per plate)	1500
Power = $I \times$ Counter e.m.f.		Power = I^2R	

450. Fuses and Circuit Breakers. A fuse is used to prevent excessive currents. It consists of a small strip of alloy with low melting point. The heat generated by an excessive current melts the fuse and breaks the circuit. Fuses of 10-, 20-, or 30-ampere ratings are used in the home. Measuring instruments and electrical equipment are often protected by individual fuses of appropriate size. Sometimes *circuit breakers* take the place of fuses. These are switches which open automatically when excessive current flows. Large circuit breakers are used in the power plant to protect the line and the generators from excessive overloads.

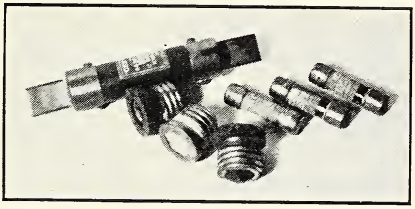

Fig. 3. Fuses.

451. Electric Lamps. The earliest electric lamp was an arc. It was invented (Sir Humphry Davy) soon after Volta discovered the voltaic cell.[1] A current is sent through two graphite rods in contact; the rods (" electrodes ") are then separated and an arc forms. In the intense heat and strong current, the gas molecules are broken up into charged " ions," and the air becomes a good conductor. (The resistance falls as the current increases; a rheostat must always be used in series with the arc to prevent the arc's becoming a short circuit.) Most of the light in the carbon arc comes not from the arc itself but from the graphite posts

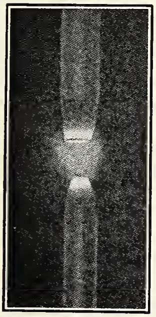

Fig. 4. Arc light.

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[1] Davy's arc lamp was, however, no more than a striking lecture demonstration. To operate it he used an enormous battery of 2000 cells. Practical electric lighting had to await the invention of the electric generator 60 years later (§509).

as they are heated white hot by the bombardment of the charged particles. When the arc is operated on direct current the positive electrode, heated by the bombardment of electrons and negative ions, is the hotter. Of all artificial light sources in common use the carbon arc is brightest and hottest and whitest.

The arc light is inconvenient. Inventors searched for a resistance wire that could be heated white hot. It cost Edison $40,000 to discover that a charred piece of cotton fiber in a vacuum could be heated to incandescence (1879). These carbon filaments were improved and the incandescent lamp was born. Decades later, after the great electrical companies developed their large research laboratories, it was found that tungsten, with almost as high melting point as carbon, made stronger filaments and could be heated to higher temperature. Tungsten lamps displaced carbon lamps. Research continued; instead of using a vacuum, the lamps were filled with inert gas; this blanketed the filament, reduced the evaporation, and allowed still higher temperatures, whiter light, greater efficiency. And so were developed the gas-filled tungsten incandescent lamps which we use today.

TABLE 39

LAMP EFFICIENCIES

	Candlepower per watt
Carbon filament	0.3
Tungsten filament	
25 watt (vacuum)	0.8
60 watt (gas)	1.0
300 watt (gas)	1.5
Sodium vapor	4.0

The modern incandescent lamp gives about 1 "candlepower" per watt. But more than 90 per cent of the energy is wasted in invisible heat radiation. Nature has been much more efficient in the cold light which she has given to the firefly. Mercury-vapor lamps (green) and sodium-vapor lamps (yellow) are being developed; they are very efficient, but the color of the light is not pleasing. A recent development is a very high-pressure mercury vapor arc for searchlights. It is an arc in a capillary tube, the size of a match stick; in temperature and brightness the arc is comparable with the sun. Research for more efficient lighting goes on.

452. Incandescent Lamps. If a number of incandescent lamps are connected in parallel across the power line (the usual connection), *the lamp with the least resistance takes the greatest current and has the greatest power* (IV or I^2R). Furthermore there must be more radiating filament surface if more power is to be radiated. Therefore a high power lamp has a coarse filament. Incandescent lamps for general lighting service are made to operate on 110, 115, or 120 volts with power ratings up to 1000 watts and usually with an average life of 1000 hours. (A cheaper class of lamp is rated at 500 hours.) Lamps for floodlighting (airports, etc.) have a power rating up to 10 kilowatts. Gas-filled lamps operate at temperatures ranging from 2400° C (for a 40-watt house-lighting lamp) up to 3000° C (for a ten-kilowatt floodlamp).

The efficiency of lamps depends upon the filament temperature. Of course all of the energy input comes out from the lamp, mostly as radiation, but less than 10 per cent of this radiation is visible to the eye (light). This visible fraction increases with the temperature and at higher temperature we get more light for our money. The "photoflood lamp" used in photography is overrated in voltage, operates at a temperature of 3200° C, with an efficiency of nearly 3 candlepower per watt, but has a life of only a few hours. . . A coarse filament can suffer more evaporation before the filament breaks; hence without unduly shortening the life the higher-power lamps are made to operate at higher temperatures. A 200-watt lamp gives as much light as six 50-watt lamps. . . Lamps of lowest power are evacuated; above 40 watts they are filled with gas (mostly argon). The inert gas reduces sublimation of tungsten; a molecule leaving the surface is likely to strike a gas molecule and rebound back to the filament. The gas introduces heat losses by convection but except with the smallest filaments this loss is more than balanced by the economy introduced by higher temperature.

453. Electric Furnaces. Nichrome wire, heated red hot, serves as the heating element in the flatirons and toasters and coffee pots and electric ranges used in the home. Laboratory furnaces are wound with nichrome wire or, for higher temperatures, with platinum foil. In the electrical furnaces of industry the heat is generated in various ways. In refining of metals, the metal may be melted in a crucible by a current of thousands of amperes sent through the metal itself, which thus acts as its own resistance coil. Or, more commonly, a giant electric arc is struck between a large post of carbon and the metal ore. Or sometimes through the center of the furnace runs a great carbon post which is heated white hot by a large current — a giant brother to the carbon filament.

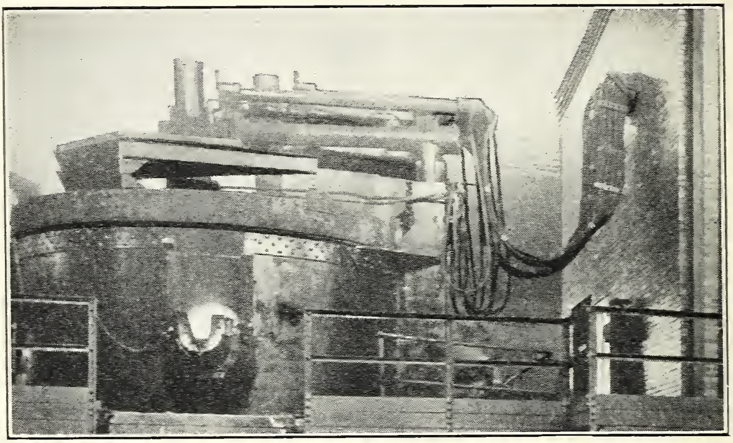

FIG. 5. Electric furnace of arc type for production of high-grade steel. (Graphite electrodes visible above furnace.)

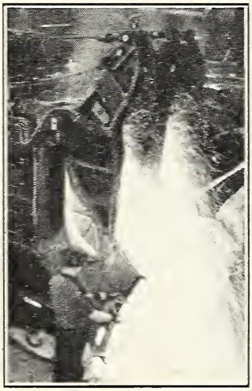

FIG. 6. Electric welding of automobile body. (One half of inverted body is visible, window under workman's arm, rear fender over his head.)

The electric arc is used for cutting and welding of metal sheets, railroad rails, etc. To a certain extent electric welding has been used instead of riveting in assembling structural-steel buildings and bridges.

454. Series and Parallel Connection of Lamps. Five lamps each designed to carry 1 ampere at 110 volts can be connected either in series or in parallel. When the lamps are connected in parallel the line voltage must be 110 volts, and 5 amperes will flow; when the lamps are connected in series, the line voltage must be 550 volts, and only 1 ampere flows. The power consumption (IV) is the same with either connection. But for two reasons the series connection is not suitable for the home: (1) when a single lamp burns out all the lamps are extinguished; (2) such high voltage would be dangerous. Nevertheless, as we shall now show, the series connection has one important advantage; it is commonly used in street lighting.

455. Power Loss in the Line. We have discussed the *IR* drop in a power line. The power loss (heating the line) is I^2R. The same electrical power (IV) can be supplied either at high current or at high potential. The engineer prefers to use high potential since this diminishes the line loss. Suppose the line resistance for the above lamps to be 2 ohms. With the lamps connected in parallel the line carries 5 amperes and the line loss (I^2R) is $25 \times 2 = 50$ watts; with the series connection the line carries 1 ampere and the line loss is $1 \times 2 = 2$ watts.

Electrical energy is supplied at as high voltage, small current as possible. If all the electrons which flow through our lamps and motors had to be sent through the distribution lines, the power loss, within even a small city, would be prohibitive. But power from large water-power plants is often sent for hundreds of miles into the surrounding territory. The energy for a whole city may be carried by two or three wires. Millions of amperes will perhaps be used in the city. It is out of the question to carry such currents through the line. In distribution lines, power is carried at high voltage — hundreds of thousands of volts in long lines, from 500 to 3000 volts in city distribution. We shall learn in Chapter 39 how this high-potential low-current power is transformed to low-voltage high-current power before delivery to the consumer.

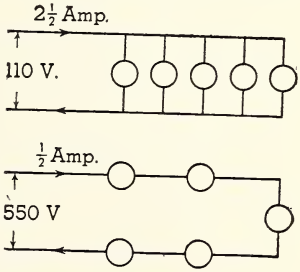

Fig. 7. Electric lamps in parallel and series.

456. Relative Cost of Electrical Energy. Although the cost and calorific values of fuels vary, Table 40 shows in a general way the relative costs of different heat sources.

To compensate at least partly for the comparatively high unit cost of electricity, every effort is expended to make electric ranges efficient; with the coal stove, of course, a large part of the heat goes up the chimney. But, most important to remember, electrical energy has no entropy. It is hardly fair to electricity to compare it in terms of its *heat* equivalent. By reducing the *IR* loss we can convert electricity with almost 100 per cent efficiency into mechanical work. There is no 70 per cent or 80 per cent tax to be paid to the second law of thermodynamics as there is for heat. When electricity is generated in a steam plant the "tax" has already been paid. In efficient modern generators run by steam turbines the fuel cost is less than ½¢ per kilowatt-hour. Other costs of operation and distribution raise the retail price — usually to about 5¢ per kilowatt-hour for the smallest consumer.

TABLE 40

B.t.u.'s bought for 1¢

	B.t.u. per ¢
Coal (at $8.00 per ton, 13,000 B.t.u. per lb.)	32,000
Oil (at 7¢ per gal.; 140,000 B.t.u. per gal.)	20,000
Natural gas (at 7¢ per "therm" [100,000 B.t.u.])	14,000
Artificial gas (at $1.00 per 1000 cu. ft., 530 B.t.u. per cu. ft.)	5,300
Electricity (at 1¢ per kw-hr.)	3,000

These figures make no allowance for relative efficiencies. Typical efficiencies for house heating: Coal 35 per cent; oil 50 per cent; gas 60 per cent; electricity 90–100 per cent.

457.* Definition of Electromotive Force. We have not as yet given an exact definition of "electromotive force." As the term has come to be used in practice a "source of electromotive force" is any agency by which another form of energy (mechanical, thermal, chemical, magnetic) is transformed into electrical energy or (counter electromotive force) by which electrical energy is transformed to another form of energy *reversibly*. The irreversible, entropy-increasing, degradation of electrical energy into heat energy in a resistance is not classed as an electromotive force. When a coulomb of charge passes from negative to positive electrode in a dry cell 1.5 joules of electrical energy is created by the chemical action. The **electromotive force** or **e.m.f.** of a cell (or other agency) is defined as being equal to the work done upon a unit charge (by the chemical or other forces); it is the electrical energy created per unit charge when current flows through the agency without heat loss.

With steady currents, where condensers are not being charged, no energy is being stored in the electrical circuit. Hence the input and output are the same, and the sum of the forward electromotive forces equals the sum of the counter electromotive forces and of the *IR* drops (Ohm's Law for the complete circuit, §443).

We have here defined the e.m.f. of a specific agency. One also refers to the **electromotive force of a circuit,** meaning the work done in carrying a unit charge around the circuit. The circuit may be the conducting circuit formed by wires, cells, and generators, etc.; the electromotive force of the circuit is then the algebraic sum of the forward and counter e.m.f.'s of the electrical agents. Or, as the term is frequently used, the circuit may be an imaginary closed path in space, for example a circular path in the electric field depicted in Fig. **31–1**. The e.m.f. about such a closed path in an electrostatic field is zero, since a repulsion at one part of the circuit is balanced by an attraction at another part. We shall find other electric fields (due to moving magnets, §§502, 525) where this is not true.

458.* Thermal Electricity. The evolution of heat as electricity flows through a resistance corresponds to frictional heat in mechanics. There is degradation of energy, and the process cannot be reversed — the I^2R heat cannot be reconverted into electrical energy by reversing the current.

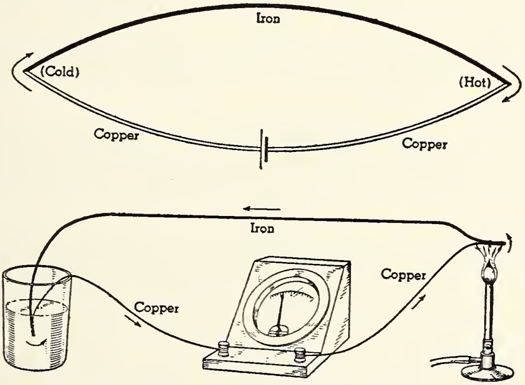

FIG. 8. Peltier and Seebeck effects.

There exists another process by which electrical energy can be converted into heat reversibly; that is, heat can also be converted directly into electrical energy. When an electric current flows across a junction of two different metals a small amount of heat is evolved or absorbed. For example, when a current flows around the upper circuit in Fig. 8 heat is evolved and the junction is warmed as the current flows from iron into copper; at the other junction heat is absorbed and the junction is cooled as the current flows from copper into iron (Peltier effect).[1]

We get the converse effect if we substitute a galvanometer for the battery and heat the right junction with a flame and cool the other. Heat is now absorbed from the flame and released at the cold junction, and there now exists an electromotive force in the circuit which sends current in the opposite direction (Seebeck effect). For any pair of dissimilar metals there exists such an electromotive force when the two junctions are at different temperatures. The effect is small; the electromotive force is usually only a few millionths of a volt for a temperature difference of 1 degree. Its important application is the thermocouple (or thermopile, a number of thermocouples in series) used for temperature measurement.

QUESTIONS

1. What are some practical sources of electrical power? How is electrical power expended?

2. Our janitor explains that by buying 120-volt rated lamps and using them in our 110-volt circuit the average life is tripled. Does this sound reasonable? Are we saving money?

3. Two lamps are designed for use on a 110-volt line. In one lamp the filament is coarse and short, resistance low; in the other the filament is fine and long, resistance high. Which will use the more power and give the more light?

4. Which of these lamps will be brighter if they are connected in series?

5. Compare the power consumption in: (*a*) a single lamp across a power line; (*b*) two of these lamps in series; (*c*) two of these lamps in parallel.

6. Power expended is equal to I^2R: (*a*) when there is no counter e.m.f.; (*b*) except in bad conductors; (*c*) always.

7. Gas is placed in incandescent lamps: (*a*) to cool the filament; (*b*) to cool the bulb; (*c*) to reduce evaporation of the filament.

Vocabulary: Joule's Law, fuse, circuit breaker, incandescent (arc) lamp, power loss, Peltier (Seebeck) effect.

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[1] Owing to their different affinities for electrons two chemically different metals in contact have a slight potential difference. (This is in contradiction to the law of equal potentials of electrostatics but the effect is very small.) Here the copper is at the higher potential, and the electrons lose kinetic energy as they pass from iron to copper (current flow from copper to iron).

PROBLEMS

1. What is the power consumption of an 11-ohm lamp on a 110-volt line?

2. In Fig. 33–8, with current as given, find the power dissipation in each resistance. How does the power vary with resistance when the resistances are (1) in series (A, B); (2) in parallel (C, D)?

3. The hot-plate of an electric range has two 10 ohm resistance elements, which may be operated separately or in parallel or in series (110-volt power line). Find the power consumption in the three arrangements.

4. The lamps in Fig. 33–13 are supposed to draw 2.5 amperes each. Find their wattage. What is the power loss in the line?

5. A motor on a 220-volt line draws 10 amperes. What is the power load on the line? If the resistance of the motor is 2 ohms, how much power is lost as heat? What is the useful output of the motor?

6. How many ergs of electrical energy are furnished by a 10-volt battery for each coulomb which passes?

7. By how many per cent will the power in a lamp decrease if the current drops by 1 per cent?

8. Prove that an alternative formula for power consumption in a resistance is: $P = V^2/R$. (This is a useful formula when dealing with pure resistances — lamps and resistors — but it holds only when $V = IR$. In contrast with Joule's Law it cannot be used to obtain the heat loss in an electromotive force.)

9. How much heat is generated per minute when a 50-ohm resistance is connected across 110 volts?

10. A 20-ohm coil, carrying 5 amperes, is immersed in a liter of water. How much will the temperature rise in a minute?

11. How much will the temperature of a liter of water be raised per minute by a 60-watt lamp immersed in it?

12. How long will it take to boil a pint (500 cc) of coffee in an electric coffee pot? Resistance of heater is 20 ohms, 120-volt line.

13. How much current is drawn from a 6-volt storage battery by a 30-watt automobile lamp? What is its resistance?

14. What is the resistance of a 7-watt 120-volt pilot lamp? of a 2000-watt 120-volt searchlight lamp? of a 30-watt automobile headlight?

15. How do the filaments differ in a 25-watt 6-volt automobile headlight bulb and a 25-watt 110-volt house-lighting bulb? Find the resistance and normal current of each.

16. An electric light company advertises that 1¢ will operate an iron for 20 minutes or a washing machine for 1 hour, or will light a lamp for 4 hours. The light rate is 5¢ per kilowatt-hour. What is the power of the lamp, motor, and flatiron? At 110 volts how much current flows through each?

17. What would you estimate as the average monthly consumption of energy for light in winter months by a student using 60-watt lamps? What would the light bill be at 5¢ per kilowatt-hour?

CHAPTER 36

ELECTROLYSIS

459. When electricity flows through certain substances (for example, through the copper chloride solution in Fig. 1) chemical action occurs where the current enters and leaves, and matter is transferred through the substance. This phenomenon is called **electrolysis.** The substance through which the current passes is called the **electrolyte.** The conductors by which the current enters and leaves the substance are called **electrodes,** the electrode at which the current (+ charge) enters the electrolyte being named the **anode**, and that by which it leaves, the **cathode.**

Often the electrolyte is water containing a salt (or an acid or a base) in solution.[1] However, electrolytic conduction also occurs in molten salts, in certain non-aqueous solutions, and to a slight extent in pure water. The chemical action at the electrodes usually becomes evident by the evolution of gas or by the deposition of metal (electroplating) or by the dissolution or corrosion of the electrode.

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[1] Definitions for the student with little knowledge of chemistry:

Ion: a charged atom or group of atoms (§384).

Radical: a group of atoms within the molecule which behaves in many respects as a single entity; for example, the OH radical in NaOH or the SO_4 radical in H_2SO_4.

Acids, Bases, and *Salts.* Many substances when dissolved in water break up (*dissociate*) into positive and negative ions. A substance which yields H^+ ions is called an *acid*; an example is hydrochloric acid, H^+Cl^-. A substance which yields OH^- ions is called a *base*; an example is sodium hydroxide, Na^+OH^- (common lye). A *salt* is a combination of ions other than H^+ and OH^-; examples are common table salt, Na^+Cl^-, and copper chloride, $Cu^{++}Cl_2^-$.

Valence. The valence of an atom (or radical) is the number of hydrogen atoms which it will either unite with or replace. For example, referring to Fig. 3–1, the valence of oxygen is 2 and the valence of nitrogen here is 3.

Combining weight (or *chemical equivalent*)*:* the atomic weight of an atom (or formula weight of a radical) divided by its valence. For example, the combining weight of oxygen is 8; that of SO_4 (valence 2) is 48.

Gram-combining-weight: a quantity with a mass in grams numerically equal to the combining weight. For example, 8 grams is the gram-combining-weight of oxygen.

460. Faraday's Laws of Electrolysis. In 1812 Michael Faraday was a bookbinder's apprentice; Sir Humphry Davy was the leading chemist in England. Faraday had performed his first electrolysis experiment when he constructed a "voltaic pile" of seven copper half pence and seven sheets of zinc separated by six pieces of paper moistened with salt water, and with this battery decomposed magnesium sulfate. The young Faraday brought himself to Davy's attention and was appointed his assistant in the Royal Institution; there he lived and worked the rest of his life, achieving fame but always a man of singularly modest habits and the simple ways of thought.

Twenty years after his first experiment with the voltaic pile Faraday announced two quantitative laws of electrolysis: The mass of substance liberated at an electrode (1) is jointly proportional to the current and the time, or in other words to the quantity of electricity which has passed through the cell, and (2) is proportional to the *combining weight* of the liberated substance, or in other words to its *atomic weight divided by its valence.*

In Chapter 32 the deposition of copper was described. Six times as much metal is deposited by a current of 2 amperes flowing for 3 hours as by 1 ampere for 1 hour; this is Faraday's first law. Instead of copper (atomic weight 63.6, valence 2) we may (from a silver nitrate solution) plate out silver (atomic weight 108, valence 1); the weights of the two metals deposited (for the same current and time) will be in the ratio of 31.8 : 108; this is Faraday's second law.

461. Nature of Electrolysis. When electricity flows through a wire, electrons enter one end, changing the potential here; this starts a movement of electrons along the wire, and electrons leave the other end. The atoms themselves do not move, and no chemical change occurs. But no free electrons exist in an electrolytic solution; the current here is carried by ions (charged atoms or charged groups of atoms).

Into a beaker containing a solution of copper chloride (cupric chloride, $CuCl_2$) let us place two platinum strips (electrodes) (Fig. 1). If we now connect this to a 6-volt battery, current will flow through the solution from one electrode to the other. The current (+ charge) flows from high potential to low, from left to right in the figure; thus the high-potential ("positive") electrode is anode; the low-potential ("negative") electrode is cathode. While the current flows copper is deposited on the cathode and chlorine gas is given off at the anode.

To explain this we must recall the nature of the solution (§384). When the copper chloride crystals are dissolved in water the copper atom and the two chlorine atoms in the molecule break apart; the chlorine atoms carry with them each a negative charge (one electron each) and the copper atom carries two positive charges (two electrons lost). We have then throughout the solution these positive and negative ions (Cu^{++} and Cl^-). There are twice as many of the singly charged negatives as of the doubly charged positives. The solution is neutral. The stage is now set for electrical conduction. When the electrodes are at different potentials the positive copper ions are repelled from the positive plate and attracted to the negative. They drift to the right in Fig. 1 and reaching the cathode give up their charge and form the copper deposit. The negative chlorine ions drift to the left and lose their charges, and bubbles of chlorine gas are liberated on the anode. The solution remains uncharged: two (singly charged) chlorine ions reach the anode for every (doubly charged) copper atom which reaches the cathode.

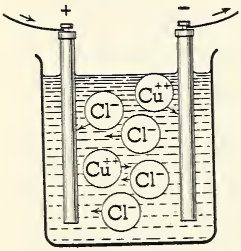
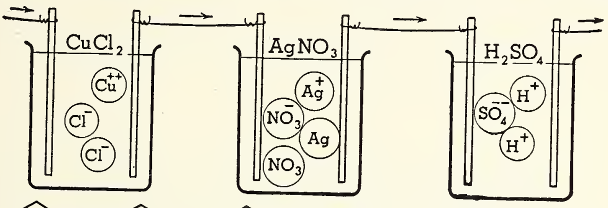


FIG. 1. Electrolysis of $CuCl_2$.

462. Electrolysis and Combining Weight. We now see the significance of Faraday's laws. Quite evidently, since each atom of copper carries a definite charge — as its price of passage across the solution — the quantity deposited will be directly proportional to the electrical charge passing through the solution — to the current times the time. This is Faraday's first law. Now compare the quantity of chlorine and copper deposited. There are twice as many chlorine ions as copper ions deposited in a given time. But the atomic weight of each ion is 63.6 for copper and 35.5 for chlorine. Hence the two deposits are (by weight) in the ratio of $\frac{63.6}{2} : 35.5$. This is Faraday's second law: The electrolytic deposit is proportional to the atomic weight divided by the charge on the ion (valence).

The significance of Faraday's laws will be seen more sharply if we compare the deposits in a number of cells, with different electrolytes, connected in series. The same current flows through each of the cells in Fig. 2. From the direction of the current flow, in each vessel, the anode is evidently at the left, the cathode at the right. The positive ion in each case goes in the direction of the current, and the negative ion goes against it. Each hydrogen and silver ion carries a single charge, and the same number of atoms of each is deposited on the cathode in a given time; the same number of singly charged negative ions (Cl^- and NO_3^-) go to the anode. Half as many doubly charged copper and sulfate (SO_4^{--}) ions are transported. Hence the number of grams deposited at each electrode is proportional to the weights of the atoms for the monovalent ions, to half the atomic weight for bivalent ions. While 1.008 grams of hydrogen is being deposited there will be 108 grams of silver, 31.8 grams of copper, etc. This is the significance of Faraday's laws.



Deposit	Cl	Cu	O*	Ag	O*	H
At. Wt.	35.5	63.6	16	108	16	1
Valence	1	2	2	1	2	1
Deposit by 96,500 Coul.	35.5 gm.	31.8 gm.	8 gm.	108 gm.	8 gm.	1 gm.

* Secondary Reaction

FIG. 2. Electrolytic cells in series.

"But," the student must ask, "what is the numerical value of the proportionality constant? How long must a current of 1 ampere flow (or how many coulombs must pass) to deposit one gram-combining-weight of any substance, i.e., 1.008 grams of hydrogen, 31.8 grams of copper, etc.?" When the above experiments are repeated with precision it is found that *96,500 coulombs of electricity deposit a gram-combining-weight of any element (or radical)*. This is one of the important constants of physics and should be remembered by the student. It is called the *electrochemical constant.*

We may now summarize Faraday's laws in the proportion

$$\text{Mass (in grams) deposited : Gram-combining-weight} = \text{Charge (in coulombs) transported : 96,500 coulombs.} \quad (13)$$

463. Secondary Reactions; Electrolysis of Water. The whole story is seldom told simply in terms of the ions transported through the solution. Frequently, as soon as the ion reaches the electrode a chemical reaction occurs. For example, when in the electrolysis of brine the sodium atom is deposited and loses its charge the neutral sodium attacks water, and it is a hydrogen instead of a sodium atom which is liberated at the cathode; similarly, in Fig. 2 oxygen from the water takes the place of the negative radicals SO_4, NO_3 at the anode.

This explains the electrolytic dissociation of water. Pure water does not conduct electricity readily. There are enough impurities in ordinary tap water to make it conduct, but in practice a little sulfuric acid is usually added. Then when a current passes, oxygen forms at the anode, hydrogen at the cathode, in the ratio of 8 : 1 (the combining weights). The sulfate radical (SO_4) is not deposited; as soon as it loses its charge it changes place with the oxygen in the water — oxygen bubbles are formed and the sulfate ion stays in solution (Fig. 2).[1]

464. Practical Use of Electrolysis. The student has seen many examples of electroplating: silver-plated knives and forks, gold-plated watches, chromium-plated automobile parts and nickel-plated articles of endless variety. . . Another example of electrolysis familiar to all is the charging of storage batteries (§471) . . . " Electrotyping " is an important process in modern printing. It is used for the duplication of the finest photogravure and printing plates. (This book is printed from electrolytically deposited plates.) A wax impression of the original is taken; this wax mould is then dusted over with graphite to make it conducting, and a copper plate is electrolytically deposited. The process has a number of other applications; it is used for the reproduction of medals and similar objects and, in the making of phonograph records, for producing the metal master mould from the original soft wax record. . . Electrolytic processes are also used extensively in the purification of ores — gold, silver, copper, lead — and in the manufacture of many chemicals. Hydrogen and oxygen are obtained by electrolysis of water, chlorine by the electrolysis of brine. Sodium, calcium, and (most important) aluminum are obtained by electrolyzing the molten salts of these metals. Electrolytic processes demand large currents, and the industries are usually located near waterfalls where power is cheap. Some of the largest of them are located at Niagara Falls.

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[1] The reaction depends also upon the nature of the electrodes, particularly of the anode. At a platinum anode oxygen is released as described; with a copper anode instead of negative oxygen ions coming out of solution, positive copper ions go in, surrounding the anode with a blue solution. On a tantalum anode an insulating oxide layer is formed which then blocks the further flow of current. (Aluminum shows the same effect in some electrolytes.) This anode blocking layer is the insulating dielectric in the electrolytic condenser (§403).

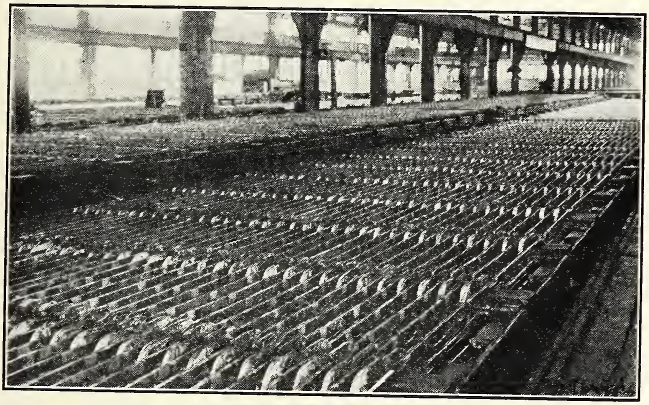

FIG. 3. Electrolytic refining of copper.

The electrolytic deposition of sodium can be shown on a small scale in an interesting lecture demonstration. A light bulb is partially immersed in a bath of molten sodium nitrite which is maintained at a potential somewhat above that of the filament of the bulb. Electrons leave the incandescent filament and reach the inner surface of the bulb. The charge then passes through the hot glass by electrolysis. Na^+ ions in the glass pass into the bulb and the sodium metal appears on the inner walls as a silvery coating (see Fig. 52–6).

465. The Mass of the Atom. The student must have wondered how the mass of the atom — so inconceivably small — is measured. The problem is not so difficult as it seems.

From the electrochemical constant we can find at once the ratio of the charge on the atom to its mass. Millikan measured the charge on the electron. Changing the value given in §388 into practical units, $e = 1.60 \times 10^{-19}$ coulomb; this is the value of the positive charge on the hydrogen ion. Hence from eq. (13) we have

$$m : 1.008 = 1.60 \times 10^{-19} : 96{,}500,$$

giving $m = 1.67 \times 10^{-24}$ gram as the mass of the hydrogen atom.[1] The periodic table gives the relative weights of the atoms; with one weight known all others are obtained by proportion.

ELECTROMOTIVE FORCES AND VOLTAIC CELLS

466. Electrical and Chemical Energy. In an electrolytic cell there is usually chemical change. The chemical energy[2] may be increased (as when water is decomposed into hydrogen and oxygen) and the electrical energy decreased; or there may be a decrease in chemical energy (as when zinc dissolves in sulfuric acid) and an increase in electrical energy. We can either turn electrical energy into chemical energy or the reverse. Conservation of energy shows that (aside from the *IR* drop) in the first case the electrical potential energy must decrease as the current goes through the cell (back electromotive force) and in the second case, where the chemical energy of the cell is decreased, the potential energy of the charge must be raised (forward electromotive force). This second case gives us our dry cells, storage batteries, etc., and it is these "voltaic cells," turning chemical energy into electrical energy, that we now study.

467. A Simple Primary Cell. A strip of zinc and one of copper dipped into dilute sulfuric acid form the simplest voltaic cell. If we join the electrodes by a wire, current will flow around the circuit; from the copper to the zinc outside the cell, from zinc to copper in the electrolyte. The zinc goes into solution, and bubbles of hydrogen form upon the copper. When the wire is removed all action stops but the electrodes have a difference of potential of 1.08 volts.

Metal ions from the electrodes tend to go into solution. There is thus a rivalry among the positive ions (Zn^{++}, H^+, Cu^{++}) for place in the solution. This tendency for ions to dissolve (" solution pressure ") is greatest for zinc, least for copper. When the electrodes are placed in the cell positive zinc ions enter the solution and (by their charge) drive out hydrogen ions upon the copper. This dissolving of zinc ions must occur momentarily, even with open circuit. But almost immediately the electrodes become charged, zinc negatively, copper positively. A potential difference of 1.08 volts is enough to check the motion of ions, and the action ceases. The wire between the electrodes allows the positive charge to pass from the copper back to the zinc, and the chemical action proceeds — zinc dissolving, hydrogen gas forming. And with each dissolving ion (Zn^{++}) two positive charges are boosted from the low- to the high-potential electrode.

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[1] Or, perhaps more simply, since one elementary charge is carried by one hydrogen atom and a gram-atomic weight (1.008 grams) is deposited by 96,500 coulombs, there must be as many atoms in a gram-atomic weight as there are electrons in 96,500 coulombs (6.02×10^{23}).

[2] This refers to what is technically known as "free" chemical energy.

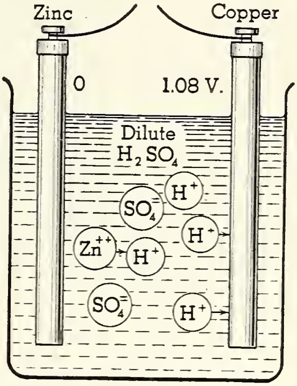

FIG. 4. Simple primary cell.

468. Defects of the Simple Cell. There are two reasons why this simple cell is not used practically. Unless the zinc is extremely pure, hydrogen bubbles will form on the *zinc* electrode and the zinc will dissolve even when the electrodes are not connected. This is called *local action*. Impurities on the surface of the zinc in reality act as minute positive electrodes, focal points upon which the gas bubbles can form. To remedy this the zinc is amalgamated with mercury; the zinc dissolves in the mercury, and is brought to the surface; the impurity is left behind and the local action disappears.

The other defect is *polarization*. As the action proceeds the copper electrode becomes covered with hydrogen, and we have effectively, instead of a cell with zinc and copper electrodes, one with zinc and hydrogen electrodes with a much lower electromotive force. There are several ways of preventing polarization. One method is to substitute copper sulfate ($CuSO_4$) for sulfuric acid (H_2SO_4) as the electrolyte; copper rather than hydrogen is now deposited. We shall now describe a practical cell of this kind.

469. Gravity Cell: 1.08 Volts. In this cell the negative electrode (amalgamated zinc) is placed near the top; the positive (copper) electrode is at the bottom. The electrolyte consists of two layers: copper sulfate solution in the lower half and sulfuric acid or zinc sulfate in solution above. Except for a slow diffusion, the two solutions are kept apart by the greater weight of the copper sulfate. (Hence the name of the cell.) Since the copper electrode is surrounded by copper ions there is no polarization. The electromotive force of the cell is 1.08 volts; its resistance rather high. This cell was once widely used as the power supply in telegraphy.

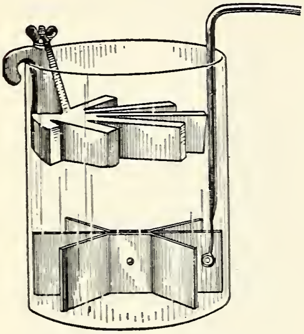

FIG. 5. Gravity cell.

470. Dry Cell: 1.5 Volts. The electrodes are zinc (negative) and graphite (positive); the zinc also forms the case of the cell. The electrolyte is a pasty solution of ammonium chloride (NH_4Cl). An oxidizing agent is added as a depolarizer (attacking the free hydrogen). The cell is then capped with sealing wax to prevent drying out. The electromotive force of the cell is about 1.5 volts, and its resistance low. The dry cell is convenient and has found wide use; batteries of thirty small dry cells in series (45 volts) have been widely used as " B batteries " in radio.

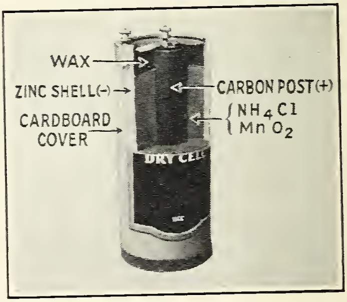

FIG. 6. Construction of dry cell.

471. Storage Cell: 2 Volts. After the chemical elements in the gravity cell or the dry cell are exhausted, we cannot restore them by simply reversing the current.[1] In the storage cell this can be done. The electrodes in the storage cell (when fully charged) are of lead (negative) and lead peroxide (PbO_2) (positive). To increase their effective surface the active material is present in both plates in a spongy form impressed into a rigid frame. The electrolyte is dilute sulfuric acid. The action of the cell depends on the fact that a negative ion is carried from the peroxide plate to the lead plate. When the cell is completely discharged the two plates are alike. (At the same time there is a replacement of O^{--} ion by SO_4^{--} so that in the discharged cell the plates become $PbSO_4$ instead of PbO.) As a consequence of the reaction sulfuric acid in the electrolyte is changed to water and the specific gravity decreases. As the student knows, the density of the electrolyte is an indication of the state of charge of the cell.

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[1] This is because of secondary effects which have occurred — diffusion in the gravity cell and depolarization in the dry cell.

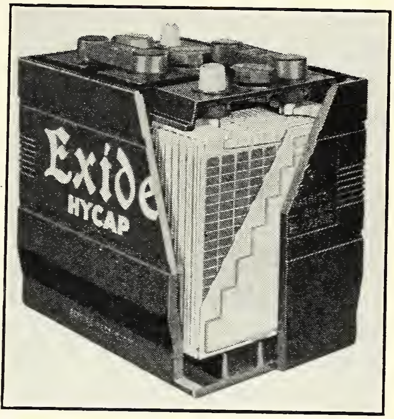


FIG. 7. Three-cell storage battery showing positive and negative plates and separator.

ACTION OF STORAGE CELL

	− Plate	Electrolyte	+ Plate
Charged........	Pb	H_2SO_4	PbO_2
Discharged......	$PbSO_4$	H_2O	$PbSO_4$

The electromotive force of this cell is about 2 volts, slightly higher when completely charged and often slightly lower when

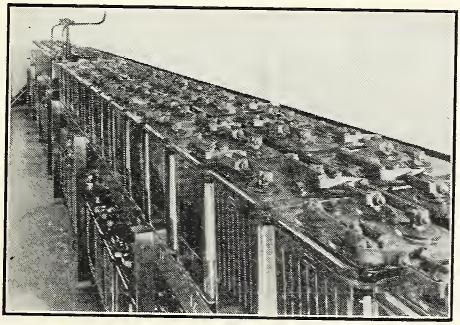


FIG. 8. Storage battery for auxiliary power in industrial plant.

partially discharged under ordinary service conditions. The internal resistance is very low but increases somewhat during the discharge as the electrolyte becomes less concentrated. In the automobile a battery of three cells is commonly used, giving an electromotive force of about 6 volts.

This is the widely used lead-acid storage cell. An alkaline cell, with potassium hydroxide (KOH) electrolyte, with iron and nickel oxide electrodes, is in less general use. Its electromotive force is about 1.2 volts. It was invented by Edison.

472. Standard Cell: 1.0188 Volts. The Weston standard cell (Fig. 9) has a closely reproducible electromotive force of 1.0188 volts at 20° C. It is used as a laboratory standard of potential. The cell is not designed to give appreciable current (its internal resistance is several hundred ohms).

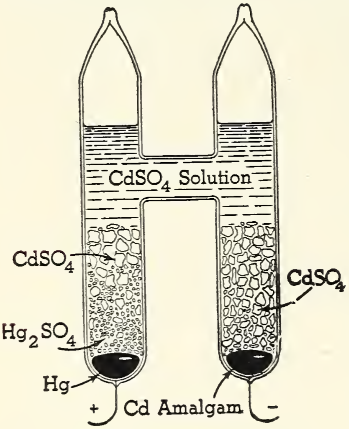


Fig. 9. Standard cell.

A standard cell has very much the same place in the electrical laboratory that a standard meter has in the mechanics laboratory, as a starting point for precise measurement. By means of the potentiometer (§442) other electromotive forces are compared with the standard cell. Then, with the aid of a standard resistance, definitely known currents can be obtained and electrical instruments calibrated.

473. Electromotive Forces in the Body. In 1780, twenty years before Volta's discovery, the physiologist Galvani found that two different metals touched to the leg of a dead frog caused the leg to contract. The frog leg was the first electrolyte and at the same time served, by its twitching, as the first current meter. Galvani did not understand the effect; it was called "animal electricity." Volta showed that a stronger current could be obtained by substituting a salt solution for the frog leg.

Galvani's discovery showed that nerve impulses are essentially electrical in nature. Many studies have been made of such phenomena. Measurable changes of potential accompany nerve activity in the living body; "brain waves" are recorded on the electroencephalograph and then are analyzed by the brain specialist or the psychologist as possible indicators of nervous instability. An electromotive force is generated in the heart as it contracts.

Accompanying the heart beat potential differences of several millivolts (much larger than the brain-wave voltages) can be observed between the two hands (or a hand and a foot). The voltage can be recorded (either directly or after amplification by vacuum tubes) by a special galvanometer designed to follow the rapid fluctuations (electrocardiograph).

QUESTIONS

1. Compare the number of hydrogen and oxygen atoms deposited in the electrolytic dissociation of water. Compare the two deposits by weight.

2. Which electrode is anode in a dry cell as normally used? in a storage battery being discharged? in a storage battery being charged?

3. Explain how the density of the electrolyte shows the state of charge of a storage battery.

4. Electrolysis occurs: (*a*) only in liquids; (*b*) in any solution; (*c*) in molten salts.

5. If the electrolyte of a storage battery is diluted by adding water the primary effect will be (*a*) to reduce the e.m.f.; (*b*) to increase the internal resistance; (*c*) to produce polarization.

Vocabulary: Combining weight, electrode, anode, cathode, electrolyte, electrochemical constant, primary (secondary) cell, local action, polarization, gravity cell, standard cell.

PROBLEMS

1. Find the weight of H^+ and of Zn^{++} deposited by 1 coulomb.

2. Suppose that a current of 2 amperes flows in Fig. 2 for 1 minute. Find the weight of deposit on each cathode.

3. A dry cell delivers 2 amperes for half an hour. By how much does the weight of the zinc case decrease?

4. In nickel plating compute the number of ampere-hours required for one gram of deposit (valence 2).

5. How many coulombs of electricity must pass to release a liter of hydrogen at 22.4 atmospheres pressure at standard temperature? a liter of oxygen?

6. A current of not more than 1 ampere per 100 sq. cm is recommended for copper plating. How long will it take to make a deposit 0.2 mm thick?

7. With a current of 100 amperes passing through acidulated water, how many liters of water are dissociated per day?

8. A storage battery is charged with a 10-ampere current for 8 hours. By how much will the weight of the electrolyte increase?

9. In the experiment described in §464 a current of 10 milliamperes flowed for 10 minutes. How much sodium was liberated?

10. In copper refining the potential drop across the cells ranges from 0.1 to 0.3 volt (depending on the strength of the electrolyte and the current used). Assuming a power cost of 0.3¢ per kilowatt-hour, find the power cost per pound.

CHAPTER 37

ELECTROMAGNETISM

474. Experiments in Electromagnetism. (1). Dip a wire carrying a strong current into a pile of iron filings. The filings cling to the wire, or rather they encircle it. They fall away when the current stops.

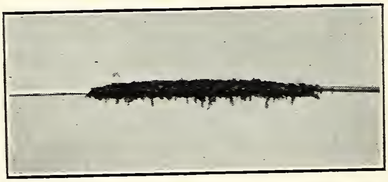


FIG. 1

(2). This is no simple attraction as can be shown by tracing the lines of force. Thrust a wire vertically through a hole in the center of a piece of cardboard. While a strong current flows in the wire, shake iron filings on the cardboard. Observe the lines of force which are depicted encircling the wire.

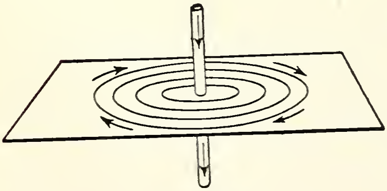


FIG. 2

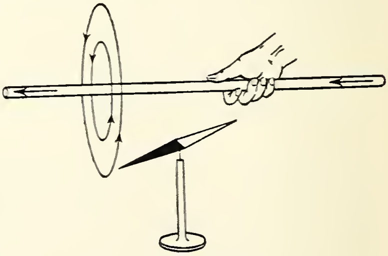


FIG. 3

(3). Follow these lines of force with a compass needle. Observe that if we look along the wire in the direction in which current flows, the lines of force are directed clockwise. A convenient rule: *Grasp the wire with the right hand, thumb in the direction of current; the fingers show the direction of the magnetic lines.*

475. Oersted. Magnetism and electricity were confused by the old natural philosophers. The electrical pioneers in the eighteenth and early nineteenth century sought for a relation between the two. But in 1819, when Oersted found the relation, it proved to be a very surprising one. Oersted was a popular lecturer in the University of Copenhagen. The story is that he made his discovery during a lecture demonstration. He was attempting to show that there was no relation between electricity and magnetism. A current from his voltaic cell was flowing through a wire; he had shown that a magnet pole was neither (*a*) attracted to the wire nor (*b*) dragged along with the current; and then he observed that there was indeed a force, but at right angles to both these directions. When his compass needle was just below or just above the wire, Oersted found that the north pole was pulled sidewise; when in front of the wire or behind it the force was up or down — the exact direction the student can find from the "grasping rule" of the last section.

476. Magnetic Field Near a Coil. When like currents flow in two parallel conductors (Fig. 4) the grasping rule shows that in the space between them their fields tend to oppose. (There is a neutral point, where the two fields cancel, midway between them.) When the currents are in opposite directions the two fields cooperate to produce a strong field between them. If a conductor is bent into a loop, the currents in all parts of the loop conspire to produce a field in a single direction within the loop. Grasp the conductor in Fig. 5*a* at any point; our rule shows that all lines of force as they pass through the loop go from left to right. When a current flows in a loop of wire, the loop of wire is

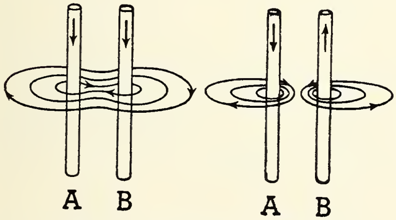
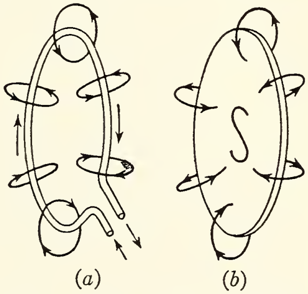


FIG. 4. Field about parallel currents.



(*a*) (*b*)

FIG. 5. Current loop (*a*) equivalent to magnetic shell (*b*).

equivalent to a magnet, with lines of force entering one face, leaving the other. This single turn of wire is equivalent to a magnetized sheet, with north and south poles on its opposite faces. (Such a magnetized sheet is called a *magnetic shell.*)

This "electromagnet" becomes stronger if we add more turns to our coil. If the wire is wound as a "helix" or "solenoid" (Fig. 6) we in effect stack these magnetic shells one on the other; the magnetic field of a solenoid is equivalent to that of a bar magnet. With a compass needle we can follow the course of the lines of force as they enter one end (the south pole) and leave the other (the north pole) of the coil. Or from our grasping rule we can predict their behavior. (Notice that the currents flow in the same direction in neighboring turns, tending to neutralize the field between them; hence in a closely wound solenoid the lines do not escape between the windings.) Either of the following serves as a useful rule for determining the poles of an electromagnet: (1) Grasp the coil with the right hand, fingers wrapped around it in the direction of current flow; the thumb points to the north pole; or (2) as one faces a north pole he observes the current flowing in a counterclockwise direction.

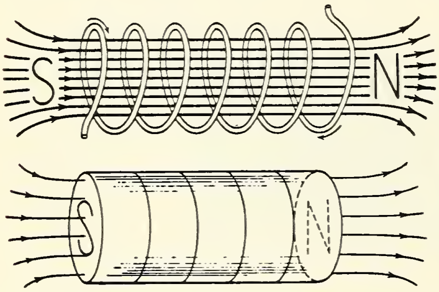


FIG. 6. Solenoid equivalent to bar magnet.

This should make somewhat clearer the explanation which was given of the wattmeter (§448). In what direction is the field produced by the field coils of Fig. 35–2?

477. Force on a Wire in a Magnetic Field. Having discovered that a current exerts a force on a magnet, Oersted could reason that a magnet must also exert a force on a current. Suppose that the wire in Fig. 7 carries a current directed into the page. We can trace out the field; it is such that both poles are pushed upward (south pole of course contrary to the field). By the law of reaction an opposite force (downward) is exerted upon the wire. *A force is exerted upon a wire carrying a current across the lines of force of a magnetic field.* We have already been introduced to this force (Chapter 32). It was indeed used to define the electromagnetic unit of current. This is the force which deflects our galvanometer coil and turns our electric motor. The left hand "three-finger rule" has been given to find its direction (§419). We may call this the "motor rule." Show that this rule predicts the downward force in Fig. 7.

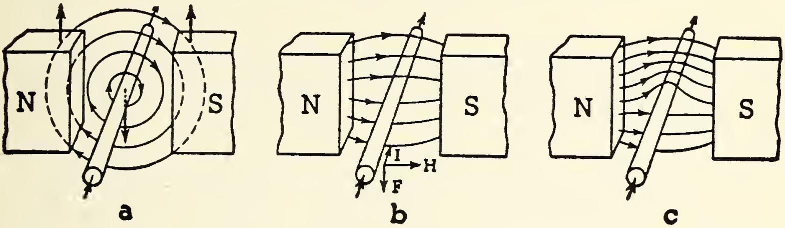


FIG. 7. Current in magnetic field. (*a*) Field of wire. (*b*) Field of poles. (*c*) Combined fields.

This force can be easily demonstrated with a carbon-filament lamp. Bring a bar magnet near a lamp carrying direct current. The glowing filament bends in the magnetic field — it bends more and more as, with the approach of the magnet, the field about it becomes stronger; it bends more and more if the current in the wire is gradually increased. The force is jointly proportional to H and to i[1] and to the length of wire (l). If we use *absolute electromagnetic units* (§421) the constant is unity and

$$F = Hil. \tag{14}$$

(This holds for a conductor *transverse* to the field.)

478. Field Strength. (1) It is rather easy to show that a single circular turn of wire (radius r) carrying current of i abamperes creates a magnetic field at the center of strength

$$H = \frac{2\pi i}{r}. \tag{15}$$

Proof: Suppose a unit magnet pole to be placed at the center of the loop. This pole will produce a field (say H') at the wire of strength $H' = 1/r^2$ (from eq. 3, p. 346); hence the force on the wire ($H'il$) is equal to il/r^2 or (since $l = 2\pi r$) $F = 2\pi i/r$. An equal opposite force must be exerted by the current on the unit pole, and by definition this is the strength of the field.

---

[1] The small letter i will be used to represent the current in electromagnetic (rather than practical) units. *When currents are expressed in amperes the right-hand member of all equations in this chapter must be divided by 10.*

If there are N turns in a *compact circular coil* the field at its center is

$$H = \frac{2\pi N i}{r}. \qquad (15')$$

Sometimes eq. (15) is used as the defining equation for unit current — an electromagnetic unit (abampere) of current flowing in a circle of unit radius creates a field of 2π gauss at its center.

479. It is also rather important to know the field strength in two other situations: near a straight wire and within a long solenoid. We shall give the relations without attempting to derive them:

(2) The magnetic field at a point a distance r from a *long straight wire* is

$$H = \frac{2i}{r}. \qquad (15'')$$

(3) Except near the ends, the field in a long solenoid is uniform. (The same number of magnetic lines passes through each unit area.) If n' is the number of turns of wire per centimeter, the field within a *long solenoid* is

$$H = 4\pi n'i. \qquad (15''')$$

These relations are all for electromagnetic units. When the current is given in amperes its value must be divided by 10.

Example. Find the field within a long solenoid with 100 turns per centimeter when a current of 1 ampere flows. $H = 4\pi \times 100 \times 0.1 = 126$ gauss. This is the number of lines of force per square centimeter (§353). Suppose that in this example the cross-section of the coil is 10 sq. cm; then a total of 1260 lines of force pass through the coil. This is called the *magnetic flux.*

480. Magnetic Moment of Solenoid. A solenoid carrying a current is equivalent to a bar magnet (§476). Its "poles" are those points near the ends, not well defined, from which the lines of force begin to bristle outward. There are $4\pi n'i$ lines of force per unit area or a total flux of $4\pi n'iA$ lines of force entering one end, leaving the other, of this electromagnet. Since 4π lines leave a unit pole, each pole is $n'iA$ units strong. If L is the length of the solenoid, its magnetic moment is $n'iAL$ or NiA, where $N(= n'L)$ is the total number of turns.

If we think of each of the N turns of the coil as corresponding to a magnetic shell, the whole magnet is a stacking of these shells. Since for the whole solenoid the magnetic moment is NiA, *the magnetic moment of the imaginary shell due to a single turn of wire is equal to the current times its cross-sectional area.*

Examples. A single turn of wire carrying 1-ampere (0.1 abampere) current has the same magnetic moment as a thin sheet of iron of the same shape magnetized with a magnetic moment of 0.1 unit over each square centimeter of its area.

Suppose that a coil is made by wrapping 50 turns of wire about this book (the shortest way round). The dimensions of this coil are 15 cm by 4 cm; area 60 sq. cm. Find its magnetic moment when it carries a 10-ampere current.

$$M = NiA = 50 \times 1 \times 60 = 3000 \text{ units.}$$

It thus has the same magnetic moment as the "typical" permanent magnet which we have used in several problems (p. 342).

481.* Magnetomotive Force. Observe that there is one fundamental difference between the nature of the magnetic field about a current and the electric field about a static charge or the gravitational field about the earth. In an electrostatic field it requires a definite amount of work to carry a unit charge from one given point to another. (This is the potential difference.) It makes no difference what path is chosen. Hence if a charge is carried from one point to another and then by some different path back to the original point altogether no work is done. Going out we may go with the field; coming back we move against it. The work in carrying the charge about a closed path is zero. But when a magnetic pole is carried about a current the situation is different. As it passes around the wire, say against the lines of force, the field opposes the motion at all times and a definite amount of work is done. Such a field is said to be "rotational." This work done on a unit pole as it goes about the closed path is called the **magnetomotive force** (m.m.f.) of the path (it might more properly be called the "magnetomotive work").

Now there is a certain definite value for the magnetomotive force about a conductor carrying a current i, irrespective of its size and shape. Consider carrying a unit magnetic pole in a circular path around a straight wire, moving against the magnetic field. The magnetic field is equal to $2i/r$, and the distance is $2\pi r$. Hence the work done in carrying the unit pole once around the current is $4\pi i$. This is the magnetomotive force of the path

$$\text{M.m.f.} = 4\pi i.$$

The same value is found for any other path around this conductor, or for a closed path around any other conductor, for example, around a wire bent in a loop (the path now passing around and through the loop). Consequently the magnetomotive force for a path passing through and around a coil or a solenoid of N turns (any length) is $4\pi Ni$. It is proportional to the number of amperes and the number of turns or (in the language of the engineer) to the number of " ampere-turns."

PRACTICAL USES OF ELECTROMAGNETISM

482. The Electromagnet. When current flows through a solenoid and a magnetic field is excited, an iron core placed in the coil becomes magnetized by induction. We must think of the lines of force due to the solenoid as extending through the iron itself, and along with these are now many more lines arising from the magnetism induced in the bar. This coil with its iron core constitutes an electromagnet. Its intensity of magnetization may be much stronger than in a permanent magnet, and the magnetization can be turned on or off by the throw of an electric switch.

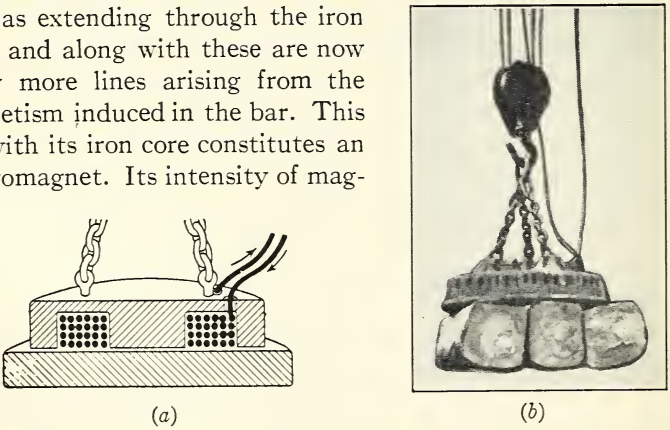

(a) (b)

FIG. 8. Iron-clad lifting magnet. The magnet in *b* is lifting 30 tons of steel ingots.

The electromagnet seems a rather obvious application of Oersted's discovery. Today with a little insulated wire and an iron bar we can make a small electromagnet which we can excite with a dry cell. We can operate a larger one from the power line. It is hard to picture the difficulties which faced Faraday and Henry and Ampère and others as they worked out the discovery of Oersted. A decade elapsed before Joseph Henry, later distinguished American scientist but at the time a young teacher in the Academy at Albany, learned to insulate wire (with his wife's silk petticoats), to wind it on his " spools " or " bobbins," and so invented the first electromagnet.

Henry was the outstanding physicist in America in the period from 1825 to 1875. At Albany he invented the first primitive electric motor and the telegraph, and he discovered electromagnetic induction (about the same time as Faraday) and self-induction (§522). He became professor of natural philosophy at Princeton and taught physics, mathematics, chemistry, mineralogy, geology, astronomy, and architecture, and discovered the principle of the transformer and of the oscillating spark. Leaving Princeton he organized the Smithsonian Institution and created our Weather Bureau, and his experiments with fog signals made the U. S. Lighthouse Service preëminent.

The young Henry had been taken from school at the age of 13; he seemed unable to learn.

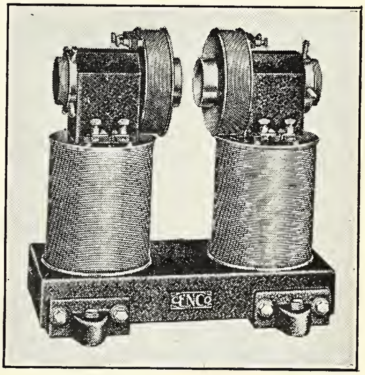


Fig. 9. Electromagnet.

Small electromagnets are used in electric bells, electric relays, telegraph sounders, and telephone receivers. Larger ones are employed in electric motors and generators. Figure 8 shows the " iron-clad type " widely used in lifting large iron masses (girders, castings, scrap iron, etc.). Figure 9 shows a type of magnet used in physical research for producing strong fields (in the gap between the poles).

Electromagnets are usually designed so that the magnetic lines of force are carried almost entirely through iron. In Fig. 9 there are large iron cores extending through the lower vertical coils (and obviously through the upper coils); these are yoked together at the base by a heavy iron bar. There is a complete circuit of iron except for the gap where the field is formed. In the iron-clad lifting magnet shown in Fig. 8 the iron shell acts as a return path for the lines of force. The lines encircle the coil and opposite poles are produced at the center face and at the rim. When the iron plate (in *a*) or the pig-iron billets (in *b*) are attracted to the magnet they link the two pole faces and complete the circuit of iron for the lines of force. The iron plate in *a* is called the *armature* of the magnet.

Dr. Gilbert (Queen Elizabeth's inquiring physician) discovered the importance of furnishing a return path for the lines of force and increased the strength of his lodestone magnets by " arming " them with an iron shell which he called the " armature." Today the term is applied, not to the shell of the magnet itself, but to a separate iron part attracted to the poles of the magnet, for example, this lower plate in Fig. 8*a* or the iron clapper in the doorbell (Fig. 10) or in telegraph instruments. In electric motors and generators the armature may be an iron cylinder rotating between stationary poles or it may be a stationary shell within which the field magnets revolve.

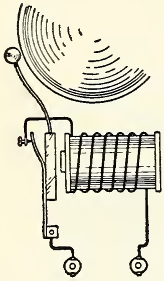


FIG. 10. Doorbell.

483. The Doorbell. One of the first electrical problems for many of us was: how does electricity operate the doorbell? The doorbell contains a small electromagnet which attracts the armature and the hammer. But on its way to the magnet coils the current flows through a contact at the back of the armature; no sooner is the armature attracted than the current is interrupted and the hammer flies back. This completes the circuit again, and the armature is once more attracted. Hence the vibration of the hammer and the ringing of the bell.

484. Telegraph Sounder and Relay. As a rather obvious application of the electromagnet came the telegraph sounder. Joseph Henry had made a crude telegraph system between his home and his laboratory. Samuel Morse, an artist turned scientist, had a vision of telegraphy on a larger scale; with help from Congress a line was built, and in 1844 the first telegraph message was sent between Washington and Baltimore. In the simple sounder, when current flows the armature is attracted to the electromagnet which it strikes with a click; by means of a code of dots and dashes the letters are transmitted. This simple sounder has been developed until today messages, sent from a keyboard hundreds of miles away, are typed out directly.

Because of line resistance Morse could not operate his receiving instrument directly over the complete distance from Washington to Baltimore. He invented the relay. This was a slight modification of the sounder. A contact was placed on the armature so that, when attracted, it closed a second circuit, acting as a key for a second section of line which was operated by its own battery. A relay station midway between the two cities automatically repeated, retelegraphed, the message from one section of line to the other.

Electromagnetic relays have many important applications outside of telephony. The thermostatic contact (Fig. **19**–3*a*) or the photocell (§587) may operate a sensitive relay, quite like the telegraph relay, which in turn closes the power circuit to start a furnace, perhaps, or to light a room. A simple push button may control the lights of a large hall — not directly but by operating a large magnetically controlled switch.

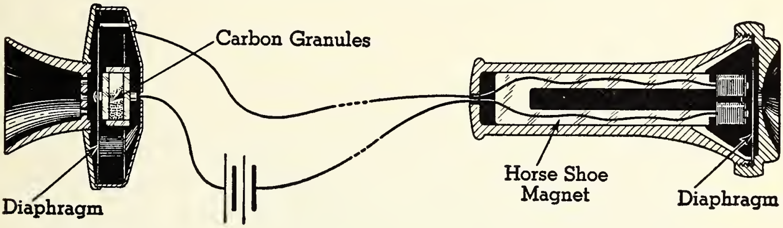


FIG. 11. Simple telephone circuit.

485. The Telephone. In 1876 Alexander Graham Bell invented the telephone. In the telephone receiver a flexible iron diaphragm rests close to the ends of a permanent horseshoe magnet. Wound about each pole is a small coil, and as the current in these coils varies it increases and decreases the pole strength. The diaphragm is thus attracted now more, now less, and its vibration produces sound waves in the air.

Figure 11 represents a very simple telephone line, giving one-way transmission. The transmitter (carbon microphone) contains a pile of carbon granules in a chamber shaped like a pillbox, and through these granules the current must pass. Sound waves of the voice vibrate the transmitter diaphragm; this diaphragm is fastened to the flexible lid of the " pillbox " and as it vibrates alternately compresses and loosens the granule pile and so changes its resistance. In this way the sound waves effect variations in the current in the line; passing through the receiver these current fluctuations reproduce the sound. . . Using several dry cells the student can construct a telephone circuit in the laboratory.

This is not the simplest possible telephone circuit. The student can make a telephone circuit simply with two receivers (without battery) using one of them as a transmitter. The voice vibrates the diaphragm, and this changes very slightly the air gap and so varies the magnetic flux. This flux change causes a current (by electromagnetic induction, Chapter 39). The original Bell transmitter was of this kind.

Crystal microphones are also used, depending upon the piezoelectric effect (§383). Piezoelectric and electromagnetic microphones are used in radio transmission since they give somewhat greater fidelity than the carbon resistance type. The carbon resistance type is more sensitive, however, and is in general use in telephone service.

486. The Direct-Current (D-C.) Motor. Figure 12 shows the principle of the d-c. motor. Current flows through a coil in a magnetic field. In the figure the coil is represented by a single loop. Contact is made with the power lines by *brushes* which bear against a split ring (the *commutator*). When current flows around the loop in the direction shown, the magnetic field exerts a force upward on the left side, downward on the right (by the three-finger motor rule), turning the coil. After the coil has turned through 90° and just as the wires are about to reverse sides, the direction of the current flow through the loop is automatically changed by the commutator. The result is that current is always flowing outward in the left-hand wire, inward in the other, and there is always the couple turning the coil clockwise.

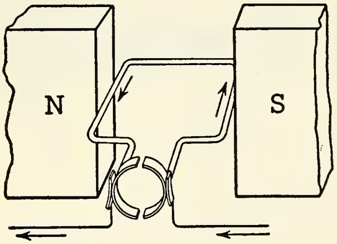


FIG. 12. Principle of d-c motor.

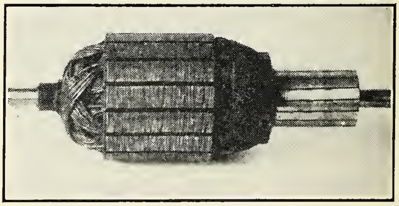


FIG. 13. Armature and commutator in demonstration motor (or generator).

In practical motors the field is established by an electromagnet; this is called the *field magnet.* The rotating coil is wound upon an iron core (the *armature*). There are usually not one but many coils each wound on the armature, each wrapped at a different angle, and the commutator consists of as many pairs of corresponding segments. As the armature turns, the current about it is reversed coil by coil; at all times the current is flowing *in* in the wires on one side, *out* on the other side of the armature.

The action of the motor can be explained in a slightly different way. As current flows about the armature winding (the loop in Fig. 12), into the right side, out from the left, magnetic lines of force are set up, passing upward in the figure. The armature is thus an electromagnet with north pole at the top. This pole is attracted by the south pole and repelled from the north pole of the field magnet; hence the turning of the armature is reduced simply to action between poles. The commutator insures that there will always be a north pole on the upper side and a south pole on the lower of the rotating armature.

487. Types of D-C. Motors. D-c. motors are classed as series wound, shunt wound, or compound wound depending upon the way in which the windings of the field magnet are connected to the armature windings and power line. In the series-wound motor the field windings consist of a few turns of heavy wire, and these field coils are connected (through the commutator) in series with the armature (Fig. 15). Field and armature are excited by the same current, and (because of the counter electromotive force of the armature) this current decreases at high speed. Hence the series-wound motor has a stronger field and exerts a much greater torque at low speeds than at high. Armature and field are connected in parallel in the shunt-wound motor. The

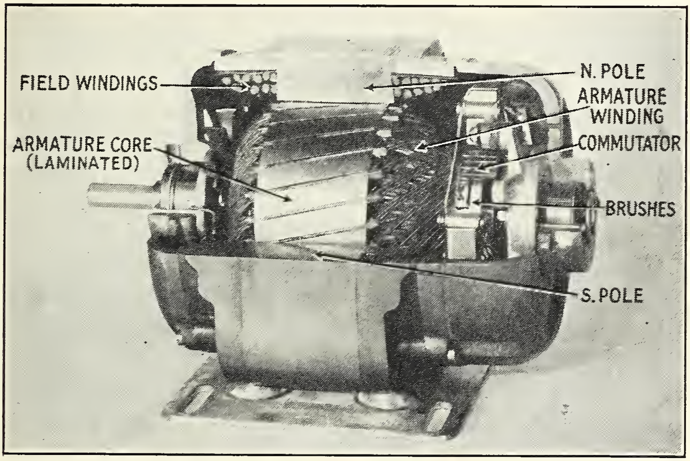


Fig. 14. 4-pole d-c motor.

field coils now have rather high resistance and take a small, nearly constant current, and the motor exerts a nearly constant torque at all speeds. The compound-wound motor has a combination of both windings. Where there is heavy starting duty, as in streetcars and electric locomotives, series motors are used; the shunt or compound type is in most general use.

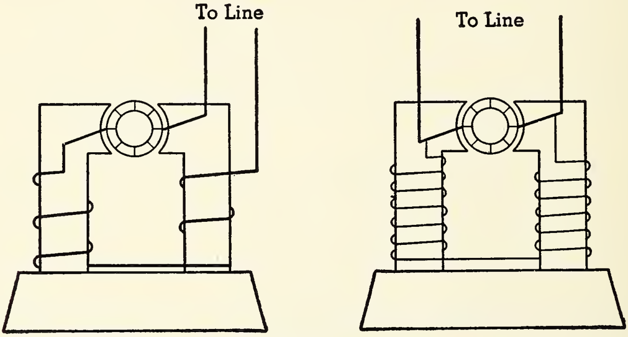


FIG. 15. Series and parallel field windings.

Instead of two, the field of the motor may have four, six, or more poles, but except for these rather superficial differences all d-c. motors are much alike. A-c. (alternating-current) motors will be described later. The chief advantage of the d-c. motor is that with it variable speeds can be readily obtained (§515).

488. Comparison of the Iron Magnet and the Solenoid. It has been shown that current flowing in a solenoid excites a field quite similar to that of a bar magnet. But the lines of force of a solenoid do not begin and end on " poles " — they form continuous lines threading their way through the coil. The field is " rotational " (§481). We could adopt the same point of view in respect to the iron magnet. Indeed, we often do so. Let us imagine a field of a bar magnet with lines continuing unbroken through the iron. (Draw six lines passing *upward* in the bar magnet of Fig. 16.) The intensity of this field is represented by B. So we have two kinds of fields, alike in air but different in iron: our H field originating in north poles and terminating in south poles, and now our B field for which there are no poles. In air there are no poles in any event, and here B and H are the same; they are very different in iron. Indeed, within a permanent magnet the two fields are opposite; there is a weak *H* field which tends by induction to demagnetize the bar (§493).

The *B* field is called the " induction field " to distinguish it from our earlier (*H*) " force field " and we call these endless lines, " lines of induction." But these are only names. Since a free test pole cannot be placed within the bar itself we cannot decide whether it is the *H* field or the *B* field which here really represents the force which would act on a unit pole. Nor shall we care. Both fields are used. In the next chapter we shall consider *H* as the field which tends to magnetize or demagnetize a bar by induction and the *B* field as the field which results from the induction. And we shall find that the iron magnet is in fact an electromagnet.

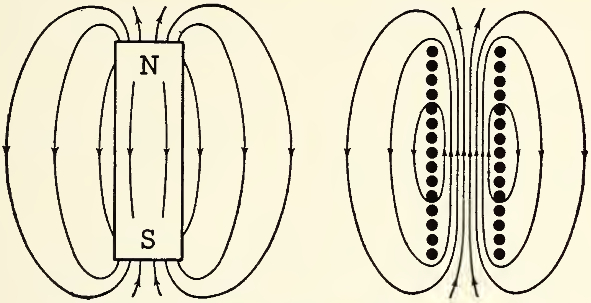


FIG. 16. *H* field of bar magnet and solenoid.

489. Résumé. A current is encircled by a magnetic field. The grasping rule gives its direction. Its value is $2i/r$ near a straight wire; $2\pi Ni/r$ at the center of a compact coil; $4\pi n'i$ within a solenoid. (Divide by 10 if the current $[I]$ is in amperes.) But about a wire of any shape the magnetomotive force is $4\pi i$.

There is also a force on a current which passes across lines of force; its direction is given by the left-hand three-finger rule; its magnitude is *Hil*.

Each coil of a solenoid acts as a magnetic shell; the magnetic moment of this shell is equal to the current times the area.

QUESTIONS

1. What is the direction of the magnetic field above a current flowing east, to the east of a current flowing north, above an electron moving north, in the middle of a coil about which current flows counterclockwise?

2. What is the field strength 2 cm from a long straight wire, at the center of a single turn of 2-cm radius, in a long coil of 10 turns per centimeter — all with 1 ampere flowing?

3. A current flows through a loop of wire. In what direction should the plane of the loop be placed in the earth's field to experience the greatest couple?

4. A telegraph wire carries current to the east. What is the direction of the force on it in the earth's field?

5. What is the essential difference in the operation of a galvanometer and that of a motor?

6. Which of the following phenomena is *not* used in a microphone (telephone transmitter): (*a*) variation of resistance; (*b*) piezoelectric effect; (*c*) electrolysis?

7. The series-wound motor is notable for its (*a*) high efficiency for heavy duty; (*b*) large torque at low speeds; (*c*) constant angular velocity.

Vocabulary: Electromagnet, solenoid, magnetomotive force, d-c. motor, armature, field coil, *H* field, *B* field.

PROBLEMS

1. What is the direction of the magnetic field (*a*) in the center of Fig. 1, p. 430; (*b*) below Fig. 8, p. 402; (*c*) within coil, Fig. 6, p. 400 (due to coil alone); (*d*) at the Bunsen burner, p. 439; (*e*) just above the battery, p. 439; (*f*) at the center of the field coils, p. 431 (due to these coils alone)?

2. Assuming current flowing into right-hand wire, label magnet poles in Fig. 10; in Fig. 15; in Fig. 8.

3. Indicate direction of flow of current around each coil in Fig. 9 to produce a field from right to left between pole pieces.

4. Indicate direction of force acting on the left-hand wire in Fig. 12, p. 464; Fig. 6, p. 400; Fig. 4, p. 455.

5. If the direction of flow of current is reversed, will the direction of rotation of either the series- or shunt-wound motors in Fig. 15 be reversed?

6. A power line extending east and west carries a current of 100 amperes. Find the force exerted by the earth's field on a meter length of the wire.

7. Assume in Fig. 7 a current of 7 amperes flowing in a field of 100 gauss; assume the pole pieces 12 cm square. Estimate the force on the wire.

8. Assume the dimensions of the wire loop (armature) in Fig. 12 to be 5 cm by 10 cm. If a current of 50 amperes flows and the field strength is 2000 gauss, estimate the couple acting on the armature.

9. The suspended coil in a galvanometer is 1 cm by 3 cm and has 200 turns. The magnetic field is 5000 gauss. Find the couple exerted upon it when a current of 10^{-8} ampere flows.

10. Suppose that the armature coil in Fig. 12 consists of 50 turns (instead of one) and is 5 cm wide and 7 cm long. Suppose that a current of 20 amperes flows through it and that the field strength is 8000 gauss. What is the force on each side of the coil, and what is the torque acting on it?

11. One hundred turns of wire are wrapped around the rim of a circular disc, 20 cm in diameter. What is the field at the center when a current of 5 amperes flows?

12. A compass needle is placed at the center of a circular loop of wire 1 meter in diameter, containing 50 turns. How large a current must flow to neutralize the earth's field at the needle? How must the coil be oriented?

13. A glass tube is wrapped with wire, one turn to the millimeter. A current of 50 milliamperes flows. Compare the field in the tube with the earth's field.

14. On a stick 2 cm by 2 cm by 20 cm are wound 1000 turns of wire. Find the field strength in the stick when 10 amperes flows. How many lines of force pass through the stick? (See §353.)

15. If each wire carries 5 amperes, what is the strength of the field at bottom of Fig. 1, right edge of Fig. 2, midway between wires Fig. 4 left, center of Fig. 5, within coil Fig. 6? (Estimate or measure distances on page.)

16. Find the couple acting on our " typical bar magnet " (p. 342) placed parallel to and 5 cm below a straight horizontal wire which carries a current of 15 amperes.

17. A wire carries a current of 20 amperes toward the east in the earth's field. Find the position of the neutral point (resultant field zero). What is the field strength at the same distance on the opposite side of the wire?

18. Two wires are suspended vertically, 1 cm apart (Fig. 4). A current of 10 amperes flows downward through the right-hand wire. What will be the direction and magnitude of the field at the other wire? Suppose that an equal current also flows downward through this left-hand wire; what will be the force on a meter length of it (direction and magnitude)?

19. What is the magnetic moment of the coil in problem 14?

20. How much current must flow through a 100-turn coil, 4 cm in diameter, to give it the same magnetic moment as the " typical bar magnet " (p. 342)?

21. One hundred turns of wire are wrapped around a pencil, $\frac{1}{2}$ sq. cm in cross-section. What is the magnetic moment of the coil when 10 amperes flow? What is the field strength at a point along the extension of the pencil, 1 meter from the center?

CHAPTER 38

MAGNETIZATION OF IRON

490. An Experiment in Magnetic Induction. A small glass tube, a little longer than a knitting needle, is wrapped with wire. Through the wire we shall send a current which we can increase in small steps up to about 5 amperes. Knowing the number of turns per centimeter we can compute the field strength in the coil corresponding to each current (§479).

A knitting needle is placed in the coil; as the field is increased the needle will become magnetized, more and more strongly, by induction. The coil and needle should be properly located a little distance east or west of a magnetometer, so that the magnetic moment of the needle in each field can be determined by the magnetometer deflection (§358). Dividing the magnetic moment by the volume gives the intensity of magnetization (§346).

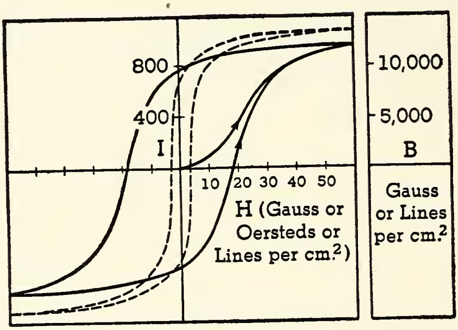


Fig. 1. Hysteresis loop for knitting needle and (broken line) for soft-iron wire. (The scale given for the B field is approximate, drawn equal to $4\pi I$.)

As we strengthen the field step by step and watch the deflection of the magnetometer, it becomes very apparent that the intensity of magnetization which is induced is not proportional to the strength of the inducing field. In Fig. 1 the inner branch of the solid curve, starting from the origin, shows the increase in intensity of magnetization (I) when the field is applied to an unmagnetized needle. The iron is but slightly affected by a small field; the intensity of magnetization increases rapidly for fields of 15 to 25 gauss[1] and finally, when most of the dipoles become lined up in the field, approaches a limiting value. Now an increase in the field can produce little additional magnetization. The iron is said to be *saturated.* This shows that the **susceptibility** (the ratio of the intensity of magnetization to the H field [§356]), is not a constant; it is small for weak fields, then becomes larger, and then decreases.

491. Hysteresis. Not only is the induced magnetism not proportional to the field; its values are different for increasing and decreasing fields. If, after a bar is magnetized to saturation, the field is gradually weakened the curve is not retraced and when the field is removed the bar retains some of its magnetization. (Upper branch of the curve; *residual magnetization* [§344], 800 units.) It requires a certain reversed field to demagnetize the bar, about 18 gauss in the figure; this is called the *coercive force.* If the field is further increased in this reversed direction the bar again becomes saturated.

As the field is increased or decreased the change in induced intensity appears to lag behind; this lagging effect is called *hysteresis.* It recalls the relation between stress and strain after the elastic limit is exceeded; a strain remains after the applied stress is removed. And hysteresis like inelasticity causes a generation of heat. Energy is required to magnetize a bar; this energy would be returned when the bar lost its magnetism if it were not for hysteresis. Hysteresis appears to be due to a kind of frictional force between the dipoles opposing either magnetization or demagnetization. The energy loss may become quite serious if the electromagnet is excited by alternating current.

A hardened piece of steel (such as our knitting needle) shows high hysteresis. Soft iron shows much less. A great deal of research has been necessary to develop the " transformer iron " with small hysteresis frequently required in electrical apparatus. In motors, generators, and transformers silicon steel is used, with coercive force of about 0.4 gauss; its hysteresis curve if plotted in Fig. 1 would have a spread of less than a millimeter. Refer to Table 34 and indicate along the H axis of Fig. 1 the coercive forces of several iron alloys.

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[1] The unit of the H field here is frequently called the " oersted " (App. 3).

492. The H Field and B Field. Often it is convenient to describe magnetization of iron by the field produced rather than by the actual intensity of the magnetization. Since 4π lines leave a unit north pole and enter a unit south pole, the number of lines of force passing through the iron in virtue of its induced magnetization is $4\pi I$ per square centimeter;[1] in addition there are the H lines of the inducing field; hence the value of the magnetic induction is

$$B = 4\pi I + H.$$

The H field is to be considered as the cause of the magnetization; it is the inducing field. The B field is the result, the sum of the direct field H of the solenoid itself and of the field created by the lining up of the dipoles in the iron (Fig. 2).

The ratio of B to H in the iron is called its *permeability* (μ); or

$$B = \mu H. \qquad \text{(Def.)}$$

An iron core multiplies the number of lines of force of the H field by μ to give the lines of the B field (the " lines of induction "); for iron the value of μ is usually several hundred or several thousand. For air the intensity of magnetization is practically zero; therefore $B = H$ and the permeability is practically 1.

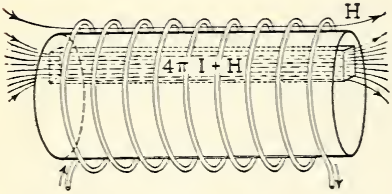

FIG. 2. $B = 4\pi I + H.$

If B (the number of lines of induction per square centimeter) is multiplied by the cross-sectional area of the solenoid, we get the total magnetic flux. The **flux** is the total number of lines of induction passing through a solenoid or a bar magnet or any given area in space:

$$\text{Flux } (\Phi) = B \times A \qquad \text{(Def.)}$$

if the area (A) is perpendicular to the B field.

Example. A long coil of 10 turns per cm is wound on an iron bar, cross-section 8 sq. cm. Find the H field when 1 ampere flows. Supposing the permeability of the bar to be 1000, find the B field, the total flux, and the intensity of magnetization of the iron. (*a*) The field strength is $4\pi n'i = 4\pi \times 10$

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[1] It has been shown (§346) that the intensity of magnetization (I) is the pole strength per unit area.

$\times 0.1 = 12.6$ gauss. (*b*) $B = \mu H = 1000 \times 12.6 = 12{,}600$ lines per square cm. (*c*) The flux then is 8 times this or approximately 100,000 lines.

493.* Field of the Poles. In treating the H field as due entirely to the solenoidal current we have been tacitly assuming that the core of the solenoid was (like our knitting needle) of such length that the effects of its free poles were negligible. In obtaining a hysteresis curve by the magnetometer method (§490) the sample must be long (in comparison with its diameter). (Better yet, the field can be applied about a ring-shaped specimen and the flux determined by electromagnetic induction [§502]; free poles are then entirely avoided.) For short bars there is an additional H field due to these poles (Fig. 37–16). This field is opposite to that of the magnetizing solenoid, and has a demagnetizing effect: it lessens the induction, delays saturation, lowers the apparent residual magnetism, and accounts for lines of force escaping along the sides of the bar.

494.* The "Magnetic Circuit." To get the maximum flux in an electromagnet, in a transformer, in the magnetic field of a generator or motor, it is essential to minimize the effect of free poles. The lines of induction are carried as largely as possible through iron. In the motor the "magnetic circuit" (as the path of the flux is called) passes through the cores of the field coils, around through the iron case of the motor and through the armature; only at the gaps between pole pieces and armature must the flux pass through air. In the horseshoe magnet the poles are close together and the greatest part of the magnetic circuit is through iron; indeed, when the horseshoe magnet picks up an iron bar the bar completes the circuit for the flux and the induced poles in the bar largely neutralize the demagnetizing effect of the magnet poles.

In the design of electromagnetic machines the computation of magnetic flux is as important as the computation of electrical current. The engineer has developed a rather simple method which avoids all mention of free poles. (1) He computes the *reluctance* of his magnetic circuit. Reluctance (speaking loosely) is a measure of the difficulty of establishing flux in the magnetic circuit.[1] It varies with the length of the magnetic circuit, inversely with its cross-sectional area and with the permeability of the material, just as the resistance of an electrical circuit depends on length and cross-section and conductivity. Because of its smaller permeability a 1-in. air gap furnishes as much reluctance as some thousands of inches of iron. (2) Acting to establish the flux is the magnetomotive force of the magnetizing solenoid. If there are N turns and the current in each is i abamperes the magnetomotive force is $4\pi Ni$ (§481), or it is $4\pi/10$ times the number of "ampere-turns." (3) The number of lines of induction (flux) in this magnetic circuit is then simply the magnetomotive force divided by the reluctance.[2]

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[1] The exact definition: Reluctance of a magnetic circuit is the magnetomotive force divided by the flux. Compare eq. (9″), p. 423.

[2] The analogy to the computation of electrical currents is obvious; the student will understand, however, that the resemblance is only in the mathematics. There is very little in common between the concepts of magnetic lines of force *established* in the "magnetic circuit" and electrons *flowing* in a conductor.

495. Ampère's Theory of Magnetism. Reading the permeability from the hysteresis curve and finding the reluctance and the magnetomotive force of his magnetic circuit, the engineer computes the flux and designs his electromagnets. But we must still ask ourselves the most interesting question — what after all is this thing which we call magnetism? At first it was supposed that the poles of a magnet between which lines of force (H) stretch were very real things. Nothing more could be said of a magnetic dipole than that it was a pair of these poles. But Ampère, following upon the discovery of Oersted, suggested that each of these little elementary magnets was itself simply a small whirl of electricity — or as we say today the dipole is an electron revolving rapidly about in an atom. The poles themselves have no more real an existence in the dipoles of iron than they have for a current flowing in a loop of wire. When iron is magnetized these Amperian currents all whirl about in the same plane. All go about counterclockwise as we face the north pole of the magnetized bar.

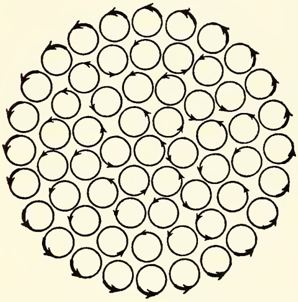

FIG. 3. Amperian whirls (north pole). Arrows represent "currents" within the atoms; actual electron motions are opposite.

These Amperian whirls take place *within* the atoms; these are no *free* electrons in random motion, free to collide with molecules and produce heat, and so these currents within the atoms continue their rotation forever without resistance. Nevertheless in other respects the effect is almost exactly the same as if an ordinary current flowed around the sides of the magnet. Figure 3 represents a cross-section of the magnet. It is seen that through the interior of the magnet the opposite currents cancel each other in their effect. Not so on the surface, and the net effect of these small counterclockwise Amperian whirls throughout the body of the magnet is the same as a counterclockwise surface current flowing around the sides of the magnet. Hence we can consider a bar magnet as essentially the same thing as an electromagnet, with current flowing around it in an infinitely thin shell; this current combines its magnetic field with that of the inducing coil to give the B field.

We are looking now into the real nature of a magnet; the axis of the dipole is simply the axis of revolution of the electrons; the poles of the magnet resemble the geographic poles of the earth, merely marking the ends of the axes of revolution of the currents in the magnet. In these respects the magnetic dipole is in marked contrast with the electric dipole (§383), in which the displaced positive and negative charges have a real existence.

496. Force between Currents. The force between magnets, either electromagnets or permanent magnets, is in reality a force which exists between currents. It is easy to demonstrate this force. If two long wires are hung about a centimeter apart (Fig. 37–4) they will attract each other when currents flow in the same direction in each; they repel when one current flows up, one down. This attraction is simply a consequence of the motor rule, as the current in one wire flows through the magnetic field of the other. (Notice that the field at wire *B* due to the current in *A* is out from the paper. Applying the three-finger rule, the force on *B* is to the left when its current is downward.) Or we can trace the lines of force encircling the two wires when the currents are parallel; acting as rubber bands they tend to pull the wires together. . . But although the magnetic field is useful in computing the force, the simple statement of the facts of magnetic attraction and repulsion can be made in the following terms:

(1) Currents flowing in the same direction attract.
(2) Currents flowing in opposite directions repel.
(3) When currents flow at an angle in two wires, the wires tend to set themselves parallel to one another.

These, rather than the laws of forces between " poles," would be taken by one interested in the direct relations between electric charges as the fundamental laws of magnetism.

497.* Magnetic Forces. When opposite poles of magnets are brought near one another as in Fig. 4, the Amperian whirls in each magnet go in the same direction and each atomic current in the one attracts an atomic current in the other. Or, if each magnet is considered as a surface current shell, these like currents attract. When like " poles " are together the currents are opposite and there is repulsion.

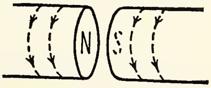

FIG. 4. Attraction between unlike poles.

We can now face a question which from the first may have seemed very puzzling: How can the force acting on the north pole of a magnet placed near a current be directed around the current rather than toward it or away from it? This seems to contradict Newton's Law — action and reaction should be in the same straight line. The answer is now clear. The " forces " acting on the poles are not truly forces: forces act on masses, and poles are only imaginary points. It is true that there is a *couple* acting on the magnet in Fig. 5 (tending to turn it counterclockwise) but this couple is furnished by the opposite forces acting on the opposite sides of the current shell, attracting on the left where the currents are parallel, repelling on the right where they are antiparallel. These forces are equivalent to the pair of forces on the poles, but their line of action is either directly toward or away from the wire.[1]

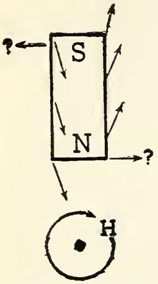

FIG. 5. Actual forces on magnet are in the plane of the wire.

498.* Permeability. We have defined permeability in terms of the magnetic field H and the induction B. Suppose that, after a field is excited in a solenoid, a bar of iron with permeability 1000 is introduced. Each line of the original field now excites 999 more lines in the iron, so that B is 1000 times greater than H. We can interpret this in terms of Amperian whirls. Suppose that there are I amperes passing through N turns on the solenoid; this is in effect the same as NI amperes passing once about the solenoid. If now we introduce the iron bar of permeability 1000, an Amperian current is set up in the surface layer 999 times greater than the current in the wire; the total current (flow current and whirl current) is 1000 times greater than the current flowing through the wire and measured in our ammeter. The permeability appears as the measure of the increase in circulating current due to Amperian whirls.

499.* Paramagnetism and Diamagnetism. Iron, cobalt, and nickel, and a few alloys are the only substances which are strongly magnetic — these substances are said to be *ferromagnetic.* But all other substances show a slight magnetic effect; they are either " paramagnetic " or "diamagnetic." A piece of platinum suspended by a fine silk fiber between the poles of a strong electromagnet sets itself along the lines of force. The magnetic effect is the same as in iron but a million times smaller. Such a substance is *paramagnetic.* But if a short glass rod is suspended in this way the rod sets itself transversely across the lines of force. Glass is repelled, not attracted, by a magnet pole. This means that the induced magnetism is contrary to the field! The susceptibility is *negative.* Such substances are *diamagnetic.* The majority of common substances — copper, water, organic compounds, etc. — are weakly diamagnetic.

Ferromagnetism appears to be an extraordinary enhancement of paramagnetism which, somewhat like superconductivity, for certain substances can take place at sufficiently low temperatures. Each ferromagnetic substance has a definite temperature (the " Curie point ") above which it suddenly loses its ferromagnetic properties and becomes merely paramagnetic. The Curie point for iron is 769° C; for nickel, 356° C.

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[1] This is not quite all there is to the matter. Though the force on the moving electron is always in the plane of the conductor, never around it, it is not always as in Fig. 5 a simple force of attraction or repulsion. When the magnetic field at the charge is changing (Chapter 39), the charge is dragged by the current either up- or downstream (parallel to the conductor). Indeed, the laws of conservation of momentum and of angular momentum are violated if we consider only the motions of the charges themselves. *We must attribute mass and momentum to moving lines of force* (Chapter 43).

500.* Magnetic Moment of the Atom. In paramagnetic and diamagnetic substances the electrons within the atom are the source of the Amperian whirls. According to the Bohr theory, which presents a simple view of the atom (§381), electrons in the atom move about in orbits, much as planets move about in the solar system. Each electron going about in a circle is effectively a small circular current, an Amperian whirl, a small electromagnet, a dipole. It also appears that each electron is spinning on its own axis — just as the planets in the solar system spin on theirs — and this again means magnetic moment. The remarkable fact is that an electron under all circumstances always has exactly the same amount of spin just as it has the same charge). We move the electron, speed it up or slow it down; we cannot increase or decrease this spinning " current " of the electron within itself. Hence each electron itself is a magnet — its magnetic moment (called a " magneton ") is 1.85×10^{-21} c.g.s. unit. It is perhaps even more remarkable that the electron, choosing its orbit of motion in the atom, is restricted to motions which will result in magnetic moments of 0, 1, 2, 3, etc., magnetons (§381). The magneton appears as the elementary magnitude of magnetism.[1]

When an atom has a magnetic moment (due to orbital motion or spin or both) the substance is paramagnetic (or, for iron, nickel, and cobalt below the Curie temperature, ferromagnetic). The elementary magnets are alined in the magnetic field and give a positive intensity of magnetization and increase the B field. But in a majority of atoms the spins and orbital motions of the various electrons exactly cancel and there are no Amperian whirls in the absence of a field. As will be shown in the following chapter, the introduction of a field induces a current (here an Amperian whirl), and the induced current is in a direction which opposes the field which produces it. Such atoms are diamagnetic.

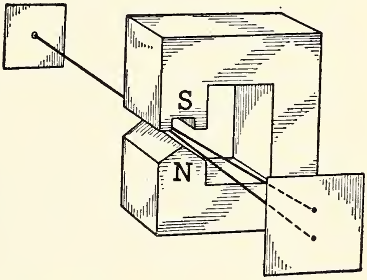

FIG. 6. Stern-Gerlach experiment.

The first direct evidence of this elementary quantity of magnetism was obtained in 1923 when Stern and Gerlach measured the drift of atoms of silver vapor in a magnetic field. Silver vapor passed into a vacuum tube through narrow slits and striking the farther end of the glass tube deposited in a small spot. But when the beam passed through the strongly divergent field near a powerful magnet pole, the beam was split as shown in Fig. 6. Half of the beam had been pulled down by the neighboring N pole, half pushed away. This showed that these silver atoms had each a single spinning electron with a magnetic moment of one magneton which set itself either parallel to the field and was attracted, or set itself opposite to it and was repelled. This experiment, measuring the magnetic moment of the electron, ranks with Millikan's experiment, measuring its charge, as one of the crucial experiments of physics.

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[1] It is not true, however, that the total magnetic moment of the atom, produced by the vector addition of spin and orbit moments, is always an integral number of magnetons; it is often a simple submultiple.

501. Résumé. Of great practical importance is the relation between the magnetic field (H) and the intensity of magnetization (I), or the relation between the field (H) and the magnetic induction (B). These quantities are not proportional; neither susceptibility nor permeability is a constant; their values vary with the field. Also there is the phenomenon of hysteresis, and because of hysteresis energy is transformed to heat each time the magnetization is reversed.

Magnetism is interpreted as due to motions of electrons. The student cannot find a better illustration of the way in which the scientist looking beneath the surface phenomenon seeks for the true explanation. Magnetism can be explained in terms of poles near the ends of the magnet; currents flowing about the sides, although at first glance quite different, actually account for the same phenomena. And this latter gives a unified view of magnetism and electricity and mechanics. This will be the view adopted by the student who wishes to see clearly the meaning of of the world about him. None the less it is convenient to summarize all the properties of Amperian currents and the forces between them, in terms of their poles, magnetic moments, and magnetic fields. It is not necessary to abandon the magnetic pole when we discover that it is a useful tool and not a material particle for force to act on.

QUESTIONS

1. Suppose that the following H fields are applied in sequence to an unmagnetized knitting needle: 20, 60, 0, -60, 20 gauss (or oersteds). From Fig. 1 give the intensities of magnetization.

2. Explain why B is approximately $12\frac{1}{2}$ times I. Does this rule hold for a non-magnetic substance, for example air?

3. Why is a horseshoe magnet less likely to lose its magnetism when a piece of soft iron (the " keeper ") is retained across its poles?

4. The permeability of a diamagnetic substance is slightly less than one; for water $\mu = 0.999990$. Will the force between two magnets, with a given separation, be increased or decreased when they are submerged in water (§340)?

5. The susceptibility of air is about: (*a*) 0; (*b*) 1. Its permeability is about: (*a*) 0; (*b*) 1.

6. For a permanent magnet coercive force is more important than residual magnetism when the magnet: (*a*) is short; (*b*) is made of a steel alloy; (*c*) has a high Curie point.

Vocabulary: Saturation, oersted, hysteresis, coercive force, transformer iron, permeability, flux, demagnetizing field, magnetic circuit, Ampère's theory, para- (dia-) magnetism, magneton.

PROBLEMS

1. Find the susceptibility of a knitting needle (Fig. 1): (*a*) when placed in a field of 10 units; (*b*) when the field is increased to 30 units. (Assume the needle initially unmagnetized.)

2. The volume of a knitting needle is 2 cc. Find its magnetic moment when it is placed in a solenoid with 10 turns to the centimeter carrying a 2-ampere current (Fig. 1). (Assume the needle initially unmagnetized.)

3. An iron ring 4 sq. cm in cross-section is wrapped with wire. Assuming a permeability of 1000, find the B field and the flux when a current produces a field of 10 gauss (or oersteds).

4. The core of an electromagnet (Fig. 37–9) has a cross-section of 10 sq. cm and a permeability of 1000. Compute the flux when the coil excites an H field of 10 gauss (or oersteds).

5. An ordinary meter stick is wrapped with a thousand turns of wire. Estimate the value of the flux through the stick when a current of 1 ampere flows. What will the value be for a steel core of the same size if the permeability is 500?

CHAPTER 39

ELECTROMAGNETIC INDUCTION

For a decade after Oersted's discovery scientists tried to produce a converse effect — to produce electric currents by means of magnets. The effect was discovered by Faraday in the Royal Institution in London and, independently, by Henry in the boys' preparatory school in Albany.

502. Electromagnetic Induction; Principle of the Generator. On October 17, 1831, Faraday made a coil of 220 feet of copper wire and connected the ends to his crude galvanometer. Into this coil he thrust a bar magnet — the galvanometer needle deflected momentarily. He withdrew the magnet, and the needle jumped in the opposite direction. . . Fifty years later came generators and transformers, and ultimately came electrical power in every home.

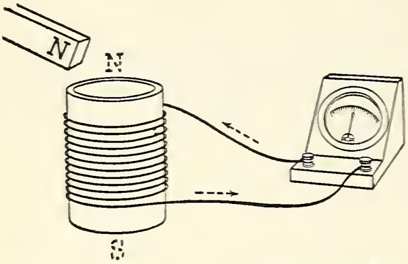


FIG. 1. Current induced by approaching pole.

Let us connect a coil of wire to a galvanometer and repeat this experiment. We thrust a north pole up to one end of the coil. We shall observe that *while this north pole is approaching a counterclockwise current flows through the coil;* a clockwise current flows when the N pole is withdrawn (or when a S pole approaches). *We have an electromotive force whenever the flux through the coil is changing.*

This is a remarkable experiment. A changing magnetic field! There lay the secret for an electrical age! And as we repeat this experiment today in our well-equipped laboratories, with measuring instruments on the shelf, insulated wire for the asking, we must not underestimate this achievement of Faraday and Henry when, with power only from Volta's cells, with measuring instruments of their own devising, with coils laboriously insulated turn by turn, they observed the deflection of a compass needle.

If our coil has many turns we can produce a galvanometer deflection simply by turning the coil over quickly in the earth's magnetic field. Again the current flows only while the number of lines of force passing through the coil is changing... And now, if we spin the coil quickly around before a bar magnet (or spin the magnet quickly around before the coil), we can generate a small current first in one direction, then in the other — we have indeed a model of an alternating-current generator.

Obviously there would have been no current in the coil if its circuit had been open; but there would still have been an electromotive force, *tending* to make current flow. *An electromotive force is induced in a coil whenever the magnetic flux through it is changing.*

503. Direction of Induced Electromotive Force. (Conservation of Energy). Now, as we push the magnet in and out of this coil, causing current to flow, we are generating electrical energy; hence we must be *doing* work. The induced current makes the coil momentarily an electromagnet, and its magnetism must be such as, in every instance, to *oppose* the motion. It must repel as a pole approaches and attract as the pole recedes. When a north pole approaches, the current must be such (counterclockwise) as to make the adjacent face of the coil north (repelling); as the north pole is withdrawn this same face must be south (attracting) — the current must be clockwise. Lenz's Law is: *The induced current in a coil flows in such a direction as to oppose the motion which produces it.*

504. Magnitude of Induced Electromotive Force. The magnitude of the induced electromotive force can, like its direction, be derived from the principle of conservation of energy. We shall give the result: In electromagnetic units *the induced electromotive force in each turn of wire is equal to the rate of change of flux through it.* Or if there are N turns in the coil

$$\text{E.m.f.} = N\frac{\Delta\Phi}{t} \text{ abvolts.} \tag{16}$$

To reduce this result to volts it must be divided by 10^8 (§422).

Example. Find the average e.m.f. when a coil of 1000 turns, 100 sq. cm in cross-section, is inverted in the earth's field, supposing that it takes $\frac{1}{2}$ sec. to invert the coil. The earth's field is approximately 0.6 gauss.

$$\Phi_1 = 0.6 \times 100 = 60;\ \Phi_2 = -60;\ \Delta\Phi = 120.$$

$$\text{E.m.f.} = \frac{N \cdot \Delta\Phi}{t} = \frac{1000 \times 120}{0.5} = 2.4 \times 10^5 \text{ abvolts} = 2.4 \text{ millivolts.}$$

505. Cutting Lines of Force. There is another way to regard electromagnetic induction. When the flux in a coil is increased its turns must cut through the lines of force; *the electromotive force induced in any part of the circuit is equal (in electromagnetic units) to the number of lines it cuts per second.* This is a particularly useful point of view when a straight conductor is sweeping through the field.

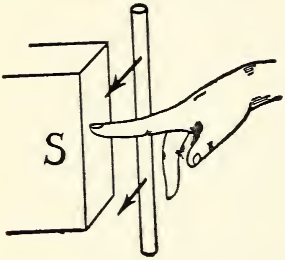


Fig. 2. Induced e.m.f. when conductor cuts lines of force. Direction of e.m.f. is given by three finger rule.

The effect can be shown by passing a wire, connected to a galvanometer, through a magnetic field. An electromotive force is induced in the wire, and a current flows in the circuit. Suppose that the field strength is 5000 gauss (lines per square centimeter), that the effective part of the wire cutting the lines is 5 centimeters long (l), that the wire moves 100 centimeters per second. Then the lines are cut at the rate of $5000 \times 100 = 2{,}500{,}000$ per second and the electromotive force is 2,500,000 abvolts or 0.025 volt. An electromotive force (equal to $H \times l \times v$) is generated in a conductor as it passes through the lines of force.

$$\text{E.m.f.} = Hlv. \qquad (16')$$

The star-shaped disc in Fig. 3 dips into mercury. If the disc is rotated it cuts the lines of force and an electromotive force is generated. This device was demonstrated by Faraday a few days after he had made the basic experiment which we have described. It was indeed the first d-c. generator.

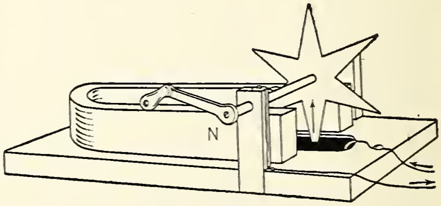


Fig. 3. Faraday disc generator.

The direction of the electromotive force in a conductor moving across a magnetic field is given by the three-finger rule on the *right* hand. The thumb now represents the motion of the conductor, the first finger the direction of the magnetic field, and the center finger gives the induced electromotive force, tending to produce current. This right-hand rule is the "generator rule"; compare it with the "motor rule" (§477). Apply this generator rule to the Faraday disc; with the electromotive force generated as shown, which way is the disc being turned?

506. In summary: An electromotive force is induced in a coil when the magnetic flux through it is changing or in any part of a circuit when this cuts lines of force. The direction of the electromotive force is obtained either from Lenz's Law for a coil or the three-finger rule for a single wire. The magnitude of the e.m.f. (in abvolts) is equal to the rate of change of the flux (for a single turn) or to the number of lines cut per second. There are 10^8 abvolts in 1 volt.

GENERATORS AND A-C. MOTORS

507. A-C. Generators. (1) **The Magneto.** When the loop in Fig. 4 is rotated in the field between the two magnet poles an alternating electromotive force is induced around it. At the instant shown the left side is moving upward, the right downward; by the three-finger rule, electromotive forces are induced in the directions shown by the arrows. A moment later the motions will be reversed and the electromotive force will be opposite. This gives an alternating electromotive force which can be fed to the line through brush contacts on the *slip rings* shown in the figure.

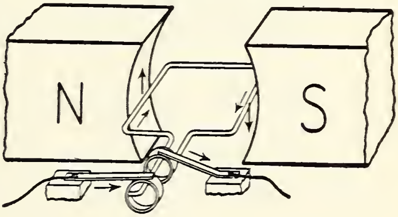


FIG. 4. Principle of a-c generator.

This figure is only schematic. In practice the rotating coil consists of many turns wrapped on an iron core (the armature). Small generators of this type, with permanent magnets and often turned by hand, are called magnetos. They are used for bell ringing in rural telephones.

508. D-C. Generator. If the line is connected to the armature by means of the commutator (as in Fig. 37–12) instead of the slip rings, the armature connections are reversed at each half cycle and the electromotive force on the line, though pulsating, is always in the same direction. This is the principle of the d-c. generator. In the practical generator, instead of one, many coils are placed at different angles about the armature; the commutator has correspondingly many segments (Fig. 37–13), and the pulsations are reduced to a hardly perceptible ripple.

D-c. generators are classified (like d-c. motors, §487) as series wound, shunt wound, or compound wound, according to the way in which the field magnets are excited. In the series-wound generator the line current passes through the field windings, and the field and generated electromotive force *increase* when more current is drawn. In the shunt-wound generator, the opposite is true. The high-resistance field windings are in parallel with the line; as more current is drawn in the line, the magnet current is somewhat reduced and the electromotive force *decreases.* The compound winding is a compromise between these simple cases which gives an electromotive force substantially constant over a wide range of conditions.

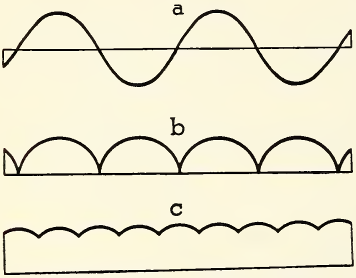


FIG. 5. Generated e.m.f.
(*a*) A-c. generator.
(*b*) D-c. simple coil armature.
(*c*) D-c. double coil armature.

509. The first practical generator was developed by the French engineer Gramme in the 1870's. It had a ring-shaped armature. At first the field magnets were excited by batteries, but soon Gramme devised his " division machine " in which the field current was picked off from the armature output. This Gramme dynamo first showed the possibilities of electrical power and was the incentive for the development of electric lights which startled the world at the Paris exposition of 1879, and of electric traction which came into being during the next few years. (Hitherto the only practical applications of electricity had been in telegraphy and electrolysis). The Gramme dynamo itself became the first practical motor, when it was found, perhaps by accident, that one of these machines would run another. The early generators were usually direct current; the great advantages in alternating currents were not realized until twenty years later.

510. A-C. Generators. (2) The Alternator. Alternating current rather than direct current is usually employed in power distribution. The large alternator of today differs in three respects from the magneto which has been described: (1) its armature (the " stator ") is stationary while the field magnets (the " rotor ") revolve; (2) instead of two it has anywhere from four to a hundred magnet poles; (3) the field magnets are electrically excited by an auxiliary d-c. generator. The corresponding advantages are: (1) slip rings are avoided for the main line currents; (2) the rotor speed is reduced; (3) stronger fields are obtained. Figure 7 shows diagrammatically the operation of a four-pole alternator. The coils on the stator are connected

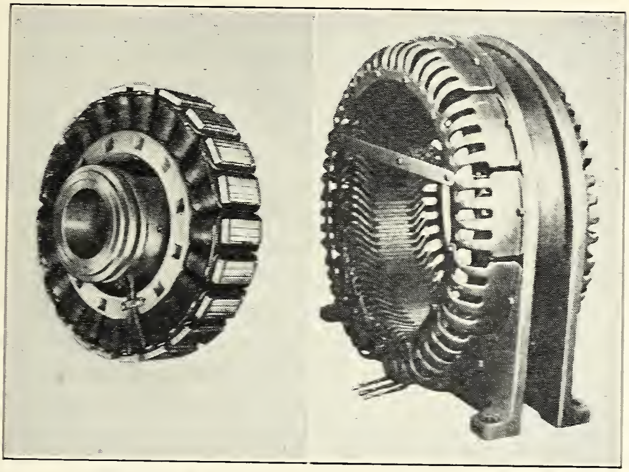
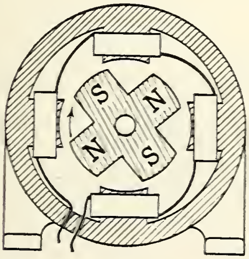
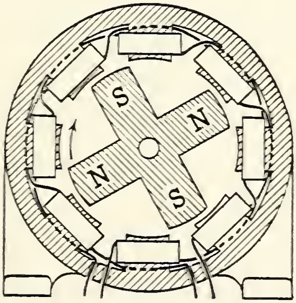


FIG. 6. Low-speed alternator. Rotor showing poles and slip rings. Stator showing armature windings and 3-phase outlets.



4 Pole Single Phase Alternator

FIG. 7



4 Pole Double Phase Alternator

FIG. 8

in series in such a way that as a north pole approaches one coil and a south pole approaches the next the induced electromotive forces add up. Then, as the field poles leave these coils and approach the following ones, the electromotive force is reversed. Since there are two poles of each sign on this four-pole machine there are two a-c. cycles per revolution. (The windings and slip rings of rotor are not shown in the figure.)

511. Two-Phase Alternator. The spaces between the armature coils are wasted in Fig. 7. Another series of coils placed here serves as a second independent current supply. Evidently the electromotive force generated in these coils is just 90° out of phase with the first (Fig. 9); the magnetic flux through these coils is a maximum when that in the other coils is zero. This is a *two-phase* generator. Often there are three such sets of independent circuits on the stator (three-phase generator). Because of economies in generation, distribution, and use, most power plants generate either two- or three-phase current.

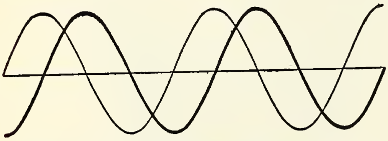


Fig. 9. Two-phase alternating e.m.f.

The motive power of the generator may be a water wheel or a steam turbine or a fuel-oil (Diesel) engine. The modern electrical generator is very efficient; the greatest cost factors are interest charges and distribution costs rather than fuel. Hence water power usually has no great advantage over the steam plant, and, because of the initial cost, may be uneconomical. The actual fuel cost in a modern steam plant is usually less than ½ cent per kilowatt-hour.

512. Eddy Currents. When a metal plate passes through a magnetic field so as to cut the lines of force, currents are set up in it. They flow in such a direction as to oppose the motion of the plate. The electrical energy is dissipated as heat. These are "eddy currents." Such eddy currents may cause serious losses in generators and motors. To

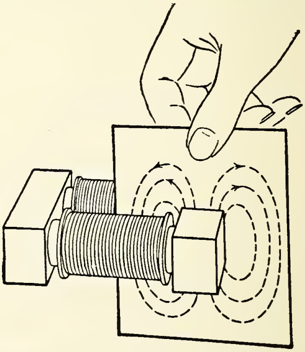


Fig. 10. Eddy currents.

avoid them the iron parts of the generator and motor are laminated — that is, constructed of many thin layers, roughly insulated from one another. This interrupts the flow of eddy currents in the iron.

Eddy currents are sometimes used to damp out oscillation. An example is the magnetometer (Fig. 29–11); there is a copper plate just below the bar magnet, and eddy currents in this plate bring the swinging magnet to rest.

513. Induction Motor. A striking demonstration of eddy currents is to spin a permanent magnet just above a copper disc, free to rotate. Eddy currents flow in the copper in such a direction as to drag back (Lenz's Law) on the magnet and (by reaction) to set the disc in motion. The disc is soon spinning with almost the same speed as the magnet.

Instead of actually spinning the magnet, the same effect can be obtained by means of two-phase currents. Coils *a*, *b*, and coils *c*, *d* (Fig. 12) are connected to the two different phases. At one moment current is flowing in the first coils making pole *a* north and *b* south; the current in *c*, *d* is zero. A quarter cycle later *c* is north and *d* is south. A quarter cycle later the current has reversed in *a*, *b*, and *a* is south and *b* is north. In this way two opposite poles chase around the disc perhaps 60 times a second. As before a disc placed in the rotating field is set spinning by the induced eddy currents.

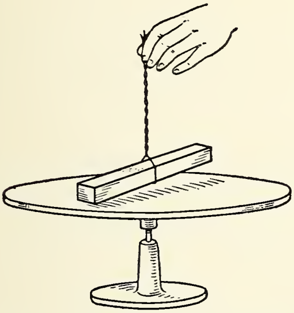


FIG. 11. Rotation produced by eddy currents.

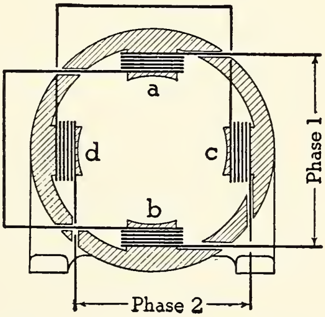


FIG. 12. Stator of induction motor.

This is the principle of the two-phase induction motor. The motor has no brushes or slip rings; the armature currents are excited by induction in a " squirrel-cage armature." This consists of copper strips embedded in the laminated core of the armature, the ends of the strips all connected together by copper rings. Through these conducting circuits flow the currents induced by the rotating field. This is the type of electric motor in most general industrial use. (The ordinary small motor used in the household usually works on this principle. Though the applied voltage is only single phase, a phase change is effected in one of the motor windings.)

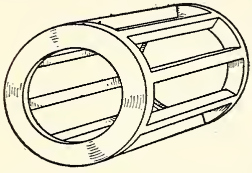


FIG. 13. Squirrel-cage rotor (without iron core).

514. Synchronous Motor. The rotor in the induction motor cannot quite keep pace with the rotating field, for then there would be no changing field in the rotor and hence no induced current. A permanent magnet can be used for the rotor, and this must either keep exactly in step with the rotating field or stop entirely. Small synchronous motors are used in electric clocks. Alternating current is usually generated at 60 (sometimes 50 or 25) cycles per second. As long as the generated frequency is maintained exactly the electric clock must be right. In large synchronous motors the rotor is an electromagnet, excited by direct current.

515. Conservation of Energy in Generators and Motors. As we watch a generator spinning quietly on its ball bearings it all seems so easy! We do not realize what a big job the steam or water is doing for us. Often it is proposed to make a perpetual-motion machine by having the generator turn the motor which then turns the generator — and perhaps have a little power left over for distribution!

To be sure, a generator does run very easily when it is not furnishing current, but when current passes through the armature its direction is such as to oppose (by the motor rule) the motion which produces it. The rotor offers a counter torque in direct proportion to the current which flows. . . And the motor when spinning idly has (by the generator rule) a back electromotive force which practically suppresses the flow of current. More and more current flows in proportion to the work the motor does.[1] To understand completely the action of a generator it must be considered at the same time as a motor, striving to turn in the opposite direction but not succeeding because of the compensating torque from the driving engine; and in the motor we must see a generator, exciting a counter electromotive force.

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[1] In a-c. motors (for example, constant-speed synchronous motors) the power input under load increases because of a phase change in the current and an increase in the " power factor " (§535).

When the d-c. motor is running with no appreciable load it assumes a speed such that its generated counter electromotive force is practically equal to the line voltage and no appreciable current flows. Thus when it idles it consumes very little electrical energy. When a load is applied, the speed and back electromotive force are slightly reduced, to allow the necessary current to flow. In either case the speed required to produce the counter electromotive force will be inversely proportional to the magnetic field strength. Hence the speed of d-c. motors can be reduced either by increasing the field current or by decreasing the potential difference applied to the armature brushes.

516. " Super Power." Today our country is linked by a great network of transmission lines. Power is transmitted for hundreds of miles, from coal fields and rivers and mountain streams, to merge with the power from city plants. The power of ten thousand horses through a single wire! Already large parts of our country are tied together, all the generators in exact synchronism, beating together like the many hearts of some strange mammoth, forcing the life blood of electricity into the great arteries and then into the smaller capillaries until it reaches our workshops and homes to turn the motor or kindle the light or tell the time. This is " Super Power." Perhaps we can look forward to the time when the whole nation will be a single network, when the coal from a Pennsylvania mine and the waterfall on a western mountain range will work together to push the electrons when I turn on my switch. The whole country one machine! Plans have been laid for this nation-wide harnessing of power, but, interest charges being what they are, it is not practical at present. It is cheaper to transport coal.

Electricity must be transmitted at high voltage (§455). The output of the generator is stepped up to some 20,000 or 50,000 or 100,000 volts for cross-country transmission, is stepped down to possibly 3000 volts as it enters the city, is stepped down again to 110 or 120 volts near the home. In the economical use of electricity the *transformer* plays a role almost as important as the generator and motor.

MUTUAL AND SELF-INDUCTION

517. Mutual Induction; Principle of the Transformer. The experiments in §502 can be repeated with a solenoid carrying a current substituted for the permanent magnet. (This coil is called the *primary*; the coil attached to the galvanometer, the *secondary*.) When the primary approaches, the direction of the current in the secondary will be opposite to that in the primary (since, by Lenz's Law, adjacent poles are now alike). As the coils separate, the two currents have the same direction.

Exactly the same results are obtained if, instead of moving the primary back and forth, the coils are placed close together and we simply increase and decrease the primary current. Again the changing flux produces an electromotive force in the secondary. The effect can be shown by merely closing and opening the primary circuit (Fig. 14), starting and stopping the primary current. This is mutual induction: the excitation of an electromotive force in one coil due to a change in current in a neighboring coil. . . The direction of the electromotive force in the secondary coil is opposite to the primary current when the latter is increasing; in the same direction when it is decreasing. For example, when the circuit in Fig. 14 is closed, starting primary current flowing clockwise (viewed from above) the electromotive force drives current counterclockwise around the secondary.

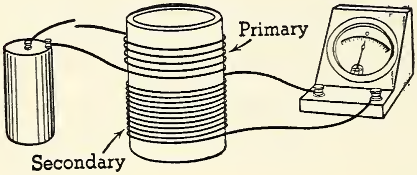

FIG. 14. Principle of transformer.

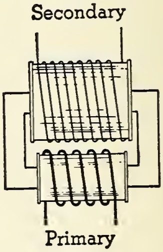

FIG. 15. Step-up transformer. Coils are linked by iron core.

518. The Transformer. If an iron core passes through the two coils the flux changes will be greater and the effects will be more pronounced. Let us now pass alternating current through the primary. Sixty times a second this current changes from clockwise to counterclockwise and back again, and each time this primary current changes an electromotive force is set up in the secondary. This is the transformer.

In the practical transformer the iron core is usually made in the form of a closed ring to decrease reluctance (§494) and increase the magnetic flux. This construction is illustrated in Fig. 15. (Since practically all the lines of flux pass around through the iron, placing the primary and secondary on opposite sides of the ring causes no appreciable decrease in the mutual induction.) The iron core is always laminated to prevent eddy currents.

There is an important relation between the voltage and current in primary and secondary. *The differences in potential across the primary (input) and secondary (output) are proportional to the number of turns in the two coils.* (The electromotive force depends upon the number of times the lines of flux cut the turns of the coil.) But on the other hand *the currents flowing in primary and secondary are inversely proportional to their number of turns.* Thus if V_p, I_p, and N_p represent the voltage, current, and number of turns in the primary, and V_s, I_s, and N_s the corresponding quantities in the secondary,

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{I_s}{I_p}.$$

It follows as a consequence of these relations that the power input in the primary (I_pV_p) equals the power output (I_sV_s) from the secondary. These relations are for ideal transformers. Actually the power output is reduced by heat losses; there are losses due to resistance of the windings (I^2R) and to hysteresis in the core and to eddy currents in the core. The efficiency of a large power transformer may be as high as 98 per cent. The transformer is cooled by circulation of oil to remove the 2 per cent of heat generated; the cooling pipes can be seen in Fig. 16.

519. The Transformer as an Electrical Machine. Transformers serve a purpose in electricity analogous to that of levers and pulleys, etc., in mechanics. By means of a lever we can apply a small force through a large distance and produce a large force acting through a small distance. By means of transformers we can change low-voltage high-current a-c. input into high-voltage low-current output, or the reverse. Input and output power are the same (except for heat losses), but the transformer changes the power to a form suitable for the purpose at hand.

Transformers are classified as " step-up " or " step-down " according to whether they raise or lower the voltage. Just as we speak of the mechanical advantage of a lever, so we might speak of the " electrical advantage " of a transformer. The step-up transformer has large " electrical advantage." (This term, however, is not conventional.) Large transformers are used in power-transmission lines to step the voltage up for transmission and to step it down for distribution. Step-up transformers are used for lighting neon signs, step-down transformers for ringing doorbells. . . Step-up transformers are used for exciting x-ray tubes and for other high-voltage work in the laboratory; for the large currents and low voltage needed in electric welding and electric furnaces step-down transformers are used.

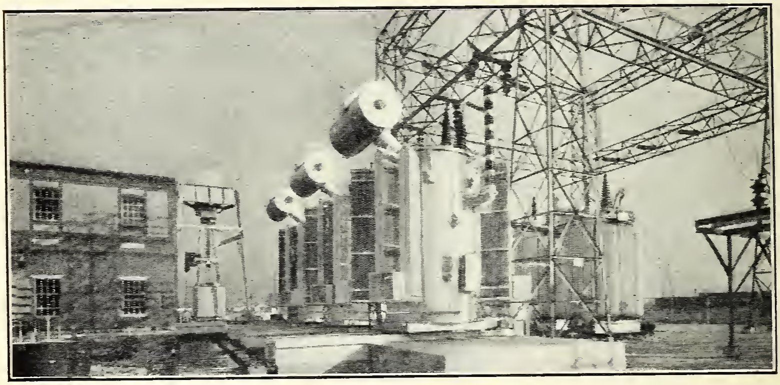

FIG. 16. Step-up transformers at power plant.

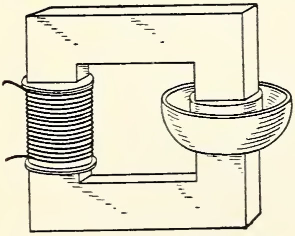

FIG. 17. Induction furnace.

520. The Induction Furnace. Here the furnace is itself a step-down transformer. As used in the refining of metals, the crucible is ring shaped and the metal in the crucible acts as a single-turn secondary and is melted by the large induced current. Another type of induction furnace, used with high frequency currents (§545), is without an iron core. The primary is a few turns of wire. A graphite crucible placed within this coil is heated to incandescence by induced current; metals placed in the coil are heated by eddy currents — the electrodes in vacuum tubes are sometimes baked out ("de-gassed") in this way. It has recently been found that currents may also be induced in the body in very much the same way, resulting in an artificial fever which has been found beneficial in treating certain diseases (diathermy).

521. The Induction Coil. The induction coil ("spark coil") is essentially a transformer operated by interrupted direct current. A current through the primary (Fig. 18) magnetizes the core and attracts the armature which breaks the circuit. The armature then flies back, and so vibrates, making and breaking the primary circuit at each vibration. (The operation of this primary circuit is the same as that in the doorbell.) The secondary coil, wrapped around the primary, has thousands of times as many turns; at each interruption of the primary an electromotive force is excited in the secondary sufficient to produce a spark across the terminals.

An induction coil is used to excite the ignition spark in gasoline engines. Here there is no vibrating armature, but, instead, *contact points*, actuated by a rotating cam, make and break the circuit for each spark; rotating with the cam is the *distributor* which connects the secondary to the different spark plugs in proper succession. (See Fig. 33-7; the distributor is not shown.)

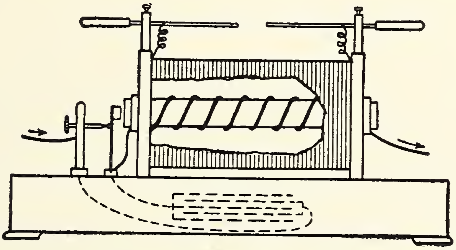

FIG. 18. Induction coil.

522. Self-Induction: Principle of the Choke Coil. In the transformer the electromotive force induced in the secondary is opposite to every change in the primary current. With only a single coil there is still a similar effect. Whenever the current in a solenoid changes, the lines of flux cut every turn within the solenoid itself and induce within the coil itself an electromotive force counter to the change. This is *self-induction*. When the current is decreasing there is a forward electromotive force; when it is increasing there is a back electromotive force: *the self-*

induced electromotive force tends to keep the current unchanged. This reminds one of the action of mass in mechanics, always opposing a change of velocity. Self-induction may be considered as electrical inertia.

Figure 19 illustrates a simple demonstration of self-induction. When a steady current flows the lamp is dark because the low-resistance electromagnet by-passes most of the currrent. But the lamp flashes whenever the current is started or stopped. Current cannot begin to flow suddenly or stop suddenly in the electromagnet because of its large self-induction. Therefore at the moment the switch is closed most of the current goes through the lamp. Then, again, when the switch is opened, the current in the electromagnet continues to flow for a moment and necessarily finds it's way back through the lamp.

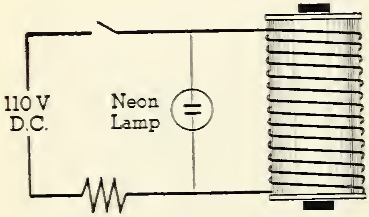

FIG. 19. Self-induction.

When alternating instead of direct current is used in this experiment practically the whole current always passes through the lamp; the alternating current is *choked out* from the electromagnet. A coil with many turns wound on an iron core, used for impeding the flow of alternating current, is called a *choke coil.*

The primary of a transformer carries very little current when the secondary is open because the primary then acts as a choke coil. Only when current flows in the secondary, can appreciable current flow in the primary. The primary and secondary currents are out of phase; when one flows clockwise the other flows counterclockwise. Both currents flow without greatly changing the flux in the core.

523. Self-Inductance and Sparking; the Cushioning Effect. The student can demonstrate for himself, very simply, the effect of self-inductance. Suppose that current from a dry cell flows through a small resistance; there is almost no perceptible sparking when the circuit is broken; there is, of course, no way that a person can get a shock from this circuit. But connect into the circuit a choke coil with low resistance and high " coefficient of self-induction." Now breaking the circuit produces a strong spark; if one holds the bare wires he receives a shock as the circuit is opened. The high voltage, sufficient to produce the spark and the shock, is caused by the very sudden stoppage of the current. It is quite analogous to the blow given by the sudden stopping of a hammer. The self-induction corresponds to the mass.

Such sparking often occurs at contact points (as in the vibrator of the induction coil) and is here highly objectionable. It is reduced by placing a condenser in parallel with the breaking points (represented in the base of Fig. 18). The stopping of the current is now not quite so nearly instantaneous; it can momentarily continue, charging up the condenser. The condenser is analogous to a spring or cushion, softening the blow of a hammer.

524. Coefficient of Self-Induction. The **coefficient of self-induction** (L) of a coil is defined (in practical units) as the voltage developed (opposite to the change) when the current in the coil changes at the rate of 1 ampere per second. The practical unit is called the *henry.* (The student can state for himself the definition in electrostatic or electromagnetic units.) This definition again illustrates the relation between self-inductance and inertia. From the definition,

$$\text{E.m.f.} = L\frac{\Delta I}{t},$$

which may be compared with the fundamental equation of mechanics, written as

$$F = m\frac{\Delta v}{t}.$$

525.* General Statement of Law of Electromagnetic Induction; the Rotational Electric Field. We have been interested in the very practical matter of the e.m.f. in a wire loop when the magnetic flux through it was changed by the motion of a magnet or by a changing current, or in a straight wire as the wire cut lines of force. But fundamentally the law of electromagnetic induction has nothing to do with the presence of conductors. There must still be the e.m.f. about a glass ring, or even about a closed path which we imagine described in air or in free space, when a magnet pole approaches and the flux through the ring or through the imaginary path increases. Of course (because of practically infinite resistance), there is no current in these cases.[1] The law of electromagnetic induction is: When the magnetic flux within a given closed path changes (or, in other words, when lines of force cut through the path) there is an e.m.f. around it equal (in abvolts) to the rate of change of flux (or in other words, equal to the number of lines of force cutting the path per second). The e.m.f. around the path is by definition (§457) the work done in carrying a unit charge around it. Observe that this is another example of a *rotational* field (§481). In a varying magnetic field the electric lines of force are closed rings, not beginning and ending on charges. This type of field stands in sharp contrast to the irrotational fields of electrostatics; this difference is typical of the fundamental difference between electrostatics and electrodynamics.

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[1] However, in a partially evacuated vessel electric discharge occurs rather readily (Chapter 44). When such a vessel is placed in a strong alternating magnetic field a ring-shaped discharge occurs. This "electrodeless discharge" is a beautiful example of induced e.m.f.

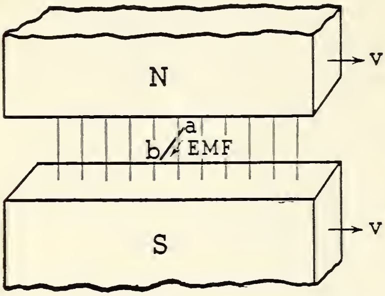

FIG. 20. E.m.f. in moving magnetic field.

526.* Electric Field in a Moving Magnetic Field. In Fig. 20 the magnet poles are supposed to be moving to the right; the moving field excites an e.m.f. perpendicular to the paper. For an imaginary line ab perpendicular to the plane of the paper is cut by Hlv lines per second (where l is the length of the line and v the velocity of the field), and this is the e.m.f. (in electromagnetic units). Now the e.m.f. is by definition the work done in carrying a unit charge between a and b, and this equals the electric field strength times the distance (El; eq. (7), p. 379). Hence the strength of the electric field itself is

$$E = Hv.$$

It is in the direction ab, perpendicular to the plane of the paper.[1] If a unit charge (abcoulomb!) were placed between the moving poles it would experience a force of Hv dynes in the direction ab.

“MAGNETOELECTRIC” INDUCTION

This chapter has dealt with the electromotive force induced by a changing magnetic field. The converse effect is the induction of a magnetomotive force by a changing electric field. It was deduced by mathematical reasoning by James Clerk Maxwell in 1867.

527.* Magnetomotive Force about a Changing Electric Field. Suppose a current to be flowing momentarily in the conductor in Fig. 21, charging up the condenser and building up a field between the plates. It was shown in Chapter 37 that a magnetic field would exist about the wire with a magnetomotive force of $4\pi i$ about the path a. But suppose that we take for our path the circle b, such that its plane cuts not the wire with its moving electrons but the gap between the condenser plates; will there be a field here? No current passes directly through the circle *b*. Can we suppose that the magnetic field suddenly ceases here? Maxwell showed that the magnetomotive forces around *a* and *b* must of necessity be equal. He showed that here where there was no current the magnetomotive force about the path was proportional to the *rate of change of electrical flux* through it. (*Electrical flux* means the total number of electrical lines of force between the plates; i.e., the field strength [*E*] times the area.) The exact relation is

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[1] The rotational character of these lines is not here apparent; they turn back as they pass beyond the edge of the magnet poles.

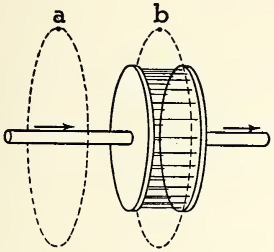

FIG. 21. M.m.f. by changing electric field.

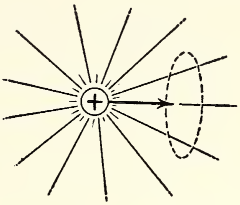

FIG. 22. M.m.f. near moving charge.

$$\text{M.m.f.} = \frac{1}{c^2}\,\frac{\Delta\Phi_E}{t},$$

where Φ_E represents the electrical flux and c is 3×10^{10}, the ratio between the fundamental electrostatic and electromagnetic units. (The factor $1/c^2$ enters into this equation and not into that for electromagnetic induction because the electromagnetic units which we are using were specifically designed to eliminate any such factor in the latter case. See §566.)

Although this law, which we are calling the law of "magneto-electric induction," has no important application to such things as motors and generators and is not usually honored with a name, it is necessary if we are to complete our picture of electromagnetism; it led directly to Maxwell's prediction of electromagnetic radiation. It is to be considered as an addition to the law for the magnetic field about a current (Chapter 37):

$$\text{M.m.f.} = 4\pi i + \frac{1}{c^2}\,\frac{\Delta\Phi_E}{t}.$$

It is needed to describe exactly the magnetic field about a circuit with rapidly varying current and about moving charges. When the charge (Fig. 22) approaches there is a magnetomotive force about the closed path $\left(\frac{1}{c^2}\frac{\Delta\Phi_E}{t}\right)$ exactly as there is an electromotive force when a magnetic pole approaches $\left(\frac{\Delta\Phi_B}{t}\right)$.

528. * A Magnetic Field Accompanies a Moving Electric Field. The law of magnetoelectric induction can be stated (like the law of electromagnetic induction) in terms of the cutting of lines of force. The magnetomotive force about the closed path in Fig. 22 equals $1/c^2$ times the number of electrical lines cut per second.

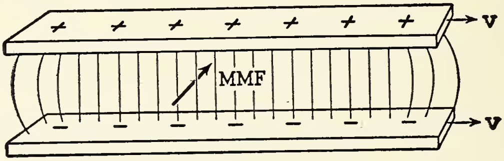

FIG. 23. M.m.f. in moving electric field.

And we can prove (compare §526) that if the charged plates in Fig. 23 move to the right there is, in virtue of the motion of the electric field, a magnetic field (into the page):

$$H = \frac{1}{c^2}vE\cdot$$

A cross magnetic field always exists when there is a moving electric field. The mutual interaction of the two fields results in progressing radio waves (Fig. 42–7).

QUESTIONS

1. What is the direction of the e.m.f. (1) when an east-west telegraph wire falls to the ground (in earth's field); (2) when a wire hoop is inverted on the table; (3) when a magnet is dropped, N pole down, into a coil?

2. What is the e.m.f. when the flux in a 100-turn coil is changed at the rate of a million lines per second; when a wire 1000 cm long moves 1000 cm per sec. across a field of 100 gauss?

3. A windmill with metal vanes turns in the earth's field. Will there be a potential difference along the vanes? Will it depend on the direction of the axis of rotation?

4. Do the current to run a motor, the force to turn a generator, and the current in the primary of a transformer depend upon the energy output? Explain.

5. A 110-volt d-c. generator is charging a 90-volt storage battery. If the power driving the generator fails, the generator will be driven by the battery as a motor. Will the direction of rotation be reversed, (*a*) for a series-wound, (*b*) for a shunt-wound generator? (Circuit breakers are placed in the charging line to prevent this possibility.)

6. A box has two terminals. When these are connected in the line it will allow alternating but not direct current to flow. Another box will allow direct but not alternating current to flow. What is in the boxes?

7. How can a solenoid and iron core be used as a choke coil for the gradual dimming of lights in a theater?

8. A dry cell sends current through a choke coil. When the circuit is broken there is a strong spark. Explain.

9. Iron is laminated in order to suppress: (*a*) eddy currents, (*b*) hysteresis, (*c*) resistance loss in windings.

10. Synchronous motors differ from other a-c. motors in having (*a*) induced currents in the armature; (*b*) definitely located magnet poles in the rotor; (*c*) no commutator or slip rings.

Vocabulary: Electromagnetic induction, induced e.m.f., Lenz's Law, magneto, dynamo, alternator, two- (three-) phase generator, slip rings, eddy current, induction motor, synchronous motor, mutual (self-) induction, transformer, induction furnace, induction coil, choke coil, coefficient of self-induction.

PROBLEMS

1. In what direction will current flow to the galvanometer in Fig. 14 (*a*) if a *S* pole is thrust into the upper end of the coil; (*b*) if a *N* pole is thrust into the lower end; (*c*) if the primary circuit is closed?

2. Which will be the high-potential side of the secondary in Fig. 15 if current suddenly begins to flow into the right-hand wire of the primary; what will be the direction of e.m.f. around the crucible, Fig. 17, if current flowing into the upper primary wire decreases; in Fig. 19 how will current tend to flow through the neon lamp if a north pole approaches the upper end of the core?

3. How would current tend to flow if a straight piece of wire were passed down between the *N* and *S* poles on p. 344, if a wire perpendicular to the page at *a* p. 346 were moved toward the top of the page, if a wire were thrust in between the pole pieces of Fig. 5, p. 399?

4. A 1000-turn coil has a resistance of 25 ohms and a diameter of 7 cm. The magnetic field in the coil is reduced from 10,000 gauss to zero in $\frac{1}{5}$ sec. Find the induced e.m.f.

5. Find the current which flows in problem 4 if the terminals of the coil are short-circuited.

6. Find the average e.m.f. if the coil in problem 4 is placed with its plane perpendicular to the earth's field and then suddenly inverted. Assume that $\frac{1}{2}$ sec. is required to invert the coil.

7. Find the average e.m.f. in the coil in problem 4 if a bar magnet with pole strength of 2000 units is thrust into it in $\frac{1}{5}$ sec.

8. A 5000-turn coil of wire in the form of a square, 20 cm. on an edge, is in a half second inverted in a field of 2000 gauss. Find the original flux Φ; find $\Delta\Phi$, $\Delta\Phi/t$ and the average e.m.f. induced.

9. What e.m.f. is produced when a wire 20 cm long sweeps with a speed of 10 meters per sec. across a field of 5000 gauss?

10. A long east-west telegraph wire falls 16 ft. to the ground. Find the e.m.f. induced by the earth's field in each meter length of the wire just before it reaches the ground.

11. A square coil, 10 cm on a side, with 5000 turns, makes 60 revolutions per sec. in a field of 5000 gauss. What is the maximum rate at which lines of force are cut? What maximum e.m.f. is induced in the coil?

12. An all-metal airplane with 15-meter wing spread travels 360 kilometers per hour. What is the difference in potential between the tips of the wings? (Vertical component of the earth's field, 0.5 gauss.)

13. A bicycle wheel with 40 spokes 30 cm long, supported with its axis parallel to the earth's field, spins with 20 revolutions per sec. What is the difference in potential between rim and hub?

14. Suppose that the transformers in Fig. 16 have 200 turns on the primary, 3000 turns on the secondary. Power is generated at 2300 volts. What is the potential difference across the high-tension distribution line?

15. A doorbell transformer, for use on a 120-volt line, has a 4000-turn primary and a 200-turn secondary. What is the output voltage? When 2 amperes flows through the bell what current flows in the power line?

16. The 4000-turn primary of a transformer is connected to an a-c. 120-volt line; a 2-ohm auto lamp is connected across the 200-turn secondary. What current flows in the lamp? What current flows in the power line? (Neglect the internal resistance of the transformer.)

17. If the coefficient of self-induction of a choke coil is 5 henries, what is the counter e.m.f. at a moment when the current is changing at the rate of 100 amperes per sec.?

18. Find the force on an electron when moving with a speed of 10^9 cm per sec. across a magnetic field of 20,000 gauss. (Consider the electron at rest and the field moving.)

CHAPTER 40

FLOW OF ALTERNATING CURRENTS

Alternating rather than direct current is generally used in power distribution, primarily because of the ease with which alternating voltage can be stepped up and stepped down for economical transmission. Many new phenomena appear in the flow of alternating currents. When two equal alternating currents are fed into a conductor, the resulting current may be anything from zero to the sum of the two, depending on their relative phases. . . The voltage may be out of phase with the current; there can be both current and voltage, yet no power. . . Alternating currents can flow where direct currents cannot, and vice versa. . . But though the subject is fascinating, it cannot be given more than brief mention in an elementary discussion of electricity.

529. Alternating Voltage and Current. The alternating voltage of the power line can be demonstrated by means of a neon glow tube. If the tube, excited by 60-cycle[1] alternating current, is viewed in a rotating mirror, it is seen to glow first on one electrode then on the other, as the voltage reverses 120 times a second. Or the alternation of the current can be shown by bringing a magnet near a carbon lamp lighted by alternating current; in the magnetic field the filament vibrates back and forth 60 times a second (compare §477).

But for an accurate representation of the alternating voltage and alternating current one uses an *oscillograph*. In one form of the oscillograph, current flows in opposite directions in a pair of neighboring wires in a strong magnetic field. The wires are pushed in opposite directions by the " motor force " and rotate a minute mirror. By means of a beam of light the motion is recorded upon a rapidly moving photographic film, and the result, if the current is alternating as in simple harmonic motion, is a simple sinusoidal wave form.

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[1] Strictly, the frequency should be given in *cycles* (complete oscillations) *per second*; however, in general usage this is shortened to *cycles*, the time unit being understood. In the United States power is usually generated at 60 cycles, occasionally at 25 cycles.

This oscillograph is essentially a galvanometer with a coil of such low moment of inertia that its deflection can follow the rapidly changing current. For very high frequencies a still lighter mechanism is needed, and this is found in the *cathode-ray oscillograph*, where the moving element is a beam of electrons in an evacuated tube (§579). This seems to be the ideal oscillograph and has generally displaced the older instrument of the galvanometer type.

By means of the oscillograph alternating currents and alternating voltages can be studied in detail as to amplitude, wave form, and phase relations. (See Fig. 2.)

530. Effective Values of Alternating Current and Voltage. What shall we take as a measure of the alternating current? We cannot take its average value, for this is zero. (Nothing will happen if alternating current flows through an electroplating cell — or through a d-c. galvanometer.) But the heat developed equals I^2R, and I^2, though varying, is always positive. The *square root of the average value of* I^2 is taken as the effective value (I_e) of alternating current (" root mean square current "). The effective value of a-c. voltage (V_e) has a similar meaning. It is these effective values which are read directly on a-c. ammeters and voltmeters. The root mean square values are approximately three-fifths of the maximum values for the voltage and currents shown in Fig. 2.

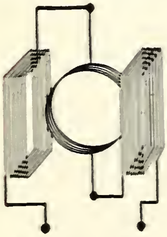

FIG. 1. A-c. Galvanometer.

531. A-C. Galvanometers, Ammeters, and Voltmeters. The moving-coil galvanometer (Fig. 32–6) can be adapted for measurement of alternating current by substituting a field coil for the permanent field magnet. The alternating current flows through the field coil and moving coil in series (Fig. 1); thus, as the direction of the current in the moving coil reverses, the direction of the field also reverses and the direction of the turning couple is unchanged. (Compare Fig. 1 with the wattmeter, Fig. 35–2.) This couple is jointly proportional to the current in the suspended coil and in the field coil and hence is proportional to I^2. The magnitude of this couple is rapidly changing, but the coil cannot follow the rapid variations and gives a steady deflection proportional to the average value of I^2. With a suitable scale (proportional to the square root of the deflection) it gives the effective current (I_e).

Exactly as with d-c. instruments this galvanometer is shunted to make the a-c. ammeter, connected with a high series resistance for the a-c. voltmeter, reading I_e or V_e directly. The a-c. instruments can be used for direct current, but because of the rather awkward square root scale they are not very convenient.

There are several other types of a-c. ammeters and voltmeters. This electrodynamic type, just described, is used for ordinary power frequencies. For much higher (radio) frequencies thermal galvanometers are used; the alternating current either heats a fine wire (causing expansion) or heats a thermocouple. In some instruments an ordinary d-c. galvanometer is used, and before passing into the instrument the alternating current is changed to direct (rectified) by the suppression of half the wave (§548).

532. Condensers. The student should read again the earlier discussion of condensers (§§402, 403).

Unlike direct current, alternating current can flow through a condenser. Perhaps we should say that alternating current can *effectively* flow through the condenser, because it simply charges and discharges the plates without any free electrons actually moving through the dielectric (insulating medium). Though the condenser is really an open circuit, even direct current flows into it momentarily while it charges the plates, one positive, the other negative. If the direct voltage is reversed there is another momentary current as the condenser is discharged and then recharged in the opposite direction. Operating a reversing switch several times per second we can (with a large condenser) get enough reversing current in this way to light a small lamp. With alternating voltage, changing its direction say 120 times a second, a continuous alternating current will flow; the plates are no sooner charged than they begin to discharge. At each reversal a small fraction of a coulomb of charge is transferred; the exact amount depends upon the voltage and upon the capacitance of the condenser.

533. Practical Unit of Capacitance. Capacitance is Q/V. In Chapter 31 this was expressed in electrostatic units (statfarads). The practical unit is the *farad*. It is the capacitance of a condenser in which 1 coulomb of charge produces a potential difference of 1 volt. A coulomb is an enormous charge, and consequently the farad is an extremely large unit. The capacitance of condensers is usually expressed in microfarads (millionths of farads), of small (radio) condensers in micro-microfarads (millionths of microfarads).[1]

534. Resistance, Capacitance, and Inductance in Parallel. With an oscillograph we can analyze the current through a resistor, a choke coil, and a condenser in parallel when an alternating potential difference is applied. These three elements shown in Fig. 2 were connected directly to the 110-volt a-c. line. The taller curve represents the 60-cycle applied voltage.

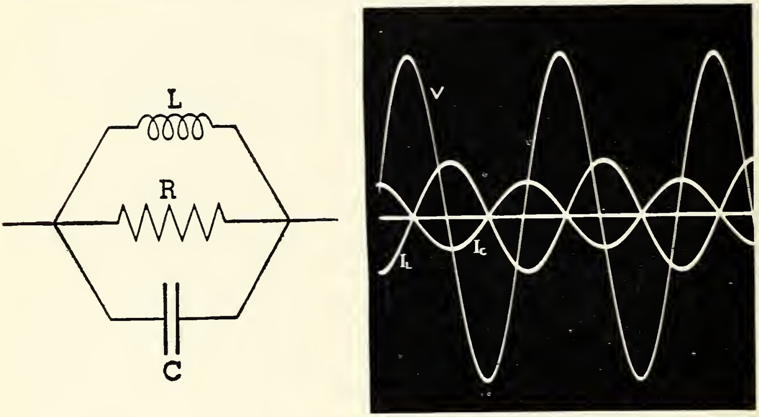

FIG. 2. Alternating current through pure inductance and capacitance. Current through resistance (not shown in oscillogram) is in phase with voltage. I_L and I_C are opposite in phase and each is 90° out of phase with the potential difference. (*Oscillogram.*)

To avoid confusion in the figure the current through the resistance is not shown. It is in phase with the impressed potential difference and can be drawn in by the student as a curve similar to the potential-difference curve but higher or lower, depending on the resistance. Observe that the current through L and C are both out of phase with the voltage. When the voltage is a maximum both I_L and I_C are zero, I_L increasing and I_C decreasing. Later, at the moment when $V = 0$ and both sides of the circuit are momentarily at the same potential and there is no current through the resistance, then the maximum current is flowing through the choke coil and through the condenser! But at every moment I_L and I_C flow in opposite directions, the one to the right when the other flows to the left. And each is out of phase with the current through the resistance.

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[1] A micro-microfarad is not greatly different from a statfarad.

535. A-C. Power. When alternating current flows through a simple resistance, voltage and current are in phase. Voltage is greatest when current is greatest and the power is I_eV_e (as it is for direct current). But when alternating current flows through a choke coil the voltage is not in phase with the current; it is greatest when the current is changing most rapidly (see Fig. 2); half the time it is in the direction of the current, half the time opposed to it; half the time it is a forward e.m.f., half the time a counter e.m.f. Compare gravity pulling a pendulum as it comes down, pulling it as it goes up. On the whole no work is done. There are an I_e and a V_e, but the power is zero. (This assumes the choke coil to have no resistance.) Choke coils with variable self-inductance can be used instead of rheostats for the control of alternating current (as in the dimming of lights in the theater); they have the decided advantage that (neglecting their resistance) they consume no power, do not heat up.

Power equals I_eV_e only when current and voltage are in phase. In a pure self-inductance or capacitance they are 90° out of phase. In general the power depends on the *power factor*, which is the cosine of the phase-angle difference (θ) between current and voltage:

$$P = I_eV_e \cos \theta.$$

In incandescent lamps, electric furnaces, and rheostats (resistances with practically no self-inductance or capacitance) $\theta = 0$, and the power factor is unity. The power expended and heat developed in these appliances is, as for direct current, I_eV_e or I_e^2R.

The wattmeter described in §448 operates with alternating as well as with direct current. *At each moment* the couple upon the coil is jointly proportional to the current in the line and the voltage across the line; the deflection depends upon the average value of IV and thus measures the power. D-c. power can also be measured with a voltmeter and ammeter; with alternating current this is not possible unless the power factor is known. There may be a large alternating current and a large alternating voltage, but if they are 90° out of phase the average value of IV is zero.

536. Impedance of Condensers and Choke Coils. When alternating current flows through an incandescent lamp, an electric furnace, or a rheostat, the relation between current and voltage (effective values) is given by Ohm's Law:

$$I_e = \frac{V_e}{R}.$$

But in general Ohm's Law cannot be used for alternating currents. The flow of alternating current differs from the flow of direct current in two respects: (1) Alternating current is impeded by the self-induction of a choke coil; direct current flows readily. (2) Direct current does not pass through a condenser; alternating current does (effectively at least). We introduce a property of the circuit, called its *impedance*, which plays the same role for alternating currents that resistance plays for direct currents. Instead of Ohm's Law we have in general the relation

$$I_e = \frac{V_e}{\text{Impedance}}. \tag{17}$$

A condenser in a line is essentially an open circuit, and direct current cannot flow continuously. We have seen that a current can flow momentarily, charging up one plate and inducing an opposite charge on the other; in other words an alternating current can effectively flow through a condenser. The opposition which a condenser offers to the flow is less the greater its capacitance (C) and the higher the frequency (n) of the current. There is infinite impedance for direct current ($n = 0$). The impedance of a condenser is[1]

$$\text{Imp.}_C = \frac{1}{2\pi nC}. \tag{17'}$$

We have seen that a coil of many turns on an iron core has a large coefficient of self-induction and opposes a rapidly varying current. Such a choke coil in a line acts as electrical inertia. The opposition which it offers to the flow of alternating current varies in proportion to its self-inductance (L) and to the frequency of the alternations — it offers no impedance for direct current. The impedance of a self-inductance is

$$\text{Imp.}_L = 2\pi nL. \tag{17''}$$

As a concrete illustration, if the frequency used in Fig. 2 were twice as great (120 cycles) the I_C curve would be twice as tall, the I_L curve half as tall; the current through the resistance would be unchanged.

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[1] This equation is not derived here but one can readily see why the 2π enters the formula. The frequency (n) represents the number of *revolutions* made by a simple (two-pole) generator per second; $2\pi n$ is its number of *radians* per second. Similarly n represents the number of complete cycles per second of the phase angle and $2\pi n$ its number of radians per second. Of course in our consistent system of units it is the latter which is used. The practical unit of capacitance, which must be used to get the impedance in *ohms*, is the *farad*.

For alternating current, as for direct current, the impedance to flow offered by a pure resistance (no capacitance and no self-inductance) is simply equal to the resistance

$$\text{Imp.}_R = R.$$

The current through an incandescent lamp is given by

$$I_e = \frac{\text{Voltage}}{\text{Impedance}} = \frac{V_e}{R}.$$

In an electromagnet (neglecting for the present its resistance) the current is given by

$$I_e = \frac{\text{Voltage}}{\text{Impedance}} = \frac{V_e}{2\pi nL},$$

and through a condenser

$$I_e = \frac{\text{Voltage}}{\text{Impedance}} = \frac{V_e}{1/(2\pi nC)}.$$

Example: The impedance for 60-cycle alternating current of a choke coil with self-inductance of 1 henry is $2\pi \times 60 \times 1 = 377$ ohms.

When 110 volts alternating current is impressed upon it the current is $I_e = 110/377 = 0.29$ ampere.

537. General Expression for Impedance. We shall attempt to show (by analogy) in the next paragraph that capacitive and inductive impedances work against each other and tend to cancel each other, and that both, in their effects, are 90° out of phase with the resistance. For a circuit containing a condenser and a choke coil in series the impedance is

$$\text{Impedance} = \text{Imp.}_L - \text{Imp.}_C$$

or

$$= 2\pi nL - \frac{1}{2\pi nC},$$

and in the most general case for a circuit with self-inductance, capacitance, and resistance in series (Fig. 3) the impedance is

$$\text{Impedance} = \sqrt{(\text{Imp.}_L - \text{Imp.}_C)^2 + \text{Imp.}_R{}^2}$$

or

$$= \sqrt{\left(2\pi nL - \frac{1}{2\pi nC}\right)^2 + R^2}. \qquad (17''')$$

These relations show that, by balancing with proper values a choke coil against a condenser, their impedances can be made to cancel. A circuit containing both can be "tuned" to carry alternating current with no impedance whatever except that due to resistance. Resistance can never be entirely eliminated but may be quite small and then when inductive and capacitive impedances balance a very large alternating current is produced by a small a-c. electromotive force.

538. A Mechanical Analogue. Consider a block on a table. If the block is not constrained we can move it with constant velocity. Force is necessary to overcome the frictional resistance of the table (R), but, for steady motion, the inertia (analogous to L) is of no consequence. Suppose that the block is constrained by a spring (analogous to a condenser). Continued motion with constant velocity is now impossible, but we can stretch the spring (charge the condenser) or we can shake the body back and forth. At high frequencies there may be a great deal of motion even for small displacements; the impedance of the spring becomes less. But now there is acceleration and the inertia of the block can no longer be neglected; a large mass tends to choke down high-frequency oscillation; the impedance offered by the inertia increases with the frequency. The mass acts like self-inductance in the electrical circuit.

As a matter of fact, the spring stretching force and the mass accelerating force are exactly opposite in direction (the acceleration is proportional to the displacement [stretch] but is opposite to it in sign). Since one effect increases and the other decreases with frequency, at a certain critical frequency, when $n = (1/2\pi) \cdot \sqrt{k/m}$ (eq. 12′, p. 165), the magnitudes of the two forces are equal; they cancel in effect, and a small applied force, alternating with this frequency, can produce a very large oscillation. The amplitude would be infinite if there were no friction. This is resonance.

Our analogy will be more exact if the resistance is due to viscosity (of air or liquid). This, unlike surface friction, depends on velocity. The viscous resistance is then greatest as the body moves rapidly through the equilibrium position. The spring force and (opposite) inertia force are greatest at the extremities; they are opposite to each other (180° out of phase) and are 90° different in phase from the resistance. An equation exactly analogous to (17‴) is obtained in this mechanical case.

539. An Oscillating Circuit. These analogies — the charged condenser with the stretched spring, the choke coil with a heavy bob — are exemplified by the following experiment. A very large condenser (say 100-microfarad capacitance) and an electromagnet (say 100-henry self-inductance) are connected in series as shown in Fig. 3. A neon tube will indicate by its glow the charge on the condenser (when the switch is closed). The condenser is charged to 100 or 200 volts. (The " spring " is now " stretched.") When the switch is closed the condenser discharges through the coil of the electromagnet. But, owing to the coil's self-inductance, the current cannot build up to its full amplitude at once; it gradually builds up (like a mass getting into motion) and reaches its greatest value just as the condenser plates are discharged. And now the current cannot stop at once; it continues, and the condenser is charged up in the opposite direction! Then the current reverses, again gradually builds up in the coil, and recharges the condenser to almost its original voltage. So the current continues to flow back and forth, but gradually is damped out by the resistance. At each charging of the condenser the neon lamp glows. It flickers on and off two or three times a second until, after several oscillations, the voltage falls too low to excite the glow.

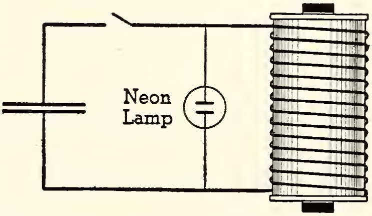

FIG. 3. An oscillating circuit.

This is obviously an exact parallel to the bob oscillating on a spring. Or it may be compared to a torsion pendulum rocking back and forth (as the balance wheel of a watch); hence this condenser-inductance combination is sometimes called a " flywheel circuit." The frequency of oscillation (n) is that for which the capacitive and inductive impedances exactly compensate each other.

$$2\pi nL = \frac{1}{2\pi nC},$$

giving

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

or

$$T = 2\pi \sqrt{LC}. \tag{18}$$

This is the *natural period of oscillation* of the circuit.

If the capacitance and inductance are made smaller, a shorter period and a higher frequency are obtained. These electrical oscillations are used in radio (Chapter 42) where the frequency of oscillation is between a hundred thousand and a hundred million cycles.

QUESTIONS

1. On an a-c. line why is the average value of IV equal to zero in a pure self-inductance or capacity, but equal to I_e^2R in a resistance?

2. When the potential difference is a maximum across the parallel elements in Fig. 2, what is the direction of the current in each? (Suppose the left side to be at the higher potential.) What is the direction a quarter cycle later?

3. What is the impedance (*a*) of a choke coil and (*b*) of a condenser at zero frequency? At very high frequencies?

4. The power factor of a motor depends upon: (*a*) its efficiency; (*b*) the phase difference between current and voltage; (*c*) the mechanical equivalent of heat.

5. The impedance of a circuit is always increased by an increase of: (*a*) inductance; (*b*) capacitance; (*c*) resistance.

Vocabulary: Root mean square current (voltage), impedance, power factor, flywheel circuit.

PROBLEMS

1. Find the impedance for a frequency of 160 cycles per second of (*a*) a resistance of 500 ohms, (*b*) a self-inductance of 1 henry, (*c*) a capacitance of 1 microfarad.

2. Find the impedance if the resistance, inductance, and capacitance of problem 1 are placed in series.

3. If 110-volt, 160-cycle potential difference is applied to the combination in problem 2, what will be the root mean square value of the current?

4. What is the root mean square value of the potential difference across each element in problem 3?

CHAPTER 41

THERMIONIC TUBES

540. Thermionic Emission. When a metal is heated in a vacuum, electrons evaporate from the surface. The temperature required varies with the substance: for some substances (sodium, cesium, thorium, etc.) the emission occurs at dull red heat; for others in which the electrons are bound more firmly (such as pure tungsten) appreciable emission occurs only at much higher temperatures. In the earliest radio tubes electrons were emitted from a tungsten filament. Today thorium is usually alloyed with the tungsten or the emitting surfaces are coated with certain oxides which increase the low-temperature evaporation of electrons.

In many respects the evaporation of these *thermions* resembles the ordinary evaporation of vapors. The electrons have a latent heat of vaporization, and the emission tends to cool the metal. The rate of emission increases with temperature; each substance has its own emission curve, which in shape very closely resembles the vapor-pressure curves in Fig. 23–5. But, unlike molecules, the evaporated electrons strongly repel one another and an extremely small number of thermions produce saturation in the tube; their presence charges the space, and this negative, repelling *space charge* prevents the evaporation of further electrons. Continuous evaporation occurs only when the tube contains a positively charged collecting plate (anode) to remove the electrons as rapidly as they are emitted.

541. Two-Electrode Tubes; Rectification. The simplest thermionic tube (or " vacuum tube ") consists simply of filament and plate (grid of Fig. 1 omitted). The filament serves as the cathode.[1] If it is heated to suitable temperature electrons will be emitted. When the plate is made positive it attracts these electrons, and current flows across the tube. Current cannot flow in the reverse direction; when the plate is negative the electrons will simply be driven back into the filament. The tube thus acts as a valve, permitting the flow of current in one direction only, changing alternating current to direct (rectification). Such a two-electrode tube — filament (cathode) and plate (anode) — is sometimes used to rectify alternating current for charging storage batteries (§547); large tubes of this kind (called kenotrons) are also used for rectifying the high-voltage alternating current obtained from step-up transformers (Chapter 39).

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[1] It is now convenient to follow the motion of electrons rather than the flow of positive " current." By definition electrons leave the *cathode*, go to the *anode*. (Compare the definitions in §459.) In contrast with an electrolytic cell a vacuum tube is not a source of electromotive force and hence the negative electrode is always the cathode. . . In the figures of the text the cathode is represented as a filament. Actually in many radio tubes the cathode is not the filament itself but is a minute tube which the filament, inside, heats by radiation.

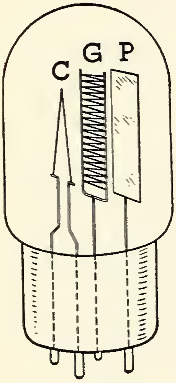

FIG. 1. Three-element radio tube.

542. Three-Electrode Tubes; Rectifier and Amplifier. The familiar use of thermionic emission is in radio tubes. The simplest radio tube has three electrodes; there is a wire screen, or "grid," between the cathode (filament) and plate. The plate is always positive, attracting electrons; but the flow of electrons to the plate is controlled by the grid. When the grid is sufficiently negative it repels the electrons and blocks the current; when more positive, current flows.

Figure 2 shows how the current passing from the cathode to the plate, in a certain type of radio tube, varies as the potential of the grid is changed. When the grid is at the same potential as the cathode, electrons can reach the plate — more if the grid

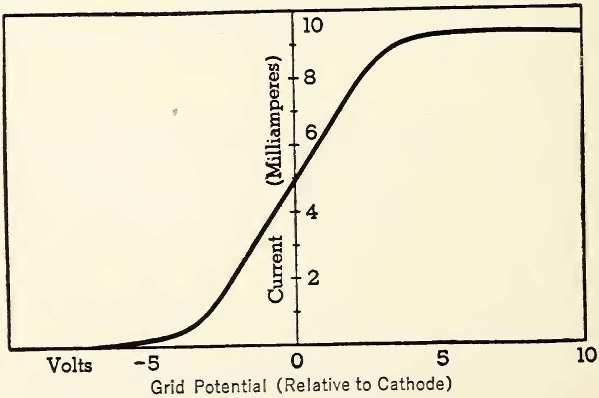

FIG. 2. Characteristic curve of radio tube.

is made a little positive. But when the grid is made a few volts negative relative to the cathode it stops the current entirely.

Let us make the grid potential 3 volts negative, so that the electron flow is practically blocked. Suppose that an alternating potential now reaches this grid, so that the grid potential fluctuates above and below this mean value. Each negative pulse can do nothing; each positive pulse allows current to flow. The tube now acts as a rectifier (responding to only half the alternating wave). The plate current is shown in Fig. 5*b*.

On the other hand, if the grid voltage in Fig. 2 is adjusted to a mean value of zero the tube responds to both halves of the alternating cycle; very small currents to the grid now control much larger currents to the plate, and the tube acts as an amplifier.

543. General Use of Vacuum Tubes. Vacuum tubes, controlled by the grid, act very much like switches, opening and closing the circuit. In an a-c. circuit this switch may alternately open and close every half cycle, allowing current to flow in one direction, stopping the opposite flow, thus producing direct current from alternating (vacuum-tube *rectifier*). By periodically making and breaking a d-c. circuit, it can change steady current into pulsating or can produce alternating from direct current (vacuum-tube *oscillator*). The tube may act as a relay, in which weak electrical impulses release large power outputs (vacuum-tube *relay*). The light from a rising star falls on a phototube and releases a very little energy; applied to the grid of a vacuum tube this releases more energy, and then this energy is further increased in other vacuum tubes until the feeble beam from the star turns on the lights of a World's Fair (Chicago, 1933). But, unlike the switch or the relay which simply opens or closes a circuit, the vacuum tube can also *control* the current, releasing an output proportional to the input but greatly amplified (vacuum-tube *amplifier*). It acts as an electric valve.

Moreover, this switch or valve is of almost instant response — it can interrupt the current millions of times per second. It thus has a great variety of uses — as rectifier, oscillator, amplifier, or relay — in the laboratory, in industry, as well as in the radio set. Perhaps there is nothing in electricity which is likely to be of greater practical usefulness to the worker in present-day laboratories than some familiarity with radio tubes.

544. Vacuum-Tube Amplifier. Figure 3 shows a typical amplifying circuit. The filament heating current is furnished by the "A battery" (2–6 volts) controlled by a resistance in the circuit; the B battery gives the plate its positive potential (20–1000 volts). The input voltages are applied between *c* and *g*, making the grid voltage higher or lower and increasing or decreasing the current from filament (cathode) to plate. The output current from the plate passes through a galvanometer or a telephone receiver (loud speaker) or a relay or other device which the output is to operate.

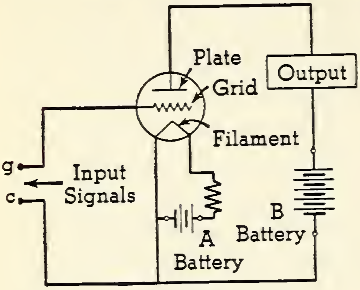

FIG. 3. Amplifying circuit.

Experiment I. Two dry cells (A battery) and rheostat furnish current for a type-30 radio tube. A 22-volt B battery is used on the plate. The flow of electrons to the plate is measured by a milliammeter (placed at "Output" in Fig. 3). In this experiment we shall leave the grid insulated but attach a short length of wire to it (at *g*), extending a foot or two upward ("antenna"). The grid will assume a potential of approximately zero, and a current of some 5 milliamperes will flow (Fig. 2). Bring a negatively charged rubber rod near the antenna, lowering its potential. Observe the decrease in plate current. The grid controls the current flow from cathode to plate but consumes no more power than an electroscope.

Experiment II. Characteristic Curve. Now set the grid at definite negative potentials by connecting between *c* and *g* one, two, or three dry cells. As the grid becomes more negative the plate current decreases. The student can plot the relation between grid potential and plate current (characteristic curve, Fig. 2).

Experiment III. Vacuum-Tube Amplifier. A telephone receiver can be used as a *transmitter* (§485). But if the student has tried it he has found the sound intensity rather low.

Connect such a receiver-transmitter between *c* and *g* and substitute earphones for the milliammeter. The weak alternating potentials impressed upon the grid produce amplified changes in plate voltage and give an increased output. Though still not very efficient this illustrates the use of the amplifier in public-address systems and in long-distance telephone lines. It suggests a wide variety of laboratory uses as an intensifier of weak electrical impulses.

With two tubes, a two-stage amplifier can be made, the plate of the first tube being connected (through a transformer) to the grid of the next.

545. Vacuum-Tube Oscillator. We have just seen how tubes are used as rectifiers and amplifiers. The third important function of vacuum tubes is to furnish electrical oscillations. The grid acts as a valve, in effect opening and closing the circuit between filament and plate, regulating the escape of power from the B battery. By periodically raising and lowering the grid potential we can release a pulsating current or (passing through a transformer) an alternating current.

In the vacuum-tube oscillator these periodic grid voltages are supplied by an oscillating flywheel circuit, the grid and cathode taking the place of the neon lamp in Fig. 40–3. In Fig. 4 the timing coil (secondary) and the condenser form a flywheel circuit. Given a slight initial oscillation in this circuit, the alternating potentials are fed (through the small condenser) to the grid, producing an amplified pulsating current to the plate. The pulsating energy output from the plate is fed back through the transformer into the oscillating flywheel circuit; here it (1) builds up and maintains the oscillation, preventing damping, and (2) supplies an output of alternating current.

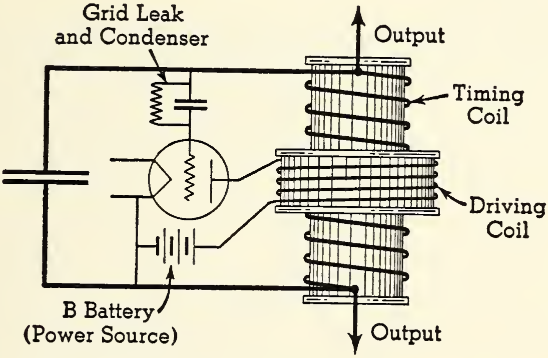

FIG. 4. Oscillator circuit. (Filament heating circuit not shown.)

Instead of consuming energy (as the neon lamp does in Fig. 40–3), the tube releases energy into the circuit and produces continuous, undamped oscillation.

This oscillating circuit is analogous to a watch: the flywheel circuit (the balance wheel) operates the vacuum tube (the escapement) and periodically receives energy from the B battery (the mainspring). Or it may be likened to a steam engine in which the flywheel operates the valve; the B battery corresponds to the boiler and is the primary source of energy.

The frequency of the oscillation depends upon the condenser and inductor used in the timing (flywheel) circuit. With a small capacitance and a small inductance (without iron core) very high frequencies can be obtained.

In medical practice high-frequency oscillations are used in diathermy (§520) and in electrosurgery. The " electric knife " consists of a platinum needle from which high-frequency currents are passed into the body. Because of the great current density at the point, the needle burns its way through the tissue, cauterizing the small blood vessels and reducing bleeding. . . But the most important use of high-frequency oscillations is in radio transmission.[1]

546.* Half-Wave and Full-Wave Rectification; Filter Circuits. Figure 5*b* shows the current output from a two-electrode rectifier. Only one half of the alternating cycle passes through the tube; this is called half-wave rectification. A second tube can be so connected as to invert and add to the output the other half of the wave (Fig. 5*c*). This is full-wave rectification.

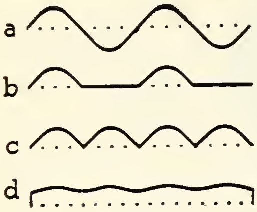

FIG. 5

Obviously full-wave rectification gives a much better approximation to a steady direct current than half-wave does, but the output current is still far from constant. The variations (ripple) can be suppressed (Fig. 5*d*) by suitable " filter circuits." For small currents a condenser can be used which becomes charged as the voltage rises to the peaks and, acting as a reservoir, continues to furnish a current output as the voltage drops. Or a choke coil can be placed in the line to suppress current changes. A combination of condensers and chokes is often used; with suitable filter an output can be obtained with negligible ripple.

For simplicity we have represented tube voltages as furnished by batteries; more often they are obtained from the a-c. power line. For the plate (*B*) voltage, the power voltage is stepped up, rectified, and filtered to give the steady potential required. This transformer-rectifier-filter combination is called a " power pack."

Electrical filters are also used with alternating current, to modify its character. Capacitances and inductances can be combined into networks to pass the higher frequencies and largely block the lower ones, or vice versa. In the radio set, high-pass and low-pass filters are often used in a tone control to emphasize either the high-pitched overtones of the voice or the low-pitched sounds of the organ.

547.* Gas Tubes. The current in vacuum tubes is usually limited to a small fraction of an ampere. Thermionic tubes which contain mercury vapor or a gas at low pressure handle larger currents. The current is largely carried by a discharge through the gas. The tungar rectifier, used for charging storage batteries, carries a current of 15 amperes. Giant mercury-arc rectifiers, carrying hundreds of amperes, furnish d-c. power, chiefly to electric railways. These tubes are simple two-electrode rectifiers, without grids. Three-electrode gas tubes, with control grids, also have a wide field of application chiefly as switches, for the opening and closing of circuits.

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[1] When extremely constant frequencies are demanded, as in radio broadcasting, the timing impulses are fed to the grid from an oscillating quartz crystal instead of from a flywheel circuit (*piezoelectric effect*, §383).

548.* Contact Rectifiers and Photocells. Another type of rectifier, with a rather limited field of use, does not depend on thermionic emission. When a properly prepared contact is made between a metal and certain semiconductors (copper oxide, selenium, galena, etc.), electrons can pass across this boundary in only one direction: from the metal into the semiconductor. Hence such contacts rectify alternating currents. The crystal detectors (rectifiers), used in the early days of radio, were of this nature. Copper-copper oxide rectifying packs are employed in certain types of a-c. meters (§531).

When light falls upon such a rectifying boundary the otherwise forbidden reverse transitions are encouraged; electrons pass from the semiconductor into the metal and an electromotive force is produced. The effect can be utilized to detect light or measure light intensity. The " Photronic cell " having a selenium layer is widely used as a light-sensitive relay and also as a light meter (particularly as an exposure meter by photographers).

CHAPTER 42

RADIO

The frequencies with which we now deal usually lie between 100,000 and 100,000,000 cycles per second. The peculiar properties of alternating currents become very pronounced. A few turns of wire, without an iron core, will act as a choke coil, with impedance of hundreds of ohms. For capacitance we need only a few millionths of a microfarad. Condensers may consist simply of two sets of interpenetrating plates (Fig. **31–12**).

549. Oscillator Circuit. Chapter 40 was concluded with an experiment showing the electrical oscillation of a "flywheel circuit," oscillation with extremely low frequency because of the large capacitance and inductance used. Let us repeat the experiment, now using a Leyden jar or a radio condenser for capacitance and for inductance a coil of only a few turns. We now charge the condenser plates with an induction coil. When we "close the switch" across the gap the plates discharge, current is built up in the coil, the condenser charge is reversed, and the charge pulsates back and forth as before but now with a period of perhaps a millionth of a second. For the period is given by

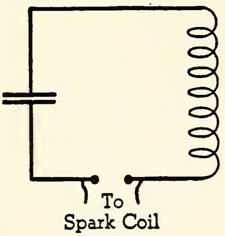

FIG. 1. "Flywheel circuit."

$$T = 2\pi\sqrt{LC}, \tag{18}$$

and the capacitance of the condenser is perhaps a hundred micromicrofarads and the self-inductance of the coil a few hundred microhenries.

Example. If the capacitance of a 10-plate radio condenser is 100 micromicrofarads and if the self-inductance of a 100-turn coil is 200 microhenries, find their frequency when connected as a flywheel circuit.

$$C = 100 \times 10^{-12} \text{ farad.}$$
$$L = 200 \times 10^{-6} \text{ henry.}$$

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{6.28}\sqrt{\frac{10^{14}}{2}} = 1{,}140{,}000 \text{ vibrations per sec.}$$

550. The Spark Closes the "Switch." Actually there is no switch (in the ordinary sense) to close in this experiment. In place of the switch there is a spark gap. Until its potential difference reaches a certain value the gap separates the condenser plates; then a spark passes, ionizing the air in the spark gap, ionization greatly reduces the gap resistance (corresponding to closing the switch), and oscillations occur. Because of the resistance, the oscillations gradually die down; we get a new charge on the condenser and a new series of oscillations with every break of the vibrator of the induction coil. Joseph Henry first showed the existence of such oscillations in the spark.

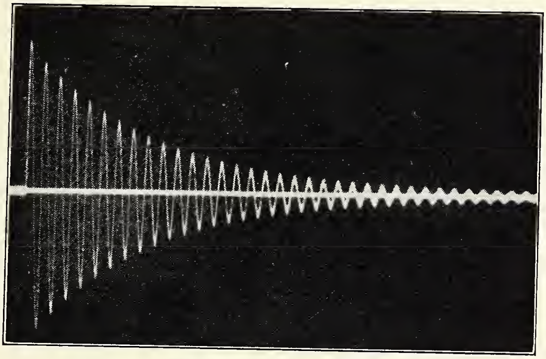

FIG. 2. Damped sine wave (*oscillogram*).

551. Electrical Resonance. The existence of these oscillations can be shown by resonance. We now need a second flywheel circuit including a variable condenser and a high-frequency milliammeter. The two coils are placed a few inches apart, and we excite the first circuit. Then, when the variable condenser

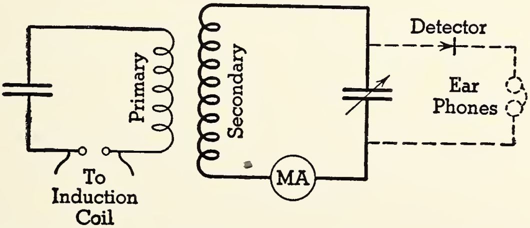

FIG. 3. Resonant circuits. (Neighboring coils indicate loosely coupled circuits. The condenser symbol crossed by the arrow represents a *variable* condenser.)

is adjusted so that the natural periods of both circuits are the same, the milliammeter in the second circuit registers a current. The coils act as primary and secondary, and, although the magnetic coupling between these two coils is rather weak, a large current is excited because of the exact resonance.

552. Rectification of High-Frequency Currents. The frequencies are far too high to produce sound in a telephone receiver (earphones). But if the oscillating current is rectified, blocking the current in one direction and allowing it to pass in the other, for each series of damped oscillations a one-way current will flow in the earphones and for each such series a single click is heard. As the induction coil vibrates and the series of oscillations follow one another several hundred times a second these clicks produce a definite tone.

There are several ways to produce rectification. If a point of wire touches the surface of a crystal of galena, current can pass into the crystal but not out. The broken lines in Fig. 3 show how a crystal "detector" (i.e., rectifier) and earphones can be used instead of the milliammeter in this resonance experiment. As the flywheel circuit oscillates, an alternating potential difference is impressed upon the detector, but only in one direction can it produce current. If we now repeat our experiment we shall hear in the receiver the buzzing tone of the vibrator when the two circuits are tuned to resonance. This is, in fact, simplest form of radio receiver.

Other types of crystal detectors have been used. But in present-day radio the detector is always a vacuum tube (§542).

553. Continuous and Modulated Oscillation. We have been considering damped oscillation. From the spark-excited flywheel circuit a series of damped oscillations is obtained at each succeeding spark. The earliest radio telegraphy utilized spark transmitters with damped oscillation. Continuous oscillation is employed in present-day radio. We have seen (§545) how such oscillation can be generated with a vacuum tube, feeding a little energy to the flywheel circuit at each oscillation, much as the mainspring (acting through the escapement) does to the pendulum of a clock.

However, perfectly uniform oscillation produces no sound (after rectification) in the earphones. To produce a tone the continuous oscillation is periodically suppressed; this is called *modulation.* The oscillation frequency may be 1,000,000 per second; the intensity changes a few hundred or thousand times a second. After rectification this modulated oscillation actuates the earphones several hundred or thousand times a second and produces sound.

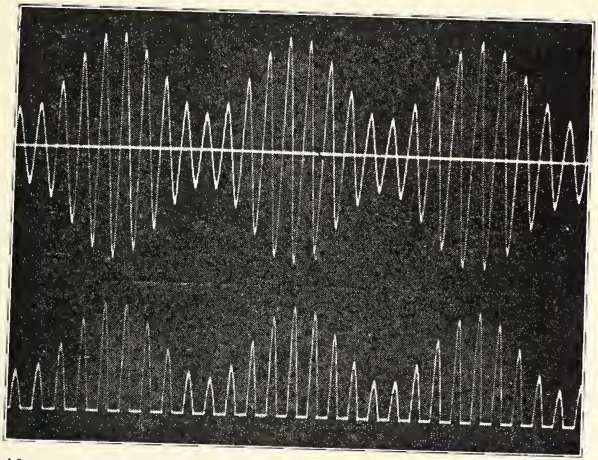

FIG. 4. *Above:* modulated sinusoidal wave. *Below:* the same rectified. Oscillograms of low-frequency oscillation; in radio the " carrier " frequency is hundreds or thousands of times greater than the modulating frequency.

554. Electromagnetic Waves. As we have watched electrical charges acting on one another at a distance, or a varying electric current inducing a current in another conductor, perhaps you have thought of the secret of electricity as residing, not in certain particles which we call electrons and protons, but in the " lines of force " outside of them. This was Faraday's view; he thought of lines of force as strains in the " ether "— and it was also the view of James Clerk Maxwell who, following Faraday, gave mathematical formulation to the laws of the electromagnetic field. At any rate, when we find these lines of force leading an independent existence without beginning (proton) or end (electron), when we have these detached lines of force thrown at us daily in a thousand broadcasting programs, we must grant them a certain amount of substantiality.

When a charge is suddenly moved the electric field is changed. *The change is not propagated through space instantly but with a* *speed of 300,000 kilometers (186,000 miles) per second.*[1] Consider a resting charge suddenly moved to the right (and again brought to rest). Figure 5 shows the field a second later. Near by the field bristles outward from the new position; at great distances (where the "news" of the change has not arrived) the field is still directed from the old position. A kink in the lines of force travels outward with the speed of 300,000 kilometers per second. A magnetic field, created by the moving charge, accompanies the kink (moving electrical field, §528). This traveling kink is a rudimentary "electromagnetic wave."

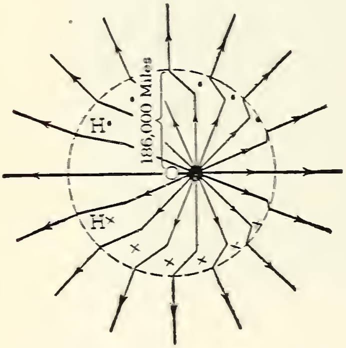

FIG. 5. Field about a displaced charge. Magnetic field in pulse represented (·) out and (x) in.

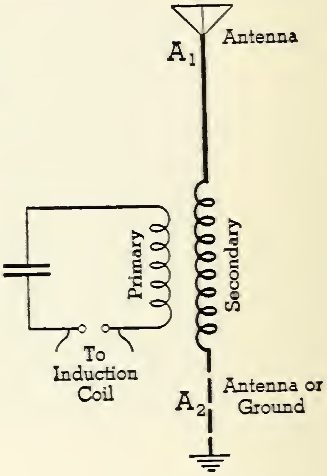

FIG. 6. Simple radio sending set. Condenser plates in Fig. 4 are "opened up" to form antennas.

Let us now cause a charge to oscillate up and down rapidly. We can do this by coupling antennas (A_1, A_2) to an oscillating flywheel circuit (Fig. 6). As the current oscillates in the flywheel circuit it will charge the upper antenna alternately positive and negative. Suppose that it does this 186,000 times a second. Evidently, at the moment when A_1 is positive, a half mile away the field will be as if it were negative, one mile away as if it were positive — this because of the time taken for the propagation of the changing field. We have a wave-length of one mile ($\lambda = v/n$).

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[1] This is the velocity of light (usually represented by c); light is electromagnetic radiation with wave-lengths much shorter than radio waves (§562).

Near the antennas there is also a magnetic field. The magnetic lines of force encircle the antennas while the current flows — they are strongest at the moment when the current is a maximum and the antennas are discharged — they are out of phase with the electric field. The electric field represents the potential energy and the magnetic field the kinetic energy of the oscillation. Being out of phase is characteristic of an oscillation which is simply a standing wave, beating back and forth without radiation. (Compare displacement and velocity in simple harmonic motion.)

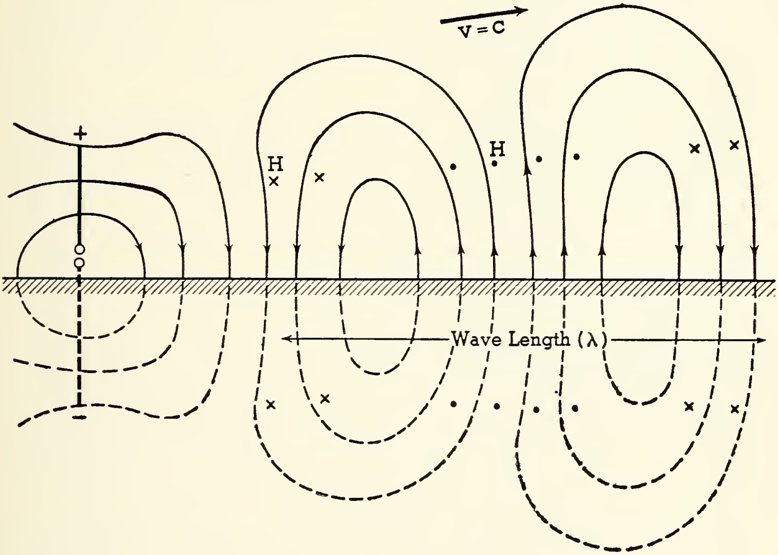

FIG. 7. Electromagnetic wave. The magnetic lines of force are circles, expanding out from antenna, alternately directed out from (•) and into (x) the page.

As the antenna oscillates — first charged, then discharged and reversed — the near-by lines of force increase and pulse out, then back, disappear and grow in the opposite direction; the magnetic field does the same thing but in different phase. But there is something more to the phenomenon than this. As the charge oscillates back and forth rapidly, and the field near the antenna is pulled in and out, reversing itself millions of times a second, the more distant lines of force, left behind, are snapped off and travel outward as closed rings. The magnetic field (shown as dots and crosses) has now somehow come into phase with the electric field, symbolic of the progressive rather than stationary character of the wave. These two oscillating mutually perpendicular fields — electric and magnetic — travel together through space with the velocity of light.

555. This description applies to waves traveling in free space. Such a transmitting set as this, with two opposite antennas, is used on airplanes. In ordinary broadcasting, the ground takes the place of one antenna and the waves travel outward with their feet resting on the ground. (Omit the dotted lines in the figure.)

This is the picture of electromagnetic waves as we can deduce it from the fundamental mathematical equations of Maxwell. How shall we think of these waves: as strains in a medium, the " ether "? We may do so, though in recent years the properties of this medium have become quite confusing. I think it would be better to avoid inventing idols of thought and confine ourselves to the experimental fact: there travels through space an oscillating electric (and magnetic) field, which means an alternating tendency for charges to move if any charges are present.

When the oscillating field reaches the antenna of our receiving set, it sets the electrons into oscillation, and with proper resonance, rectification, and amplification, we hear — perhaps grand opera and perhaps a jazz band.

556. Transmitting Set. To make the simplest transmitting set we simply couple an antenna and ground to our spark-excited flywheel circuit (Fig. 6). Alternating potentials are impressed on the antenna as the charge pulses back and forth across the spark gap. Now it is not a continuous train of waves which is fed into the " ether " but with each new spark a damped wave of gradually diminishing intensity. Rectified at the receiving station this gives the hum of the induction coil as described before. Instead of damped waves, continuous waves, suitably modulated, may be transmitted, and the receiver detects the tone corresponding to the modulation.

If a key is placed so as to open and close the circuit of the induction coil we can interrupt the hum into a series of " dots and dashes." This is wireless telegraphy. In the wireless telegraph station of a few years ago a high-power high-voltage transformer was substituted for the induction coil and a powerful spark jumped across a rugged gap.

Today continuous oscillations instead of damped waves are generally used in wireless telegraphy; they must be used in wireless telephony. At the broadcasting station one speaks into a microphone connected to a modulator tube; the sound waves successively increase and decrease the intensity of the train of transmitted waves. At the radio receiver these successive patches of waves reproduce the sound.

557. Crystal Receiving Set. To make the simplest possible receiving set it is only necessary to " open up " the condenser in the primary circuit of Fig. 3, substituting the antenna for one plate, the ground for the other. The incoming wave excites an oscillation in this circuit.

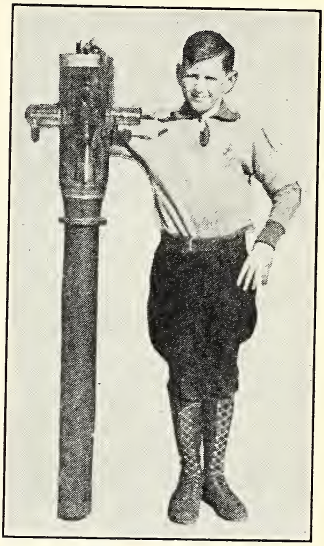

Fig. 8. Water-cooled radio tube used in 50,000-watt broadcasting station.

The coupling coils (step-up transformer, Fig. 9) can be made by winding two neighboring coils, say 20 and 100 turns, upon a cardboard mailing tube. In addition only a variable condenser, crystal detector, and earphones are needed. The 20-turn coil is the primary. It is connected to antenna and ground. The incoming wave excites oscillations in this coil. The 100-turn coil is in the flywheel circuit, and oscillations are built up here when, by adjusting its capacitance, the natural frequency of this circuit is made equal to that of the incoming wave. These oscillations are rectified and heard in the earphones.

But a better receiving set which has a radio tube as a detector will now be described.

558. A Simple Radio Set. A one-tube set which can easily be put together in the laboratory is represented in Fig. 10. The left-hand part of this circuit (neglecting the " tickler " coil) is the same as Fig. 9. The tube takes the place of the crystal detector in Fig. 9. Its grid is connected to the negative side of the filament through the grid leak which gives it the proper average voltage for rectification. The fluctuating voltages from the flywheel circuit passing through the grid condenser (shunting the leak) are impressed on the grid. Electrical oscillations from the antenna, tuned in by the flywheel circuit, rectified and amplified by the vacuum tube, pass through the earphones.

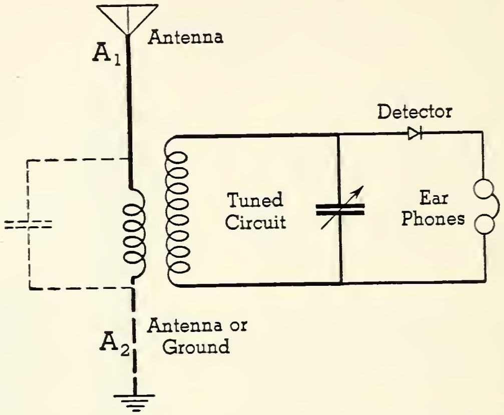

FIG. 9. Crystal receiving set. A_1 and A_2 take the place of condenser plates in Fig. 3.

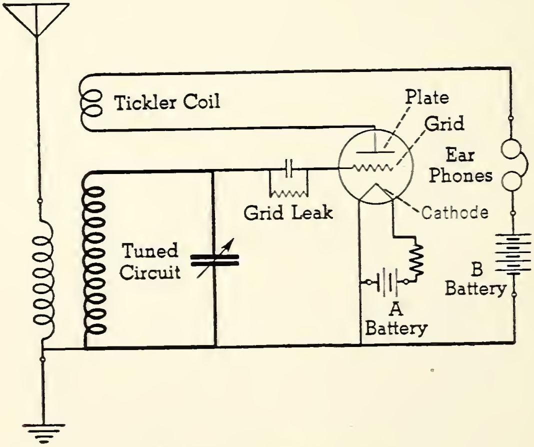

FIG. 10. One-tube radio set.

559. Regenerative Feed Back. We must now explain the important function of the tickler coil. This is a small coil which can be moved so as to increase or decrease at will its coupling with the flywheel coil. Without this the oscillations in the flywheel circuit would be strongly damped by resistance. The tickler carries the amplified pulsations from the tube and by induction feeds back just enough energy (when the coupling is properly adjusted) to neutralize the damping in the flywheel circuit. The tickler thus compensates for the resistance and increases the volume many fold.

If the tickler coil is brought so close to the flywheel coil that it *more* than neutralizes the resistance damping, the flywheel circuit will oscillate of itself. Any very slight change in the voltage across the flywheel coil will be amplified in the tube and then fed back as a larger voltage in the coil, and so on. The oscillations set up are superaudible and will not be heard except as a click when they begin, or as a beat note (§324) when the frequency nearly coincides with that of an incoming wave (the " heterodyne " effect). But while thus oscillating the set cannot receive radiation effectively.

Now although this self-induced oscillation is not desired in a receiving set it is just what is needed for the generation of undamped oscillations in the sending set. Indeed, the tickler coil, more tightly coupled with the flywheel coil, becomes the driving coil of Fig. 41–4. The student will observe that though the figures are drawn somewhat differently the transmitter (Fig. 41–4) and receiver (Fig. 10) have essentially the same circuit.

560. Radio Wave-Lengths and Frequency. In wave motion $v = n\lambda$. For electromagnetic waves $v = 3 \times 10^{10}$ centimeters per second or 186,000 miles per second. (What is the frequency of a wave a mile long?) The wave-lengths commonly employed in radio communication are given in Table 41.

TABLE 41

	Wave-Lengths (meters)	Frequency
Long Wave....................	10,000–550	30,000–550,000
Telegraphy, marine and aviation beacons, etc.		
Intermediate..................	550–200	550,000–1,500,000
Broadcasting		
Short wave....................	200–20	1,500,000–15,000,000
Telephony, broadcasting, amateurs, etc.		
Ultra-short wave...............	20–1	15,000,000–300,000,000
Television, local broadcasting		

Waves having overtones shorter than 1 mm have been generated and detected.

561. Short-Wave Radio; the Ionosphere. As has been said, the radio waves tend to travel with their " feet on the ground " (§555) and in consequence of this the longer waves follow around the curvature of the earth. This is less true for the shorter waves; these tend rather to travel in straight lines. Some leave the earth.

But in the upper atmosphere the air is strongly ionized. This region, beginning at an altitude of about 60 miles, above the troposphere and the stratosphere (Fig. 5–11), is called the *ionosphere.* It gives the atmosphere a conducting ceiling. The radio wave, striking this ceiling obliquely, is reflected back to the earth. Short waves (20–200 meter wave-length) are reflected from a height of 200 miles (where the ions are most numerous). The result is rather surprising; signals inaudible at a smaller distance (" skip distance ") may be heard again clearly at a distance of a thousand miles, as a result of this reflected beam. With the help of this reflection short waves travel very great distances; they have been detected after traveling several times around the world.

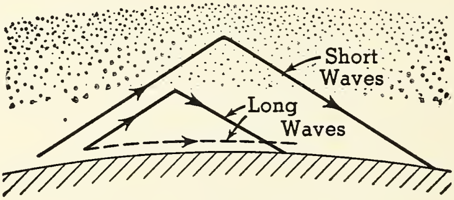

FIG. 11. Sky wave and ground wave.

The longer wave-lengths of the broadcasting range hardly penetrate the ionosphere at all. They are reflected at an altitude of about 60 miles, and this reflected wave (called the " sky wave ") joins with the direct wave (" ground wave ") to give us our observed radio reception. Because of the ground wave the skip distance is not noticeable with long waves.

562. Ultra-Short Waves. Waves shorter than 10 meters are not reflected back to the earth and cannot be used for long-distance broadcasting. As they travel very nearly in straight lines they cannot be detected much beyond the horizon — about 40 miles from a station on the top of a skyscraper. Only these ultra-short waves can be used in television (§586), and this property limits the television broadcast to a very restricted area.

In the United States the frequency range from 42,000,000 to 50,000,000 cycles (6–7.2 meter wave-length) has been allocated to local broadcasting. In these ultra-high-frequency stations a new system of modulation is being employed: the sound wave changes the frequency (instead of the amplitude) of the electromagnetic radiation, de-tuning the carrier wave (*frequency modulation*). The effective broadcast range is usually less than 50 miles (depending upon the elevation of the transmitting station) but within this range reception of extremely high fidelity is obtained, with great reduction of static and other undesired noise. It is anticipated that within the next few years there will be many broadcasting stations using ultra-high-frequencies, with frequency modulation.

Table 41 gave the wave-lengths and frequencies used in radio. These waves originate from electric currents flowing back and forth in antennas. Beyond these radio frequencies lies an enormous frequency range of electromagnetic radiation caused by the motion of electrons within atoms. Table 42 gives some of the more important frequency and wave-length intervals. (The frequencies are given in round numbers, to the nearest power of ten.) Most important is the visible radiation (light) with wave-length a little less than a thousandth part of a millimeter.

TABLE 42

Electromagnetic Radiation — Approximate Ranges

	Wave-Length	Frequency
Radio.................	10,000–1 meters	10^5–10^8 cycles
Visible light (Chap. 46)	7,000–4,000 angstroms*	10^{14} "
X-rays (Chap. 44)......	1–0.1 "	10^{19} "
Gamma rays (Chap. 45).	0.1–0.003 "	10^{20} "

* An angstrom is 10^{-8} centimeter.

563. History of Radio. James Clerk Maxwell deduced the existence of electromagnetic radiation mathematically; he presented a paper on the subject before the Royal Society in 1867.

Twenty years later (1887) Hertz produced short waves. Ten years later (1897) Marconi, utilizing much longer waves, invented wireless telegraphy. The vacuum tube appeared about 1905, first taking the place of the crystal detector and later used as amplifier. Many experimenters entered the field. Spark discharges and damped oscillations gradually disappeared, and vacuum tubes and continuous oscillations took their place. In 1915 the engineers of the Bell Telephone Company spoke from Hawaii to the mainland by wireless telephone. About 1920 came radio broadcasting.

564. Lecher Wires. When two straight parallel wires are coupled to the opposite sides of a short-wave oscillator, electromagnetic waves are propagated along the wires. The lines of force stretch across between the two wires with the direction of the field reversing every few feet (half-wave intervals) along the conductors. The wave, moving along with the speed of light, is reflected back from the end, and standing waves are set up. Corresponding to pressure and motion nodes in an organ pipe, here there are voltage and current nodes; the nodes and antinodes can be located by exploring along the wires with a neon tube, which glows brightly at the points of high potential difference and goes out at the nodes. It is a very striking experiment.

565. Tesla Coil. With high frequencies it is easy to generate high voltages — perhaps a million volts. The Tesla coil is a transformer, without iron core, in principle like the coupling coils discussed in this chapter but with many more turns on the secondary and built on a large scale. The circuit is the same as Fig. 6, with the antenna omitted. A large condenser and the primary are connected as a flywheel circuit, with a rugged spark gap. The primary voltage is stepped up 50 or 100 times in the secondary. It allows many spectacular demonstrations. Sparks snap outward in all directions from the top of the coil; wires are surrounded by a halo of light (the " corona "); an incandescent lamp glows when held in the hand near the coil — these high-frequency currents pass harmlessly through the body.

566. Radio Aids to Navigation. A wire loop may serve as a receiving antenna (instead of the straight wire). If a loop of wire is placed in the plane of the paper in Fig. 7, an oscillating e.m.f. is induced in it by the changing magnetic field through it; at right angles the e.m.f. is zero. This *loop antenna* thus points out the direction of the sending station and is used in ships and airplanes as a "radio compass." Radio transmitters operate from lighthouses and from many airports; the radio compass has become an important instrument in marine and air navigation. . . The country is also threaded with narrowly defined radio beams, directed from airport to airport, each marking an invisible highway for the aviator as he "rides the beam." At the airport are other radio aids to guide the flyer to a blind landing.

QUESTIONS

1. Compare an oscillating electrical current with a bob on a spring; compare the equations for period of oscillation.
2. Why are undamped oscillations used in radio?
3. Why is a tickler coil not used with the crystal detector?
4. Describe the three principal functions of vacuum tubes in radio circuits.
5. What is the purpose of grid leak, grid condenser, detector, tickler coil, modulator tube?
6. In the electromagnetic wave the magnetic field is: (*a*) parallel to; (*b*) perpendicular to, the electric field and the two fields are: (*a*) in phase; (*b*) 90° out of phase.
7. The primary purpose of the detector is to: (*a*) modulate the incoming wave; (*b*) produce a regenerative feed back; (*c*) change alternating to pulsating direct current.

Vocabulary: Detector, electromagnetic wave, spark (continuous) transmission, regeneration, ionosphere.

PROBLEMS

1. What is the frequency of a flywheel circuit with a 1-microfarad condenser and 1-microhenry coil?
2. What are the wave-length and frequency of your favorite radio station?
3. What wave-length is transmitted by a station with 100-micro-microfarad condenser and 100-microhenry coil?

CHAPTER 43*

ELECTROMAGNETIC THEORY; RELATIVITY

What a practical kind of thing this study of physics has been — levers and steam engines and automobiles and electric motors! The reader is forewarned that this chapter does not deal with "practical physics." We shall summarize our laws of the first electric and magnetic fields and then we shall present some of the fundamental questions concerning the ultimate significance of electricity which physicists have pondered since the time of Maxwell.

567. General Laws of the Electromagnetic Field (Electrostatic Units). Maxwell summarized in four equations the fundamental relations between the electric and magnetic fields. These stand in somewhat the same basic position in electricity that Newton's Laws of Motion do in mechanics. We shall give them in substance if not in the original mathematical form:

I. Electrical lines of force (E) begin and end only on electrical charges; 4π lines of force emerge from a unit positive charge. (This is substantially Coulomb's Law of Force, Chapter 30.)

II. Lines of magnetic induction (B) have neither beginning nor end (Chapter 37).

III. The magnetomotive force about a current is equal to $\frac{1}{c}\,4\pi i$ (Oersted, Chapter 37) or about a region of changing electric field is equal to $\frac{1}{c}\,\frac{\Delta\Phi_E}{t}$ (Maxwell, Chapter 39).

IV. The electromotive force about a region of changing magnetic field is equal to $\frac{1}{c}\frac{\Delta\Phi_B}{t}$ (Faraday, Chapter 39).

Law I describes the irrotational part of the E field, the field of electrostatics; law II states that there is no irrotational B field; laws III and IV describe the rotational part of the electric and magnetic fields; the expressions for the magnetomotive force and the electromotive force in these laws are somewhat different from those given in Chapters 37 and 39 (factor $1/c$) because *we are now using electrostatic units.* The four equations completely describe the two fields in space and give the relations between them.

568. The laws of electromagnetic and magnetoelectric induction can be stated in terms of moving lines of force. In §526 it was shown that for a magnetic field in motion (now using electrostatic units)

$$E_{\text{induced}} = \frac{v}{c} B.$$

In free space B and H are the same, and this can be written

$$E_{\text{induced}} = \frac{v}{c} H. \quad \text{(Law IV in space.)} \qquad (a)$$

Similarly a moving electric field is accompanied by a magnetic field (§528). In free space, away from moving charges,

$$H_{\text{induced}} = \frac{v}{c} E. \quad \text{(Law III in space, no current.)} \qquad (b)$$

569. Velocity of Electromagnetic Radiation. Out of his field equations Maxwell deduced the existence of electromagnetic radiation from a vibrating charge. Without attempting to follow the analysis in detail it can be said that the changing E field induces the H field and the changing H induces the E. Assuming such radiation to exist, we can easily show that it must move with velocity c. For in the radiation, the electric field does not originate in charges; the inducing field is also an induced field, and we have

$$H_{\text{induced}} = \frac{v}{c} E_{\text{induced}},$$

$$E_{\text{induced}} = \frac{v}{c} H_{\text{induced}}.$$

If v were different, say smaller, than c, the first equation would mean that H was smaller than E; the second that E was smaller than H. The equations are consistent only if $v/c = 1$ and $E = H$. Thus the velocity (v) of the wave is c. But c represents the ratio of the fundamental electrostatic to electromagnetic units and is 3×10^{10} (centimeters per second); this is identical with the velocity of light. Hence, supported by other evidence, Maxwell deduced that light was electromagnetic radiation of very high frequency and thus he bridged the gap between two great fields of physics.

570. Energy and Mass in the Field. Maxwell attributed reality to the lines of force. He considered the potential energy of a charged sphere as being localized in the electric field about it; he considered the potential energy of a charged condenser (§406) as being the elastic energy of the stretched lines of force and as located in the volume between the plates. We have found the energy of the charged condenser to be $\frac{1}{2}QV$; this can be expressed as[1]

$$\text{Energy} = \frac{1}{8\pi}E^2 \times \text{Volume},$$

and so $\frac{1}{8\pi}E^2$ is the energy per unit volume (energy density). Similarly he considered the kinetic energy of current flowing in a solenoid as stored not in the wire but in the magnetic field; this magnetic energy density can be shown to be $\frac{1}{8\pi}H^2$. As current flows back and forth in a flywheel circuit and potential energy of the charged condenser changes to kinetic energy of the solenoid, the energy can be considered as always stored in the "ether," now as $\frac{1}{8\pi}E^2$ per cubic centimeter between the plates, now as $\frac{1}{8\pi}H^2$ per cubic centimeter in the coil.

If we start our charged plates in Fig. 39–23 into motion to right or left there is a resistance to acceleration, not only that due to the mass of the plates themselves but a slight increased inertia due to the fact that the plates are charged! As the opposite charges move to the right they are in effect opposite currents, something like the opposite currents on opposite sides of a solenoid; a magnetic field is established, and there is self-induction tending to resist any change in electrical motion (i.e., any change in magnetic flux). Indeed, we must attribute *mass* to the electromagnetic field.

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[1] $Q = \frac{EA}{4\pi}$ and $V = Ed$. See §392.

571. Momentum of Electromagnetic Radiation. Although for charged plates the mass of the field is negligible in comparison with the mass of the material plates and the effect just described cannot be experimentally detected, there is other evidence for the mass of the electric field. When light (an electromagnetic wave) falls on a surface and is either absorbed or reflected, the surface is given a slight impulse. The effect though small can be observed without great difficulty (the famous Nichols-Hull experiment, §256). This means that the wave has momentum (mv). Actual experiment shows that 1 erg of light energy has mass equal to $1/c^2$ grams.[1] *Light or any form of electromagnetic radiation has mass equal to* $1/c^2$ *times its energy.*

572. Mass of the Electron. Since the electron is the lightest of particles, itself commonly used as the delicate exploring tool in probing into the nature of matter, perhaps we should not attempt to analyze it further. For where are the tools delicate enough to explore the exploring tool? Until someone discovers a "subelectron" possibly it is meaningless even to think of such a question as the distribution of charge throughout the electron, for as far as we know the electron *is* charge.

But let us think of the electron as a small negatively charged sphere, radiating lines of force. When it moves there is also a magnetic field. If we increase or decrease its speed, increase or decrease its magnetic field, there is a back electromotive force of self-induction. If the radius of this hypothetical sphere is properly chosen (2×10^{-13} centimeter) we can account completely for its mass by its self-induction! This is the *electromagnetic theory of mass.* It has recently been strongly supported by the observation that electron and positron (§602) can coalesce and become high-frequency electromagnetic radiation. But why the effective radius of the electron has this particular value and how the mass of the uncharged neutron fits into the theory — these are among the unsolved riddles of physics.

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[1] Light also "falls" in a gravitational field (weight). Astronomical observations taken during times of solar eclipse show that light from stars is deflected as it passes close to the sun. The magnitude of the effect is a confirmation of Einstein's General Theory of Relativity.

573. Variation of Mass with Velocity. By an extension of the argument of §569 by which it was shown that the *purely rotational* fields of electromagnetic waves travel with a velocity c, it can be shown that for the fields *due to a moving charge*, H_{induced} must be less than E, and so (by eq. *b*) *the velocity must be less than the velocity of light.* This has been confirmed experimentally. It has been shown experimentally that the mass of the electron increases at very high speeds. This means that it becomes harder to accelerate it; its mass and kinetic energy (no longer given by $\frac{1}{2}mv^2$) approach infinity as the velocity of light is approached. *The increase in mass is equal to the kinetic energy divided by* c^2 (9×10^{20}).

Now is all mass to be considered electromagnetic? Does all mass increase with energy? Does the inertia of a ball increase (by ever so little) when it is in motion? Does a glass of water become a little heavier when heated? Perhaps our first question will never be answered, but the answers to the others are quite definite. Mass does increase exactly in this way. 9×10^{20} ergs of energy have a mass of 1 gram. But this definite and universal answer came not from electromagnetism but from analysis of the fundamental concepts of time and space.

574. The Theory of Relativity. In mechanics motion is purely relative. I on the ground see the train move to the west; you in the train see the ground moving to the east. It makes no difference which point of view is taken; exactly the same laws of mechanics hold in a moving system as in a stationary one.

I see you moving, you see me moving; but which of us is *really* moving in an absolute sense? Is there no way to tell which of us is in motion, relative, say, to the "ether"? Electricity would seem to give the answer. Here is a charged ball on the table. If it and the earth are moving through the "ether" it should (according to Maxwell's view) have magnetic lines of force about it. The experiment was tried, and it was found that a magnetic field existed only when the ball moved *relative to the observer*. Magnetism was only a symbol for the electric field observed to be in motion.

But (according to Maxwell) electromagnetic waves travel with a velocity c relative to the ether. If I am moving through the ether toward the waves their apparent velocity should be greater than c; if I recede it should be less than c. The experiment was tried with light (the famous Michelson-Morley experiment), but under all circumstances the observed velocity of the light was exactly c. c is the speed of light (and the limiting speed of all else) *relative to the observer.* Motion is purely relative.

The results of the Michelson-Morley experiment appeared paradoxical; it was very much as if it were found that sound approached a listener with the same speed of 1100 feet a second, however fast he himself might move, toward the source or away from it. In 1905 Einstein solved the paradox by analyzing the fundamental concepts of space and time. We can ask again the questions which he asked. Does this house of yours occupy the same point in space that it did yesterday? Obviously, relative to the earth, yes; relative to the sun, no. The " same point in space " is a relative matter. But, again, are two certain events, one occurring in New York, one in San Francisco, simultaneous? Do they take place at the same " point in time "? Or is this also relative? Einstein asked, how would we proceed to determine the " same time " in these two cities. Let us send a radio wave from New York to San Francisco and immediately reflect it back. (It takes $\frac{1}{30}$ second to make the round trip.) We take then the mid-moment in New York, between the sending and the return, as simultaneous with the moment of arrival in San Francisco. We could set New York clocks and San Francisco clocks into synchronism in this way. But in doing this, in choosing the mid-moment and in setting our clocks which will henceforth define our concept of time, *we have assumed that it takes the signal the same time to travel west as to travel east.* Obviously, using *these* clocks and *this* concept of simultaneity and of time we can never detect (as Michelson and Morley attempted to do) a difference in wave velocity in the different directions. Another observer, moving, would perform the experiment differently. Time is a relative concept.

Out of these considerations the concepts of time and of (three-dimensional) space evolved as interrelated; together they act as a four-dimensional entity. Different observers moving relatively to one another see this time-space in a different perspective. This is the essence of the theory of *special relativity.* Out of this arose modifications in mechanics. It was shown that moving bodies become shorter (or, what is the same thing, bodies which appear to a given observer to be moving appear to this same observer to be shorter), and what is an electric field at rest becomes a combined electric and magnetic field in motion. And as bodies acquire more kinetic energy their mass increases. 9×10^{20} ergs has a mass of 1 gram, and nothing can go faster than light.

From a strictly logical point of view this analysis of space and time should come as a prelude to mechanics. Mechanics was greatly modified by Einstein's theory (though the modifications are extremely small except at velocities approaching that of light). In electricity the "ether" dropped out of the picture; the velocity of electromagnetic radiation was relative to the observer. But, a most significant fact, Maxwell's fundamental equations of electricity went through the revolution in physical thought unchanged. It is sometimes said that scientists do not *really* know what electricity is, as they know sticks and stones. But while fundamental notions of sticks and stones, of space and time, were being turned topsy-turvy, these laws of electricity were unaffected.

CHAPTER 44

DISCHARGE THROUGH GASES

575. Before 1900, physicists were for the most part investigating the phenomena that appeared upon the surface of Nature. But in the last decade of the last century several phenomona were discovered which opened before them the world of the atom and molecule:

(1) Discharges in gases at low pressure had been observed for some time, but their significance was hardly realized until this decade. Then the electron was discovered in the " cathode rays "; indeed, approximate measurements of its mass and charge were made at this time by J. J. Thomson.

(2) In 1895 x-rays were discovered by Roentgen.

(3) In 1896 Becquerel observed the phenomena of radioactivity, and two years later Pierre and Marie Curie discovered radium.

An era ensued in which discovery has followed discovery with unparalleled frequency. This electronic and atomic physics constitutes a very broad field. For many students it is the most interesting part of physics, penetrating as it does into the reality beneath the surface of reality. But it must lie largely outside our treatment of elementary physics, which deals primarily with the physical laws of the everyday world about us. In this chapter we shall discuss the nature of the electric discharge which occurs in a partially evacuated tube and several phenomena of the new physics which are more or less directly related to it.

576. Nature of a Spark. In a gas electricity is carried either by free electrons, broken loose from the molecule, or (as in electrolysis) by positive or negative ions. A very few ions are normally present in the air, produced by cosmic radiation or by the slight radioactivity in soil and air. When an electric field is applied across a spark gap these are quickly drawn to the electrodes. But if the field is sufficiently strong these ions knock off electrons from the atoms with which they collide and so produce new ions by collision. These in turn produce other ions, and these, still others, and so a spark passes. If the voltage is now still maintained across the gap the number of ions becomes very great, the resistance becomes quite low, and the continuous discharge is an *arc*. An example is the carbon arc.

The grandest example of spark discharge is of course lightning. For such giant sparks a potential difference of hundreds of millions of volts must be built up between earth and cloud. A faint "leader stroke" usually makes a staggering path to the ground, followed immediately by a stronger discharge down this trail of ionized gas. Several bolts sometimes occur in rapid succession along the same trail.

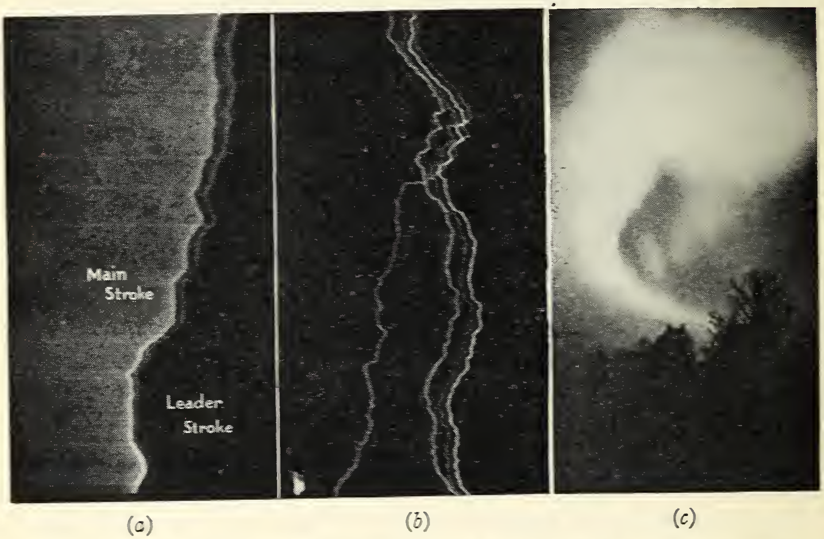

FIG. 1. (*a*) Analysis of simple lightning stroke, with rapidly rotating camera. (*b*) Multiple strokes, slowly moving camera. (*c*) Aurora borealis.

577. Low-Pressure Discharge. An electric discharge is obtained more readily when the gas is at low pressure. At atmospheric pressure several kilovolts are required to get a spark a millimeter long. The ions collide with molecules so frequently that they do not, except in a very strong electric field, acquire enough speed to produce ionization by collision. At a pressure of about 1 millimeter (of mercury) a discharge occurs with a potential difference of only a few hundred volts, and a soft glow fills a large part of the discharge tube. A dark space extends a few millimeters from the cathode; then comes the "negative glow," then another dark space ("Faraday dark space"), and then the "positive column" extending the rest of the length of the tube often showing striations, alternating patches of light and dark. The appearance of the discharge is very striking.

If the pressure is reduced still further the cathode dark space grows, pushing back the positive column. More and more of the ions go the length of the tube without colliding. With fewer collisions, the number of ions decreases and it becomes more and more difficult to produce a discharge. The tube is said to become "harder." With the highest attainable vacua no discharge occurs except at extremely high voltage when electrons are actually pulled out of the metal of the cathode.

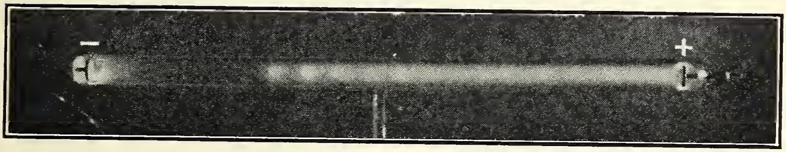

FIG. 2. Discharge tube.

Low-pressure discharges are now commonplace in neon signs. In northern lights (aurora borealis) a discharge is produced in the rarefied upper atmosphere of the earth by electrons shot off from the sun.

CATHODE RAYS AND THE MASS OF THE ELECTRON

578. Cathode Rays. The phenomena in the discharge are quite complex. There are positively charged gas atoms (and molecules) traveling to the cathode; there are negatively charged atoms (and molecules) traveling to the anode; and also, most interesting, there are free electrons projected from the cathode and moving at high speeds through the tube. These free electrons are released when the positive ions pound into the cathode. They are called "cathode rays." These beams of electrons are most conspicuous at quite low pressures, when, with comparatively few molecules present, they strike the walls of the tube and produce fluorescent light.[1]

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[1] A solid or liquid is said to fluoresce when a characteristic color is excited by the bombardment of material particles (as here) or by x-rays or by light of a different color (§682). (One speaks of light-excited fluorescence also in gases.)

When a small obstacle is interposed in the beam of cathode rays a sharp shadow is cast on the walls of the tube beyond. This shows that these rays travel in straight lines. Most of the potential difference in the tube occurs in the cathode dark space within a few millimeters of the cathode, and in this region the electrons receive their speed. They carry energy like a shower of bullets from a machine gun and heat any surface upon which they fall — often, at high voltage, they heat the anode to incandescence. (This effect is often used for de-gassing the metal parts of vacuum tubes.) They are deflected in a transverse electric or magnetic field; this deflection shows that the cathode rays are negatively charged particles (electrons) much less massive than atoms.

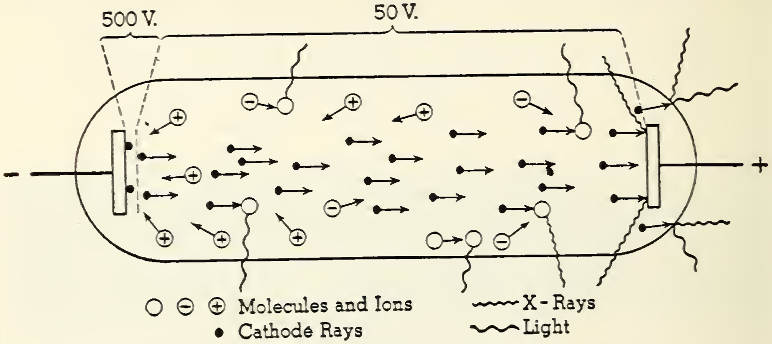

FIG. 3. Phenomena of gas discharge.

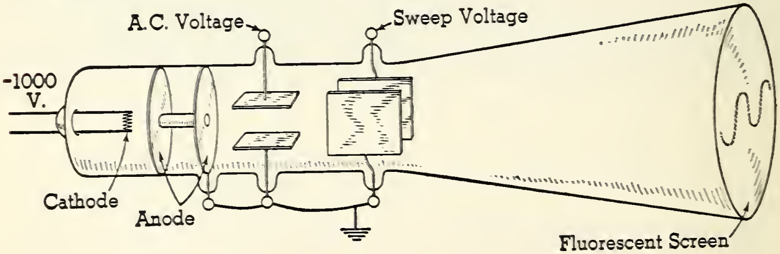

FIG. 4. Cathode-ray oscillograph.

579. Cathode-Ray Oscillograph. In the cathode-ray oscillograph, electrons, accelerated between the cathode and anode by a potential difference of several hundred or several thousand volts, pass through a hole in the anode as a narrow beam and excite a spot of light on the fluorescent screen. (Usually the tube is highly evacuated and the electrons are released as thermions from a hot cathode.) The beam passes between two pairs of plates; when a small potential difference is applied to either of these pairs an electric field is established across it and the beam is deflected, either vertically or horizontally.

The oscillograph is used to analyze electrical oscillations (Fig. 42–2, 4). When a varying voltage is applied to the first (horizontal) pair of plates the spot of light dances up and down. At the same time a potential difference (the " sweep voltage ") is periodically applied to the other pair of plates. This repeatedly sweeps the beam across the fluorescent screen from left to right, serving the same purpose as a rotating mirror in the simple mechanical oscillograph (§529). On the screen we see the wave form of the varying electrical voltage.

580. Deflection in Magnetic Field; Mass of the Electron. If a magnet is brought near a discharge tube or a cathode-ray oscillograph the cathode rays are deflected in a direction perpendicular to the lines of force. The direction of the deflection is found by the three-finger (motor) rule as for a current flowing in a wire. (The *current* is opposite to the motion of the electrons.)

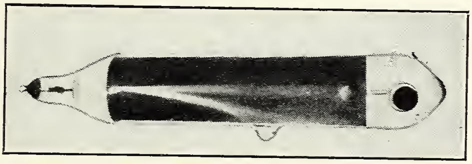

FIG. 5. Cathode rays; double exposure showing rays with and without magnetic field.

It was in the cathode ray that the electron, the ultimate constituent of negative electricity, was first recognized. By studying the deflection of the beam in magnetic and electric fields, J. J. Thomson measured the mass of the electron. The deflection of the ray in a given magnetic field depends upon the charge, the velocity, and the mass of the particle. The charge (e) of the electron being known (§388), its mass and velocity can be found. The mass of the electron is 9×10^{-28} gram — 1840 times lighter than the mass of the lightest atom (hydrogen).

581.* Force on Electron Moving across a Magnetic Field. The value of the magnetic force on the electron, moving across the field, is

$$F = evH. \quad \text{(Electromagnetic units.)}$$

Proof: When a current flows in a wire across a magnetic field H, the force on the wire is iHl (eq. 14). Therefore the force (F') on a *unit length* of the wire is

$$F' = iH.$$

This same force acts on a unit length of the cathode ray when its direction is perpendicular to the magnetic field. Let n' represent the number of electrons present at a certain instant in this unit length of the ray. If they are moving with velocity v, the number of electrons passing through a given cross-section of the beam per second is $n'v$ and the charge passing per second is $en'v$. This is the current i. When this is substituted in the above equation, the relation becomes

$$F' = en'vH,$$

or the force on a single electron is

$$F = evH.$$

(Or, alternatively, consider the electron at rest and the field moving. From §526, $E = Hv$, and the force on the electron is $F = qE = evH$.)

TABLE 43

SPEED OF ELECTRONS

Tube voltage	Speed
1 volt (radio tube)	6×10^7 cm per sec.
100 volts (radio tube)	6×10^8 cm per sec.
10,000 volts (cathode rays)	6×10^9 cm per sec.
1,000,000 volts (beta rays)	2.8×10^{10} cm per sec.
100,000,000 volts (cosmic rays)	2.99×10^{10} cm per sec.

582.* Velocity of Cathode Rays. Leaving the cathode, the electrons lose potential energy, gain kinetic. If V is the drop in the potential at the cathode (practically the potential difference of the tube),

$$\tfrac{1}{2} mv^2 = eV.$$

From this equation it is evident that the velocity varies as the square root of the voltage. The equation does not hold for extremely high voltage. At high speeds the mass of the electron increases, and the kinetic energy is no longer given by $\frac{1}{2}mv^2$. The velocity of light is never exceeded.

In this equation c.g.s. units must be used (the stat- or ab-system, not coulombs and volts). Using the values of e (4.8×10^{-10} statcoulomb) and m (9×10^{-28} gram) we can find the speed of the electron at different voltages. (Three hundred volts = 1 statvolt; §394.)

POSITIVE RAYS AND ISOTOPES

583. Positive Rays. In the discharge tube positively charged atoms or molecules of the gas are moving toward the cathode. When they strike the cathode, electrons are splashed off (cathode rays). If there is a hole in the cathode a beam of these ions will pass through into the space beyond. These are called positive rays. They consist of atoms or molecules, sometimes with a single charge, sometimes with two, occasionally with more.

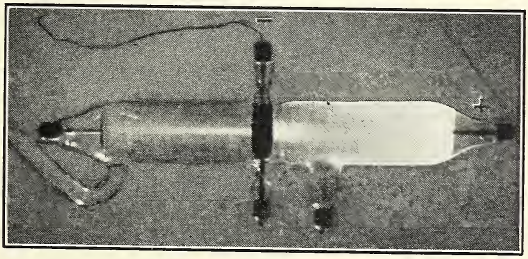

Fig. 6. Positive rays.

Positive rays are deflected in a strong magnetic field but much less than cathode rays because the charged particles are thousands of times more massive than electrons. The lightest positive ray particle is the nucleus of hydrogen (a hydrogen atom which has lost its electron). This is the proton, 1840 times heavier than the electron. The proton is the elementary positive charge.

Just as with electrons, the masses of the particles in positive rays (H^+, H_2^+, O^+, O^{++}, etc.) can be determined by magnetic deflection. A strong magnetic field is needed. Light ions are deflected more, heavy ones less. Ions of a single speed must be selected; they must be focused into a very fine beam, and this beam is then bent in a very constant magnetic field. In this way the relative masses of the atoms can be determined with an error of only a few parts in 100,000 (Table 44).

584. Isotopes. When the atomic masses are measured directly in this way it is found that the great majority of chemical elements do not consist of a single species of atom but may comprise two, three, or more atoms of different weights but with exactly the same chemical properties. These physically different, chemically identical atoms are called *isotopes*. Chlorine has two isotopes (weights approximately 35 and 37; the different isotopes exist in such proportion as to give the observed chemical atomic weight 35.46); mercury has nine known isotopes. Examples of isotopes among the lighter elements are shown in Table 44.

TABLE 44

Atomic Weights of Lighter Isotopes

	Weight		Weight
Hydrogen....	1.0081	Carbon......	12.0032
	**2.0142		**13.0069
	***3.0161		
		Nitrogen......	14.0076
Helium......	***3.0161		**15.0053
	4.0034		
		Oxygen.......	16
Lithium.....	*6.0162		**17.0040
	7.0166		**18.0065
Beryllium....	***8 (?)	Fluorine......	19.0000
	9.0137		
		Neon........	19.9967
Boron........	*10.0145		*21.9947
	11.0110		

The rarer isotopes are starred: * found by mass spectrometer; ** rarer, identified spectroscopically; *** extremely rare and discovered by investigations of nuclear disintegration.

The chemist recognizes only about 90 chemically different atomic species; actually between hydrogen (atomic weight 1) and uranium (atomic weight 238) there are some 250 different atoms.

PHOTOELECTRIC EFFECT AND TELEVISION

585. Photoelectric Effect. Electrons can be driven off from metallic surfaces in several ways. From the cathode of the discharge tube they are knocked off by bombardment of positive ions; in radio tubes they are evaporated off. Electrons are also set free by the action of light; this is called the photoelectric effect. If a zinc plate is attached to the upper knob of an electroscope, a negative charge will leak off when this is illuminated with the light (partly ultra-violet) from an arc lamp. Electrons are ejected from the surface.

The shorter the wave-length of the light, the greater is the speed given to the ejected electrons (Chapter 55). For most metals the effect does not occur at all for visible light but does occur for the short-wave ultra-violet radiation.[1] Only the alkali metals (sodium, potassium, etc.) give the photoelectric effect with visible light. These metals are used in the " photoelectric cell " which forms the electric eye of television.

The photoelectric effect must be present in the discharge tube; ultra-violet light from the discharge falling on the cathode must release electrons to add to those released by positive-ion bombardment. Probably, however, the effect is not of great importance in determining the general characteristics of the tube.

586. Television. The general principle of television can best be explained by describing an early system in which the picture was analyzed and later reassembled by means of rotating discs. In Fig. 7 a small spot of light, passing through one hole after another in the rotating scanning disc, plays rapidly again and again across an object, until the whole object has been systematically covered from top to bottom. The object is scanned in this way, over and over, thirty times a second. The light is reflected from the white surfaces, absorbed by the shadows. The reflected light falls on a photoelectric cell and permits current to flow. After being suitably amplified this electrical energy can be broadcast by a radio sending station, and so, instead of dots and dashes or music or speech, a complete record of the appearance of the object is sent out as electrical impulses in the modulated radio wave.

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[1] Light is electromagnetic wave motion, essentially a radio wave of extremely short wave-length (Table 42). Ultra-violet radiation is radiation of still shorter wave-length, which does not affect the eye.

At the receiving station each electrical pulse excites a glow in a sensitive discharge tube (neon tube). To reassemble these pulses into a picture they are observed through a viewing disc. This must operate in exact synchronism with the scanning disc at the sending station; through this disc we see the fluctuating tube, one spot at a time, until the whole area is covered. The eye sees an integrated picture.

This gives the general principle of television but in the system of today there are no moving parts. A cathode-ray oscillograph is substituted for the viewing disc, the electron beam sweeping back and forth and up and down on the fluorescent screen until it covers the area of the picture. The intensity of the electron beam is strengthened and weakened by the incoming television pulses, and this gives the picture on the screen. At the transmitting station another beam of cathode rays, sweeping back and forth within the photoelectric cell, discharging one point after another of the photosensitive surface, takes the place of the scanning disc. The sweep in the sending and receiving oscillographs must be exactly synchronized.

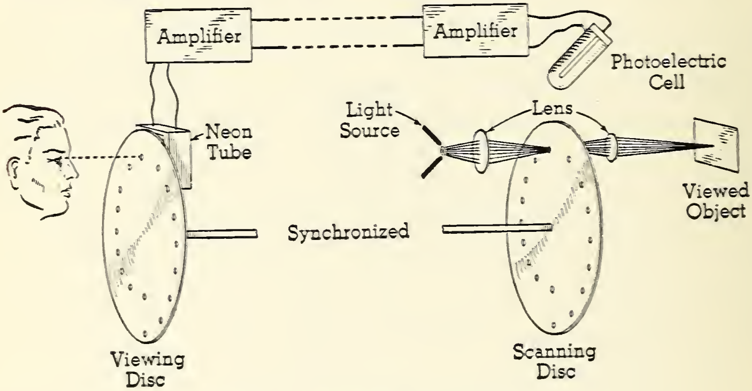

FIG. 7. Principle of television.

For a satisfactory picture something like a hundred thousand pulses, corresponding to light and dark spots of the object, must be transmitted in a thirtieth part of a second (in contrast to a modulating frequency of a few hundred or thousand per second for sound). Since each pulse must consist of a reasonable number of radio waves, very high radio frequencies are required.

Because these very high frequencies are employed the range of the television broadcast is limited by the horizon (§562).

587. General Use of Photocells. The "electric eye" (using either the photoelectric effect or the photronic cell described in §548) has found many applications. It counts automobile traffic on a highway or people entering a theater or boxes leaving a factory for shipment. It is employed in making and projecting sound movies. It is used for fire alarms and burglar alarms, for smoke indicators, for chemical controls. It opens doors, stops elevators, controls drinking fountains, operates traffic lights, sorts lemons and cigars, tests raisins and beans, candles eggs, tabulates statistics, detects poisonous gas, compares colors, and measures, inspects, and sorts factory products — it does these and a hundred more things. And it is estimated that there are still in this country a million eyes doing jobs that could be done better with the electric eye.

X-RAYS

588. X-rays. In 1895 Roentgen was studying cathode rays. He observed that something came from his discharge tube which stimulated certain chemicals on his table to fluorescence. It also affected the photographic plate. Yet it was not ordinary light because it would pass through an opaque body. The emission

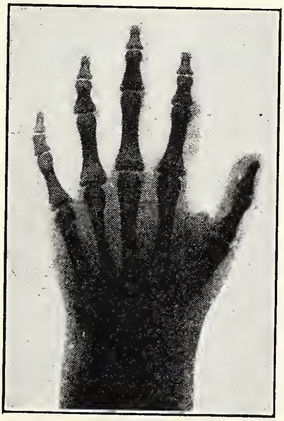

Fig. 8. X-ray photograph.

did not consist of charged particles, for it was not deflected in an electric or magnetic field. Nearly 20 years were to pass before the exact nature of these rays was understood. Roentgen called them x-rays.

Today we know that x-rays are not material particles (like electrons, atoms, ions) but rather electromagnetic radiation of very short wave-length — some million million times shorter than radio waves (Table 42). These waves are given off when the cathode rays strike the anode and are suddenly stopped. (A crude analogy is found in sound waves given off when a stone strikes the floor.) The better the vacuum in the discharge tube and the higher the voltage required for the discharge and the faster the electrons are moving, the shorter is the wave-length of the x-rays emitted and the deeper will they penetrate matter.

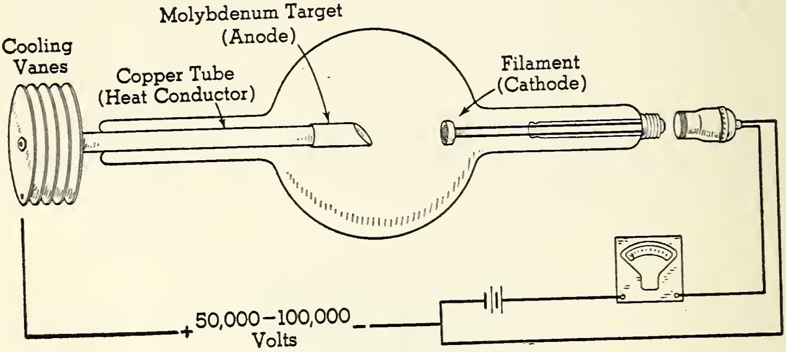

FIG. 9. X-ray tube; Coolidge type, air cooled.

X-rays are excited in an ordinary discharge tube as the cathode rays strike the anode. But except at low pressure and high voltages these rays are so " soft " (i.e., have long wave-lengths) that they are absorbed by the walls of the tube. The tube must be rather highly evacuated, though (in the gas discharge tube) some gas remains to support the discharge. More common today is the hot-cathode tube (Coolidge tube) in which there is a very high vacuum and the electrons are released from an incandescent filament.

X-rays themselves are invisible but they (1) excite fluorescence in certain substances, (2) affect the photographic plate, (3) ionize gases, and (4) produce the photoelectric effect — each of these effects has been used to detect them. In making an x-ray examination of a body, the transmitted rays fall upon a fluorescent screen (fluoroscope) and produce a visible image. When the rays fall upon a photographic plate they give an x-ray picture.

589. X-ray Therapy. X-rays are effective in treating certain skin diseases, gland enlargements, and malignant tumors. The radiation, possibly because of its photoelectric effect, is widely destructive of living cells, and casual exposure to the rays is highly dangerous. But younger cells and rapidly growing cells seem to be particularly susceptible, and this permits (in favorable cases) the selective destruction of the morbid parts without wholesale damage to healthy tissue. Gamma rays from radioactive elements (Chapter 45) act in the same way, and because of the deeper penetration and the easier control of dosage " radium therapy " has advantages in certain treatments.

590. Summary of Properties of Free Electrons. An electron is a negatively charged particle with mass 1840 times less than the hydrogen atom. Electrons can be released from metallic (or other) surfaces by heat (thermionic emission), by x-rays or light (photoelectric effect), or by ion bombardment. The electrons released (by ion bombardment) from the cathode of the discharge tube and projected in straight lines by strong fields are called cathode rays. Even at moderate speeds these electrons excite fluorescence when they strike glass or certain minerals; at higher speeds they produce x-rays when they strike a surface, the wave-length of the x-rays being shorter and their penetration greater the greater the speed of the electrons.

SUGGESTED READING. For a general survey of modern physics, see the second half of White's " Classical and Modern Physics " (Van Nostrand). Many students have enjoyed Eddington's " Nature of the Physical World " (Macmillan); it is written in a fascinating style. Jeans has written a number of books on modern physics which have been very well received. The student interested in the Theory of Relativity is referred to the first chapters of my own text: " The Physical Basis of Things " (McGraw-Hill). There is an almost unlimited choice of popular books on modern physics.

QUESTIONS

1. Name three ways by which electrons can be set free from the surface of a metal.
2. Why is a gas discharge most readily obtained at reduced pressure?
3. What kinds of particles are present in a discharge tube?

4. Why do (*a*) the mass, (*b*) the charge, (*c*) the velocity, affect the deflection of a particle in a transverse electric field?

5. What are isotopes, and how can they be separated?

6. The speed of an electron is not changed by a magnetic field. Why?

7. How are x-rays detected? What are " soft " and " hard " x-rays?

8. How does the speed of a cathode-ray particle vary with the tube voltage?

9. What is the equation for the force on an electron (*a*) in an electric field, (*b*) moving across a magnetic field?

10. Two electrons moving abreast in a cathode ray have an electrostatic repulsion equal to e^2/r^2 (electrostatic units) and a magnetic attraction equal to $\frac{e^2}{r^2}\frac{v^2}{c^2}$ (electrostatic units). Are there circumstances in which the attraction can equal or exceed the repulsion?

11. Suppose that in a discharge tube the cathode rays go from north to south, and that a magnetic field is applied from east to west. The rays will be deflected: (*a*) up; (*b*) down; (*c*) east; (*d*) west; (*e*) north; (*f*) south.

12. X-rays cannot be deflected by a magnetic field because: (*a*) they are not charged particles; (*b*) they are moving too fast; (*c*) they produce no visible effect until they strike a fluorescent screen.

Vocabulary: Low-pressure discharge, cathode (positive, x-), rays, isotope, fluorescence, photoelectric effect.

CHAPTER 45

RADIOACTIVITY AND NUCLEAR PHYSICS

591. Henri Becquerel was a distinguished French physicist, as were his father and grandfather before him. About a year after Roentgen's discovery Becquerel found that uranium (the heaviest element) was giving off rays spontaneously. The rays produced fluorescence and affected a photographic plate like x-rays, but in penetrating power more nearly resembled cathode rays.

Marie Sklodowska was a Polish girl. She had been involved in a students' revolutionary movement, fled to Paris, studied at the University, and married the already distinguished young physicist, Pierre Curie. A year or two after Becquerel's discovery Marie Curie, in cooperation with her husband, after a laborious process of fractionation discovered in the ores of uranium a new element which was several million times more active than uranium itself. They called this element " radium."

These effects of radioactivity at first seemed quite mysterious. The rays were very energetic — those from radium are sufficient to keep the element always a little warmer than the air. Where does this energy come from? Ultimately it was found that uranium and radium are decomposing — very slowly. Uranium, after passing through several intermediate stages, turns into radium — it is half converted in five billion years[1] — and radium changes into other, lighter elements and ultimately into lead. In fact, all the elements heavier than bismuth are decomposing in this way. This was a startling discovery, for it had been supposed that chemical elements were immutable, never to be changed one to another. The discovery of radioactivity was the first step in revealing the structure of the atom. It led directly to the concept of the nucleus. (The student should review §§380, 381.)

592. Alpha, Beta, Gamma Rays. Ernest Rutherford was born in New Zealand. He came to England as a graduate student and then for a number of years was professor of physics at McGill University (Montreal).[1] In the year following the Curies' discovery Rutherford analyzed the radiation and identified in it two different kinds of particles which he named alpha (α) rays and beta (β) rays — shortly after gamma (γ) rays were discovered.

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[1] This time for half conversion is called half-life (§597).

When the rays of Becquerel pass through a very strong magnetic field they separate into three components. Some of the radiation, deflected slightly to the left in Fig. 1, consists of positively charged particles. These particles penetrate only the thinnest foils, are stopped by a few centimeters of air. These are alpha (α) rays. Rutherford showed that the alpha particle had four times the mass of a hydrogen atom and carried a double positive charge. He showed that it was identical with the nucleus of the helium atom (§380).

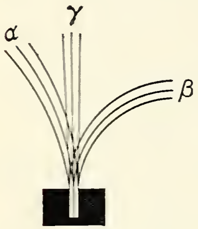

FIG. 1

Some of the radiation is deflected much more strongly in the opposite direction, indicating a much smaller mass and negative charge; it penetrates much thicker foils and consists of fast-moving electrons. These are beta (β) rays. They are the same as cathode rays in a very high-voltage discharge tube.

A third part of the radiation is undeflected in the magnetic field and has great penetrating power. It is an electromagnetic wave, like x-rays, but usually shorter in wave-length and more penetrating. These are gamma (γ) rays.

These rays from radioactive substances can be easily demonstrated. The radiation ionizes the air. An electroscope is almost instantly discharged when a minute quan-

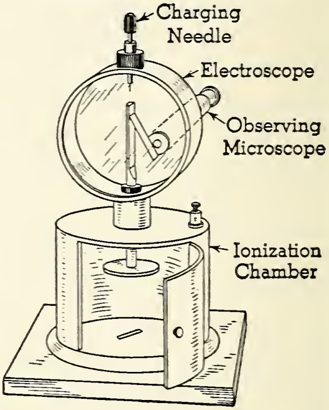

FIG. 2. Electroscope for radioactivity measurements.

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[1] He later became Lord Rutherford and the successor of Maxwell, Lord Rayleigh, and Sir J. J. Thomson as head of the Cavendish Laboratory (Cambridge University).

tity of radium is brought near. The same thing happens more slowly when a considerable quantity of uranium is used. Gram for gram, uranium is three million times less active than radium. Alpha particles produce fluorescence in many substances, notably zinc sulfide. If we bombard a zinc sulfide screen, not too strongly, with alpha particles and observe the screen with a magnifying glass, we see flashes of light, one by one, as the alpha particles strike. So we can actually count the alpha particles, each one representing the disintegration of an atom! A very minute quantity of radium mixed with zinc sulfide is used to make luminous watch dials.

593. Separation of the Rays by Absorption. Usually it is impracticable to separate the rays by a magnetic field. Alpha rays produce much more ionization than beta, and beta rays more than gamma. Hence, when a specimen emitting all three rays is brought near the electroscope, the effect can be considered as practically all due to alpha rays. If the specimen is covered with a few sheets of gold foil or several cigarette papers the alpha rays are absorbed, and the effect now is chiefly due to beta rays. The beta rays will pass through several millimeters of aluminum or of cardboard, but if the specimen is covered with sufficient cardboard, or a millimeter or two of lead, beta rays also are absorbed and the small residual ionization is due to gamma rays. Gamma rays will penetrate several inches of lead or perhaps a foot of iron.

594. Cloud Chamber Tracks. In the "cloud chamber," air, saturated with water vapor, is suddenly expanded, cools, and becomes supersaturated. When alpha particles now pass through the chamber they knock electrons from the air molecules and leave a trail of charged molecules (ions) behind them, and on these ions water droplets condense. We see spreading out in the chamber threadlike trails of mist; these are tracks of alpha particles, most striking demonstration of radioactivity. Hundreds of thousands of pictures of such tracks have been taken and have brought to light collisions with atomic nuclei, explosions of nuclei, many of the most important results of "nuclear physics."

The tracks of beta rays and of other charged particles have been studied by the same method.

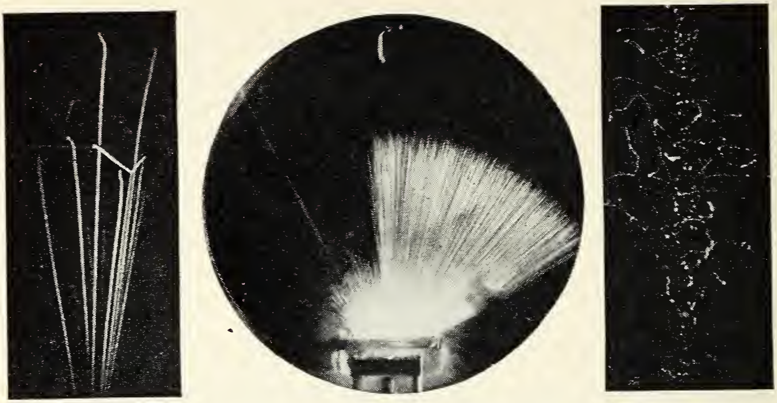

FIG. 3. Cloud chamber pictures. *Center:* alpha particles have definite range in air; they are partially stopped in passing through paraffin film in left half of picture; they eject a high-speed proton from the film. *Left:* alpha particles collide with nuclei in air. *Right:* x-rays passing through air eject electrons which leave crooked trails.

595. Ancestry of Radium. The alpha and primary beta rays come from the nucleus as it decomposes. Uranium emits an alpha particle. The atomic weight of the uranium nucleus is 238; its atomic number (nuclear charge) is 92. The alpha particle weighs 4 and has a charge of 2. So when the alpha particle leaves we have a new element of mass 234, nuclear charge 90 (Table 45). This is called uranium X_1; it is an isotope of thorium (same atomic number but two more neutrons).

Uranium X_1 emits a beta particle (electron). We can neglect the weight of the electron; its charge is -1. So the new element (uranium X_2) has a weight of 234 and nuclear charge (atomic number) of 91. Uranium X_2 emits another beta particle and gives uranium II (mass 234, charge 92 — isotope of uranium).

So the process of disintegration goes on. After two more alpha particles are emitted, we get radium (226, 88).

596. Descendants of Radium. With one or two exceptions the ancestors of radium are long lived — " half-life," billions or millions or tens of thousands of years. But after radium the disintegration proceeds more quickly. Radium itself has a half-life of 1690 years; it emits an alpha particle and becomes the gas radon (or " radium emanation "); radon itself disintegrates after a few days (alpha-particle emission) and then, after several other transmutations in the course of which three alpha particles and four beta particles leave the nucleus, the series terminates in lead.

Among these descendants of radium the longest half-life is 16 years (radium D), the shortest 10^{-6} second (radium C′). Perhaps the most interesting of the new elements is radon. It belongs to the family of noble gases (helium, neon, argon, etc.). Because of this emanation radium must be kept in a sealed tube. Radon diffuses through the tube, and radium A, radium B, and its other descendants are formed as an " active deposit " on the walls. Radon, removed from the parent radium and sealed in diminutive tubes, is the most convenient source for radium therapy.

TABLE 45

URANIUM-RADIUM SERIES

Element	Half-Life	α or β Radiation	Atomic Mass (Approximate)	Atomic Number
Uranium......	5×10^9 yr.	α	238	92
Uranium X_1...	25 days	β	234	90
Uranium X_2...	1 min.	β	234	91
Uranium II....	2×10^6 yr.	α	234	92
Ionium.......	7×10^4 yr.	α	230	90
Radium.......	1690 yr.	α	226	88
Radon........	4 days	α	222	86
Radium A.....	3 min.	α	218	84
Radium B.....	27 min.	β	214	82
Radium C.....	20 min.	β	214	83
Radium C′....	10^{-6} sec.	α	214	84
Radium D.....	16 yr.	β	210	82
Radium E.....	5 days	β	210	83
Polonium.....	136 days	α	210	84
Lead.........	....	..	206	82

We have described the uranium-radium-lead sequence of radioactive elements; there are two other sequences, one starting with thorium as parent element and one starting with an isotope of uranium (atomic mass 235) and containing actinium as its best known member. The transformations of the thorium series and the actinium series are in general very much like those described above and each series terminates in an isotope of lead.

597. Stability of Heavy Elements. The exploding of the nucleus is a matter of pure chance; no change in physical conditions — high temperatures, high pressures — affects the process. There is one chance in two that a given nucleus of radium will eject an alpha particle in 1690 years; in other words, in this time a given quantity of radium will be half disintegrated; this time is called the *half-life* of the element.

The half-life is a measure of the stability of the nucleus. It is found that the shorter the half-life the greater the speed of the ejected alpha particles. The same thing is true for the *fastest* beta particles. (Unlike alpha particles, which from a given nucleus all have substantially the same speed, beta particles from an exploding nucleus have a range of speeds.) Atomic nuclei consist of protons and neutrons. For example, in the parent uranium (U I) there are 92 protons and 146 neutrons (weight 238). Probably 92 protons and 92 neutrons can be considered as combined into 46 alpha particles leaving a considerable excess of neutrons which somehow serve to bind the alpha particles together. But the binding in these heavy atoms is always more or less unstable. As it happens the particular combination in uranium is nearly but not quite stable (half-life five billion years). It emits a slow alpha particle and so begins the train of events which ends only after the nucleus has lost 8 alpha particles and 6 electrons and reaches permanent stability in lead. Uranium X_1 has an unstable excess of neutrons and emits an electron; this loss of an electron from a neutron changes it to a proton. This is repeated again, resulting in U II with 46 alpha particles like U I but with fewer neutrons and less stability and shorter half-life. The succeeding elements have ever-decreasing stability and shorter half-lives, and they emit alpha particles of ever-increasing speeds, until at radium B there is again a neutron excess and a pair of beta rays are emitted by Ra B and Ra C. Another alpha ray is emitted, two more beta rays and an alpha ray to give the stable lead. (Observe that the beta rays here, as is usual, occur in pairs; a single beta ray leaves the nucleus with an odd number of protons, which is an unstable arrangement, probably because the protons cannot now pair off into alpha particles.)

598. Gamma Rays. Sometimes after the transformation the newly born nucleus is left with excess energy (an "excited state"). In settling down to the normal state it emits this energy as electromagnetic radiation, usually of higher frequency than ordinary x-rays. These are the primary gamma rays. (Not all the high-speed electrons [β rays] and high-frequency radiation [γ rays] in radioactivity come from the nucleus. Some of the electrons which surround the nucleus may be knocked off or excited by the primary particles and so produce secondary beta and gamma rays.)

599. Transmutation of Matter. The olden alchemists tried to make gold from other elements. But with the development of chemistry the elements came to be considered as immutable and the problem of the alchemist seemed to be as hopeless as the making of perpetual-motion machines. Then, with the discovery of radioactivity, the heaviest atoms were found spontaneously changing one into another. But these radioactive processes could be neither accelerated nor retarded by man. In 1919 Lord Rutherford found evidence that an alpha particle in passing through nitrogen gas and striking a nitrogen nucleus (7 protons, 7 neutrons, weight 14) would occasionally not rebound (as is the usual effect) but united with the nucleus and emitted a proton. The result was an isotope of oxygen (8 protons, 9 neutrons, weight 17). He found similar alpha particle reactions with a number of other light elements, and the result has been abundantly verified by tracks in the cloud chamber. This was the first transmutation of elements controlled by man.

Recently it has been found that almost any element can be transmuted — broken down to a lighter element or built up to a heavier one — by bombardment with high-speed particles. A number of very high-voltage generators (one type shown in Fig. **31**–8) have been built for speeding up protons, deuterons, alpha particles. With these an amazing array of transformations have been effected, particularly among the lighter elements. We do not know that transmutation of elements will ever be accomplished on a commercial scale, but in principle the problem of the alchemists has been solved. This is one of the outstanding successes of recent physics.

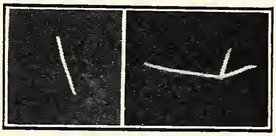

FIG. 4. Neutron collisions. Neutrons are moving upward but leave no tracks. *Left:* nitrogen nucleus projected by direct hit. *Right:* neutron unites with nitrogen nucleus which then explodes into lithium and two alpha particles.

600. The Neutron. It was among the products of this attack on the nucleus that the neutron was discovered. It was found that, when alpha rays bombarded beryllium, something was ejected which penetrated a great thickness of lead; passing through a cloud chamber it left no track of its own but occasionally by its impact projected a nucleus of a gas molecule through the chamber and so by this track betrayed its presence. Except for its momentum (which was too great for an electromagnetic wave) it resembled a gamma ray. This was the neutron. It has the mass of a proton (approximately) but no charge and hence passes through matter without producing ionization and without loss of energy except for the rather rare occasion when it strikes a nucleus. Neutrons have been widely used as the projectiles in transmutation experiments among the heavier elements; they are not, like the charged particles, repelled by these highly charged nuclei.

TABLE 46

FUNDAMENTAL PARTICLES

	Primary Particles in Matter
Electron:	negative charge, mass 9×10^{-28} gram. Recognized (J. J. Thomson) 1897.
Proton:	positive charge, mass 1840 times greater than electron; discovery of electron implied the existence of a proton.
Neutron:	no charge, mass approximately equal to that of proton. Discovered (Chadwick) 1932.
	Transient Particles
Positron:	positive charge, mass of electron. Discovered (Anderson) 1932.
Meson:	negative charge, mass about 200 times greater than electron. Discovered 1938.
	Important Complexes
Deuteron:	proton and neutron in combination. Nucleus of deuterium. Discovered (Urey) 1932.
Alpha particle:	2 protons and 2 neutrons in combination. Helium nucleus. Discovered (Rutherford) 1899.

601. Deuterium and the Deuteron. Of the seven fundamental particles listed in Table 46, three were discovered near the close of the last century and three (neutron, positron, and deuteron) discovered in the single memorable year of 1932. In that year Urey, theoretical chemist at Columbia University, concluded, from a study of the spectrum of hydrogen, that hydrogen had an isotope of weight approximately 2 (present in normal hydrogen in the ratio of about 1 part in 5000); this was confirmed by partially separating the isotope by evaporation of liquid hydrogen. Subsequently it was found possible to separate completely the " heavy hydrogen " (or *deuterium* as it has been named) by an elaborate electrolytic process. The new isotope has been an important tool in recent physical, chemical, and physiological research.

The nucleus of deuterium consists of one proton and one neutron; it is half an alpha particle. It is called a *deuteron* and is one of the projectiles used in " atom smashing."

602. The Positron and the Meson. The positron was discovered in the study of cosmic rays (§605). In 1932 Anderson, a young physicist at California Institute of Technology, observing the tracks produced by cosmic rays in a magnetic field, found a particle which had the same mass as an electron but which (as shown by its deflection) had a positive charge. (See Fig. 5.) A positive electron! The *positron* (as it is called) has since been frequently observed; it is believed that positrons and negative electrons are created simultaneously out of very high-frequency gamma rays, which are sometimes scattered from heavy nuclei in this strange way as oppositely charged particles created from the wave. The positron exists for but a brief interval; a positron and a negative electron meeting mutually annihilate each other's charges, and their energy and mass are dissipated as electromagnetic radiation (gamma ray).

In 1938, a particle was found (again in cosmic-ray tracks) with negative charge but with a mass about 200 times greater than that of the electron. It is called a *meson.* It is believed that this " heavy electron " is produced within the earth's atmosphere by the bombarding rays from outer space.

603. Artificial Radioactivity. The products of artificial nuclear transformations are often unstable, the ratio between neutrons and protons in the new nucleus being either too great or too small. When this is so, the new element ultimately emits a negative electron (changing a nuclear neutron to a proton) or else a positron (changing proton to neutron). These unstable elements have a definite half-life, and the phenomenon is similar to radioactivity, except that here positrons instead of alpha particles are emitted to relieve a proton excess.[1]

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[1] Radium was discovered by Pierre and Marie Curie; they received the Nobel Prize in 1903. Artificial radioactivity was discovered by Frederic Joliot and Irene Curie-Joliot (the latter the daughter of the older Curies); they received the Nobel Prize in 1935.

Artificial radioactivity has found practical application in hospitals as a substitute for natural radioactivity. At slight expense activated substances can be prepared in the nuclear physics laboratory which are equivalent in activity to millions of dollars' worth of radium.

Another application has been in the study of nutrition and metabolism. Radioactive sodium or carbon behaves chemically exactly like ordinary sodium or carbon. But the activated element presently distinguishes itself from its fellows by emitting an electron or positron. Activated elements eaten with the food can be traced as they are assimilated in the body. They have been described as "tagged atoms."

604. Energies of Nuclear Processes. Ten billion joules of energy is released as heat from a gram of radium in its transformation into lead — over 2 million large calories — equivalent to the heat in combustion of 600 pounds of coal! The mass of this lost energy (§573) is not negligible and makes an appreciable difference in the weight of the atom. . . For other elements the energy given up when the protons and neutrons are bound in the nucleus can be computed from the atomic weight. Inspection of a table of atomic weights (showing the weight of each isotope) indicates that the weight of the compounded nucleus is always less than that of the protons and neutrons of which it is composed. The difference is due to the heat of formation. 9×10^{20} (i.e., c^2) ergs are equivalent to 1 gram (§573).

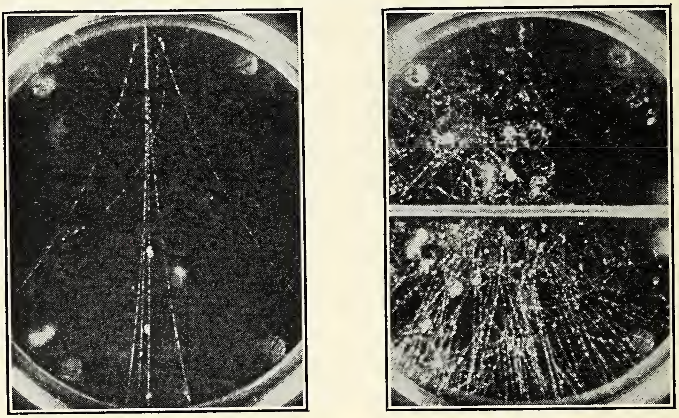

FIG. 5. Secondary cosmic rays in magnetic field. *Left:* Negative electrons and positrons produced by primary rays (positrons deflected to right). *Right:* A cosmic-ray "shower." (Cloud-chamber tracks.)

It may be that this vast store of energy in the nucleus will some day take the place of our present fuels. (This is a rather long chance and should not be used to excuse our waste of energy resources.) In a more fundamental sense nuclear transformations are probably the ultimate source of all our energy. The origin of solar energy has always been a mystery. It is probable that this energy in the sun is derived from the building up of helium from hydrogen nuclei (protons). Presumably the sun or another star begins its life as an aggregation of rarefied hydrogen gas. The gas contracts and its temperature rises (adiabatic compression, §243). When the temperature reaches some millions of degrees hydrogen nuclei can combine to form helium. From this reaction evolve over 100 billion calories per gram.[1] This process has (we believe) been taking place near the center of the sun for some billions of years and will probably continue for some billions of years more, until the hydrogen of the sun is exhausted.

605. Cosmic Rays. Into the earth's atmosphere from outer space comes a bombardment of charged particles with kinetic energies a thousand times greater than the rays of radioactivity. The most energetic of these rays penetrate the atmosphere, penetrate even several hundred feet of water. As it goes through the air each particle leaves a trail of ions behind it, occasionally leaving a " burst " or " shower," of many trails.

The presence of the rays is evidenced by the gradual leak of an electroscope, even with the best of insulation, even with the most extraordinary shielding with great thickness of lead. The rays have been studied in a variety of other ways — notably by cloud-chamber tracks. In an extensive program of investigation they have been studied in all longitudes and latitudes, in the stratosphere and in deep mines.

What is their source? No one knows. But it is generally believed that these rays carry with them a secret from the outermost parts of the universe. It is a challenging problem.

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[1] Consider the reaction for 4 gram-atomic-weights of hydrogen (1.008) uniting into 1 gram-atomic-weight of helium (4.002).

4H (1.008) = He (4.002) + 0.030 gram of radiant energy.

0.030 gram $\times 9 \times 10^{20} = 27 \times 10^{18}$ ergs $= 27 \times 10^{11}$ joules $= 6.5 \times 10^{11}$ calories. In the process half of the protons change to neutrons but this does not affect the mass balance.

LIGHT

CHAPTER 46

PROPAGATION OF LIGHT

Light is a wave motion. It is electromagnetic radiation with wave-length some millions or billions of times shorter than the characteristic radio waves. The wave-length is usually expressed in *angstroms;* an angstrom is 10^{-8} centimeter. The wave-lengths of visible light lie between 4000 and 7000 angstroms, between 0.0004 and 0.0007 millimeter.

Water waves (and sound waves) bend around an obstacle and leave no clean-cut shadows. Light is diffracted in the same way, but because of its very short wave-length the magnitude of this diffraction is relatively very small. In explaining many optical phenomena diffraction may be ignored and light treated as if it traveled exactly in straight lines (rays).

606. Shadows. The reason for shadows is self-evident in the blocking of the rays of light. Compare the shadows cast when the source of light is a flashlight bulb and a frosted household lamp. With the flashlight bulb (very nearly a point source) shadows are very sharp. With the larger light source there is not this sudden transition from light to densest shadow. The edge of the shadow develops gradually as more and more of the light source is hidden. The region of complete darkness is called the *umbra*, the transition region the *penumbra* (Fig. 1). The illuminating engineer seeks to eliminate sharp shadows; hence he avoids concentrated sources; he uses frosted shades, diffusing luminaires, and indirect illumination (Fig. 7).

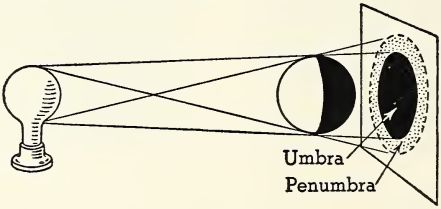

FIG. 1. Shadow.

Eclipses of sun and moon are examples of shadows on a large scale. In a solar eclipse the moon comes between us and the sun. The umbra of the moon is then a small patch on the earth's surface a few miles in diameter. For a person located here the sun is completely obscured; as the earth rotates this patch travels, describing a belt a few miles wide in which the eclipse is total. Persons in the penumbra observe a partial eclipse.

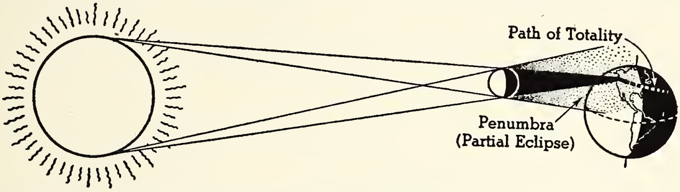

FIG. 2. Solar eclipse.

607. The Pinhole Camera. This is another example of straight-line propagation of light. The arrow *O* (Fig. 3) represents a source of light — a lamp or any brightly lighted object. Light from each point of this *object* passing through the small hole in the first screen falls as a small spot upon the second; the totality of these spots forms an *image* (*I*). If the two screens are the opposite sides of a box, light-tight except for the pinhole, and if the light falls on a piece of photographic film, we have the simplest form of camera. (The student can make one from a chalk box.) The chief fault with a pinhole camera is that it is "slow"; to take sharp pictures the hole must be small, and very little light is admitted; hence the exposure time is long.

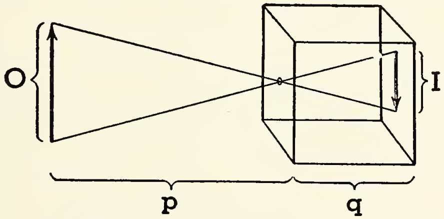

FIG. 3. Pinhole camera.

In the springtime, when the foliage is not too thick, we often find the ground beneath the trees spotted with silvery discs. Have you ever observed these during a partial eclipse? Each silver disc has now become a silver crescent. These are indeed pinhole images of the sun, formed by the light coming through the chinks in the leaves.

Observe that the image is inverted (light from the top of the object passing straight through the pinhole falls at the bottom of the image). Observe also that (since O and I form corresponding sides in similar triangles) the height of the object (O) is to the height of the image (I) as the distance to the object (p) is to the distance to the image (q):

$$\frac{O}{I} = \frac{p}{q}. \tag{1}$$

We shall find in our further study that these rules are quite general — *real images* are always inverted, and the sizes of object and image are always related by the simple formula (1).

608. Strength of Light Sources; Intensity of Illumination. The strength of a light source is measured by comparison with a standard lamp and is expressed in candlepower. The candlepower is an arbitrary unit; it is approximately the strength of an ordinary tallow candle. But candles do not themselves make satisfactory standards. The National Bureau of Standards, cooperating with similar institutions abroad, maintains incandescent lamps as light standards, and it is in terms of these that the international candlepower has been defined. Standardized lamps of specified rating may be procured. Approximately, we can expect the incandescent lamps used in the household to give about 1 candlepower per watt. (See Table 39, p. 434.)

Not to be confused with the strength of the source (S) is the intensity of illumination (L), which is a measure of the light falling on unit area of surface. It is the intensity of illumination which determines the brightness of this page as you read it. The intensity of illumination varies with the candlepower of the source and inversely as the square of its distance away. It is usually measured in terms of foot-candles; a *foot-candle* is the illumination produced by a standard candle at a distance of 1 foot. We have then the law

$$L \text{ (foot-candles)} = \frac{S \text{ (candlepower)}}{(\text{Distance in feet})^2} \tag{2}$$

Approximately what is the intensity of illumination from a 50-watt lamp 5 feet away?

Figure 4 illustrates the relation between illumination and distance. The light diverges as it leaves the source; the light in a given solid angle which at a certain distance illuminates an area A at twice the distance illuminates an area $4A$, etc.; the area illuminated increases as the square of the distance, and hence the intensity of illumination varies inversely as the square of the distance (conservation of energy).

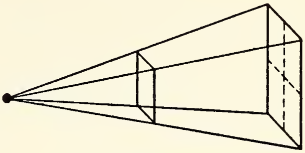

FIG. 4

The intensity of illumination can be measured by means of the "foot-candle meter"; this is a photronic cell (§548) calibrated to read the illumination directly in foot-candles.

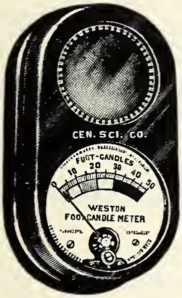

FIG. 5. Foot-candle meter.

The relation (2) assumes that the illumination is due only to direct radiation from the light source. White walls reflect light and increase the illumination. A lamp in a room with light wallpaper and light flooring will give several times the illumination of a street lamp the same distance away.

609. Measuring Candlepower. The strength of a light source is determined by comparison with a standard lamp. The instrument used for the comparison is called a *photometer.* The unknown and the standard light sources are placed at opposite ends of a graduated track. A screen,[1] movable along the track, is adjusted in position until the intensity of illumination upon opposite sides due to the two sources is judged equal. We have then from eq. (2)

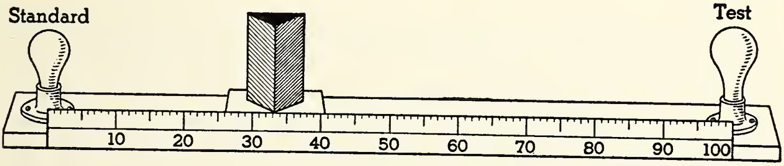

FIG. 6. Simple photometer.

$$S = L \times \text{Distance}^2,$$

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[1] A very simple screen is furnished by two blocks of paraffin, separated by tinfoil.

or since the illuminations (L) are equal

$$S_{\text{test lamp}} : S_{\text{standard}} = \text{Distance}^2{}_{\text{test lamp}} : \text{Distance}^2{}_{\text{standard}}. \quad (2')$$

610. Illumination from Various Light Sources. The intensity of illumination of the sun when directly overhead is about 7000 foot-candles. A thousand times less is the average intensity in a brightly lighted room (artificial light); moonlight is a thousand times still less intense; the light from the brightest stars is a thousand times less again; the stars just visible to the eye are a hundred times fainter still. And the brightness of the faintest stars perceived by the large telescopes is still 100,000 times less.

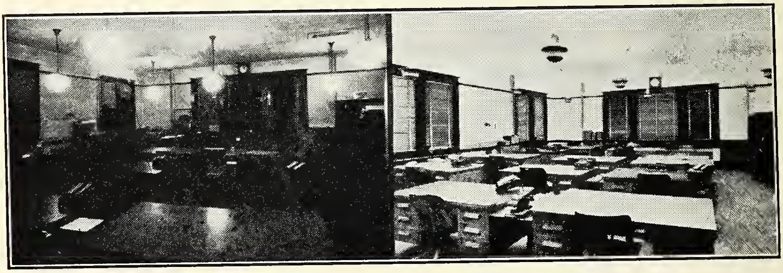

FIG. 7. Direct and indirect illumination.

An intensity of at least 8 foot-candles is recommended for schoolrooms, offices, and general reading; much greater, up to 100 foot-candles, in shops and laboratories where close observation is required. To avoid eyestrain in the study room the nature of the illumination — freedom from sharp shadows, from excessive contrasts, from glare — is quite as important as its intensity.

611. The Lumen. This unit is employed by the illuminating engineer to measure light flux. It is often necessary to consider the light flow through a window or onto a table top or to consider the light which flows in all directions from a lamp. In technical language this light flow is called *light flux*. *The lumen is the quantity of light which passes through a square foot of area 1 foot away from a 1-candlepower source.* (In other words, where the intensity of illumination is 1 foot-candle 1 lumen passes through each square foot of area.)

In analogy to electrical quantities, the candlepower of the source corresponds to the charge, the foot-candles of intensity to the field intensity or number of lines of force per unit area, and the lumens of light flux to the total number of lines of force through a given area. A 1-candlepower source radiates (in all directions) 4π lumens. Lamps are frequently rated by their lumen output; this is 12.5 (i.e., 4π) times their candlepower.

Example. With the sun overhead 7000 lumens enter a skylight 1 ft. square (§610). A 100-candlepower (approximately 100-watt) lamp emits 1250 lumens; $5\frac{1}{2}$ such lamps would be required to furnish the same light flux into the room as the skylight.

VELOCITY OF LIGHT

612. Light Velocity by Römer's Method. So great is the velocity of light that the propagation may well seem "instantaneous." (Certain Greek philosophers even thought that vision was shot outward from the eye, or, as we should interpret it today, that it was received before it was sent!) The first determination of this velocity was made by the Danish astronomer Römer (in the time of Newton).

The planet Jupiter has nine moons; Galileo had discovered four of these shortly after he invented the telescope. The innermost of these moons makes one revolution around the planet in about 12 hours. Römer, observing the time from moon set to moon set (as the moon went into eclipse behind the planet), had determined this period quite accurately. But his data seemed not entirely consistent. Comparing the time of an eclipse when the earth was nearest to Jupiter (at A) with one occurring six months later when the earth was on the opposite side of its orbit (C), he found that the second eclipse apparently occurred 1000 seconds too late. This difference Römer properly attributed to the time required for the light to pass across the diameter of the earth's orbit (186,000,000 miles). Hence, the velocity of light is 186,000 miles per second.

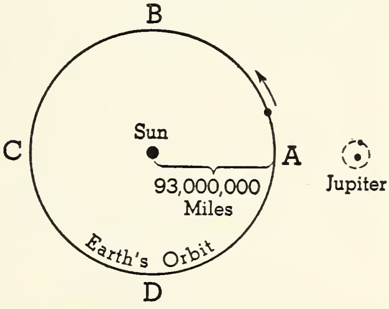

FIG. 8. Römer's experiment.

613. Aberration of Light. Some years later this result was confirmed by the English astronomer Bradley when he discovered the *aberration of light.*

Suppose that rain is falling vertically with velocity c and that we are walking through it with a velocity v. The motion relative to us is as shown in Fig. 9. If we wish the raindrops to pass down through a tube we must tip this by an angle θ. Substitute light for the raindrops, the motion of the earth in its orbit ($18\frac{1}{2}$ miles per second) for our walking, and a telescope for the tube, and we have the explanation of aberration of light. θ is the angle of aberration.

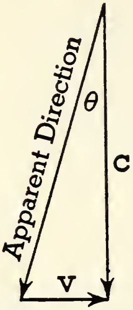

FIG. 9

Bradley observed that the apparent position of a star (seen in a direction perpendicular to the earth's motion) is different now and six months later when the direction of the earth's motion is reversed. The angle of aberration here is 0.0001 radian.

$$\frac{v}{c} = \frac{1}{10{,}000}.$$

Hence c is 10,000 v, and, since v is known to be about $18\frac{1}{2}$ miles a second, the velocity of light agrees with Römer's value.

614. Fizeau's Method. Fizeau (1849) was the first to measure the velocity of light by other than astronomical means. A parallel beam of light, after passing through one of the openings in a toothed wheel, went to a distant mirror, was then reflected back, and (when the wheel was at rest) observed through the same opening in the wheel. When the wheel was rotated at a certain speed the returning light was blocked out, because a tooth moved into the gap while the light made its journey to the mirror and back. At twice this speed the light was visible, coming to the eye now through the next gap. Thus, the speed of the wheel being known, the time of propagation was known and hence the velocity of light was determined.

615. Michelson's Method. By a somewhat similar method Michelson measured the speed of light with high precision. M represents a rotating mirror. A light beam is reflected from this to a distant mirror and then back. When the mirror M is at rest this return beam retraces its path back to the source. But when rotating the mirror will turn through some angle θ during the time of travel and the reflected beam will be turned

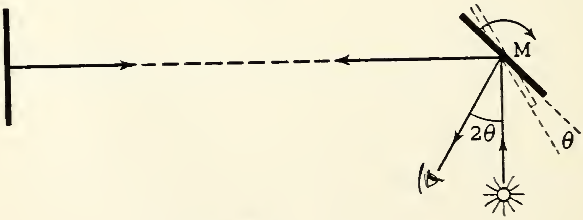

FIG. 10. Michelson's method.

through an angle 2θ.[1] The angular velocity of the mirror being known, and the angle of rotation θ being determined by observation of the reflected beam, the elapsed time can be determined and hence the velocity of light calculated.

This work was done at the famous Mount Wilson Observatory (in Southern California). The distant mirror was located on a peak in the San Gabriel Mountains some 25 miles away. (By a special government survey this distance was determined to about 1 part in 1,000,000, that is with a probable error of only 1 or 2 inches.) Michelson's value for the velocity of light was 299,796 kilometers per second.

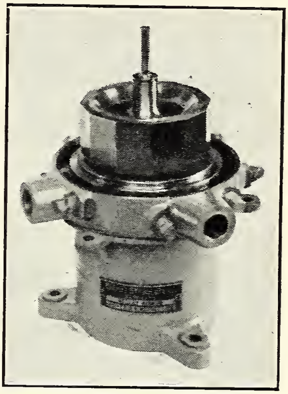

FIG. 11. 12-sided rotating mirror used by Michelson.

Just before he died Michelson constructed a vacuum tube a mile long, hoping to repeat the experiment free from atmospheric disturbances. The experiment, carried to conclusion after his death, gave the now accepted value of 299,776 kilometers per second (186,272 miles per second). Michelson's investigations stand as a monumental example of precise measurement.

This constant has the greatest fundamental significance. It is not only the velocity of light; it is likewise the limiting velocity of all else; material bodies (electrons, protons, etc.) may approach this speed; nothing can exceed it (§99). It gives the ratio of fundamental electrostatic to electromagnetic units in electricity. It seems to be related to the fundamental concepts of space and time.

The velocity of light is a constant irrespective of the motion of the observer. It might seem that if I ran toward an approaching light beam its velocity *relative to me* would exceed 299,776 kilometers per second and that if I ran away from it its apparent velocity of approach would be less. But this is not true. Michelson and Morley measured the velocity of light in different directions, at different times of day, thinking that it would depend on the motion of the earth " through the ether." But the observed velocity never changed. This is the famous Michelson-Morley experiment. It gave rise to Einstein's Theory of Relativity (§574).

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[1] When a mirror is rotated a reflected beam is deviated by twice the angle of rotation of the mirror. If this is not obvious, try it, reflecting the image of the sun or of a lamp from a mirror. When the mirror is turned through 45° the reflected image is turned through 90°. This is a result of the laws of reflection (§616).

Suggested Reading. Bragg's " Universe of Light " (Bell) is a popular general survey. Luckiesh has written several popular books on illumination and color. For interpretation of spectroscopy see White's " Classical and Modern Physics " (Van Nostrand).

QUESTIONS

1. In Fig. 1 the penumbra is represented as uniformly illuminated. Is this correct? Why?

2. Explain the eclipse of the moon. Why are complete lunar eclipses more common than complete solar eclipses?

3. Is a shadow in daylight perfectly black? Why?

4. Why must the hole be small in a pinhole camera?

5. The greater the length of a pinhole camera (and hence the larger the picture), the longer the exposure must be. Why?

6. Define candlepower, foot-candle, lumen.

7. The intensity of illumination 10 ft. from a 100-candlepower lamp is 1 foot-candle in a room with black walls but 3 foot-candles in another room with white walls. Explain.

8. Explain why in Michelson's measurement of the velocity of light the light beam was deflected by twice the angle of rotation of the mirror.

9. Suppose that the sound source or the listener is in motion or that the wind is blowing. The velocity of sound is approximately 1100 ft. per sec., relative to what: listener, source, or medium? The speed of a bullet (from a given cartridge) has a definite value, relative to what: gun, air, or target? The speed of light is 186,000 miles per sec., relative to what: source, ether, or observer?

10. When the photometer is balanced the strengths of the two balancing light sources vary: (*a*) directly as the square of the distance; (*b*) inversely as the distance; (*c*) inversely as the square of the distance.

11. The quantity of light passing into a window is measured in: (*a*) candlepower; (*b*) foot-candles; (*c*) lumens.

Vocabulary: Angstrom, umbra, penumbra, strength of light source, intensity of illumination, foot-candle, lumen, photometer, aberration of light.

PROBLEMS

1. If the photometer in Fig. 6 balances for the setting shown and if the standard lamp is 50 candlepower, find the strength of the lamp under test.

2. In comparing a 10¢ imported incandescent lamp with a 15¢ domestic product of the same wattage a balance was obtained at 57 cm. on the meter photometer. Compare the candlepowers.

3. How far away must one place a 100-candlepower lamp to give the same intensity of illumination as strong moonlight (§610)?

4. A piece of black paper absorbs 95 per cent of the light falling on it. Compare its brightness in direct sunlight (§610) with the brightness of a white object 2 ft. from a 100-candlepower lamp.

5. In a room with black walls what is the intensity of illumination 5 ft. below a 100-candlepower lamp? How much light (in lumens) falls here upon a table top a yard square? What would be the light flux to the same table in open sunlight?

6. How many 60-candlepower lamps would be required to furnish the same amount of light in a room as obtained from direct sunlight through a skylight 2 ft. square?

7. When the intensity of illumination of sunlight is 7000 foot-candles, how much sunlight (in lumens) passes through a lens 6 in. in diameter? If this light is converged to a spot $\frac{1}{8}$ in. in diameter, what is the intensity of the focal spot, and how much light falls upon this spot?

8. How long would it take light to travel around the earth? What is the speed of light in centimeters per second?

9. How long does it take light to reach us from the moon (distance 240,000 miles), the sun, the nearest star (distance 2.5×10^{13} miles)?

10. Because of the rotation of the earth at sunrise we are approaching the sun, at sunset receding, with a speed (at the equator) of 1500 ft. per sec. What is the difference in the apparent velocity of sunlight (relative to us) at daybreak and sunset?

CHAPTER 47

PARALLEL BEAMS

Light rays diverge as they leave the source. However, when the source is a great distance away a small bundle of rays (a beam) is sensibly parallel. With such parallel beams the fundamental principles of optics can be established.

616. Laws of Regular Reflection. When a sunbeam, passing through a hole in a screen, falls upon a mirror it is reflected. The angle (*i*) between the perpendicular to the mirror and the incident ray is called the angle of incidence; the angle (*r*) between perpendicular and reflected ray is called angle of reflection. It is observed that:

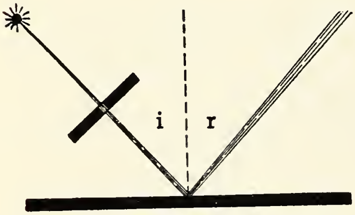

Fig. 1.

(1) The incident ray, reflected ray, and perpendicular lie in the same plane.

(2) The angle of incidence equals the angle of reflection.

617. Corpuscular and Wave Theories. What is this thing which is thrown off from the sun, travels with unparalleled speed through empty space, continually feeds energy to our earth? Since it seemed to be propagated in straight lines Newton maintained that it consisted of fast-moving particles ("corpuscles"). Such particles would be reflected from a smooth surface like billiard balls from a cushion — this explains regular reflection. Newton explained the bending of light as it entered water or glass (Fig. 5) by assuming that as the corpuscles approached they were attracted by the denser medium. This implied that the velocity of light was greater in water and glass than in air.

Huygens and Hooke maintained that light was a wave motion, but because of Newton's great authority the corpuscular theory was generally accepted for more than a century. Then (1801)

Thomas Young studied the effects of diffraction and showed that light produced interference effects very much like (though on a vastly smaller scale) the effects with sound or water waves; and finally Foucault actually measured the velocity of light in water and showed that it was not greater but smaller than in air (in agreement with the wave theory [§641]). In every respect *as regards its propagation* light acts as a wave motion. (We have seen how Maxwell identified it as electromagnetic radiation.)

618. Huygens' Construction. For Newton a ray was the actual path of his corpuscles. For Huygens (and for the physicist today) it is simply a perpendicular to the wave front which shows the direction of propagation of the waves.

Huygens considered wave propagation as the continual rebirth of new waves from old ones. Each point in the wave front *a–z* is oscillating and must be thought of as itself a source sending *wavelets* (shown as circles) out in every direction. This is for any kind of waves — water waves, sound waves, or electromagnetic waves in space. But in general the "crests" and "troughs" in the various wavelets compensate each other and on the whole produce destructive interference. Only along the envelope (*a′–z′*), tangent to all wavelets, do the wavelets from neighboring points interfere constructively. This is the new wave front. Huygens' Principle is: *the new wave is found by taking the surface tangent to all wavelets.* (There is some straggling [diffraction] at the edges of the beam where the wavelet pattern is incomplete.)

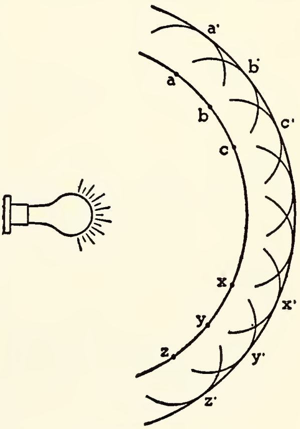

Fig. 2. Huygens' principle.

619. Wave Theory of Reflection. Let us consider the reflection of a parallel light beam from a plane mirror in terms of waves. A beam (rays *va*, *wb*, etc.) strikes a plane mirror and is reflected. *a–e* is a wave front of the approaching wave. First *a* strikes the mirror, later *b*, and finally *e*. Had the mirror not been present the wave front would now have been the dotted line *a″–e′*. But we must now construct wavelets of the wave as reflected from the mirror. By the time the incident wave has reached the mirror at e' the reflected wavelet from a has reached a'. The wavelet from b is intercepted; instead of reaching b'' it is turned back to b'. Similarly for other wavelets. The line a'–e', tangent to these wavelets, is the reflected wave front. Now aa' equals aa'' and the triangle $ae'a'$ equals $ae'a''$. Hence the angles $ae'a'$ and $ae'a''$ are equal. But these angles between the oncoming or reflected wave fronts and the surface are equal to the angles between the incident or reflected ray and the perpendicular (i and r). Therefore, the angle of incidence (i) equals the angle of reflection (r).

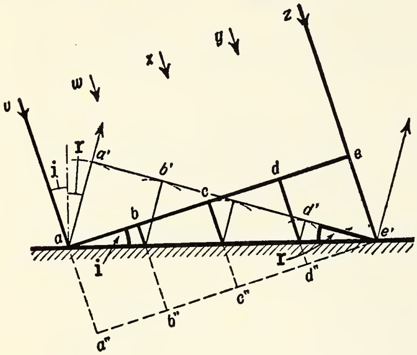

FIG. 3. Reflection from plane surface: $i = r$.

Observe that in general the angle of incidence — or reflection — can be considered as the angle between the wave front and the surface.

620. Concave Spherical Mirror. The student is familiar with concave mirrors. They are, for example, used as magnifying shaving mirrors. The reflecting surface is here a part — a very small part — of the surface of a sphere. The center of the sphere is called the *center of curvature*. The center of the mirror surface is called its *vertex*. The central line through the vertex and center of curvature (C) and extending to infinity is called the *principal axis*.

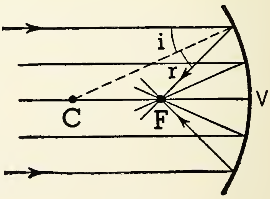

FIG. 4

The laws of reflection are illustrated by reflection from a concave mirror. Figure 4 shows the reflection of a beam which is parallel to the principal axis. A radial line is represented as passing from the center of curvature (C) to some point on the mirror (broken line). This is perpendicular to the surface. Each ray in the beam makes equal angles of incidence (i) and reflection (r); after reflection all pass through a common point which lies on the principal axis. This point (F) is called the *principal focus*.

This bundle of rays parallel to the principal axis is called a *principal bundle of rays.* For spherical mirrors which are small (constituting only a small part of a hemisphere) all the rays in a principal bundle are, after reflection, converged to the principal focus.

621. Diffuse Reflection. In snow there are countless numbers of small crystal faces; there are faces oriented at all angles. A beam of light striking such a surface is scattered in all directions. This is called diffuse reflection. Chalk, paper, any white surface has diffuse reflection. A black surface absorbs all light; a red surface absorbs other colors but reflects red diffusely.

Usually more or less regular reflection is present also. A table top, a page of this book, a new pair of shoes, or an old pair of trousers, the grass of the lawn where each blade reflects in a minute silvery spot the direct light of the sun — these are examples. (This is called glare.)

622. Refraction. When a beam of light enters the surface of water (or other transparent medium) it is bent toward the perpendicular to the surface. The angle (r) between this perpendicular to the surface and the refracted beam is called the *angle of refraction.* In Table 47 are listed the values of the angles of incidence and of refraction for light entering water. Looking at the results for small angles, it may appear that the angles of incidence and of refraction are proportional; the results for large angles show that this is not the true relation, but rather *the sine of the angle of incidence is proportional to the sine of the angle of refraction.* This law was discovered by Snell, contemporary of Galileo.

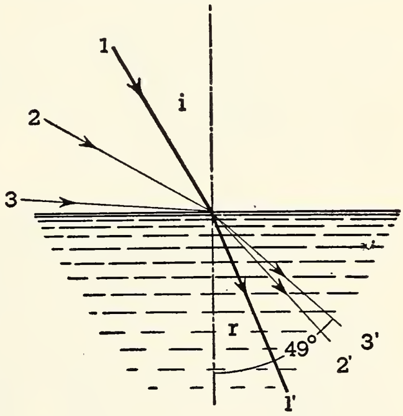

FIG. 5. Refraction: $\sin i \propto \sin r$.

$$\frac{\sin i}{\sin r} = n \text{ (a constant).} \quad \text{(Snell's Law.)} \qquad (3)$$

The constant of proportion is called the *index of refraction* (n).

TABLE 47

LAW OF REFRACTION

Angle of Incidence	Angle of Refraction (approx.)	sin i	sin r	$\frac{\sin i}{\sin r}$
10°	7.5°	.174	.131	1.33
20°	15°	.342	.256	1.33
40°	29°	.643	.482	1.33
60°	40°	.866	.650	1.33
80°	47°	.985	.738	1.33
90°	49°*	1.000	.750	1.33

* Critical angle.

Its value is $\frac{4}{3}$ (1.33) for light passing from air into water, $\frac{3}{2}$ from air into common glass. $\frac{4}{3}$ is referred to as the index of refraction of water relative to air. A substance with a large index of refraction relative to air is said to be "optically dense."

If the path of the light ray is reversed it will be found that the beam on leaving the water is bent *away from* the perpendicular. In fact the second column of the table may now be regarded as giving the angle of incidence (i') and the first column as giving the angle of refraction (r') as the light *leaves* the surface.[1] The index of refraction (now of air relative to water) is less than unity. Its value $\left(\frac{\sin i'}{\sin r'}\right)$ is $\frac{3}{4}$.

623. Passage of Beam through Plate with Parallel Sides. Figure 6 represents a light beam passing through a transparent plate. If the opposite sides of the plate are parallel the emergent ray will be parallel to the incident ray — the only effect of the glass is to jog the ray more or less to one side. This is apparent from what has just been said; for, since i' equals r, r' must equal i.

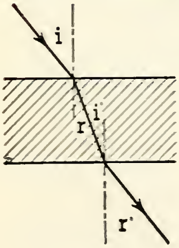

FIG. 6

624. Deviation of Light in Prism. The result is different if the sides of the plate are not parallel. Figure 7 represents the passage of light through a prism. In the figure the light is bent down toward the perpendicular as it enters the first surface, and again downward (now away from the perpendicular) as it leaves the second surface. A prism deviates the beam. (Observe that the bending is away from the edge, toward the base of the prism.) The deviation of the beam depends on the angle of the prism (the angle between the two refracting surfaces). For prisms of small angle, the deviation is approximately proportional to the prism angle.

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[1] We shall in general use primed letters when the passage of light is from a more dense to a less dense substance.

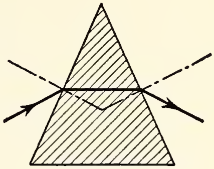

FIG. 7

625. Lenses. If a parallel beam of light falls on a lens it is brought to a focus. A lens may be regarded as a prism of continually varying angle. A light ray falling on the center (where the surfaces are parallel) passes through without deviation. The angle between the refracting surfaces increases in proportion to the distance from the center; hence light rays are deviated in this same proportion and hence (in the converging lens) are brought to a common focus.

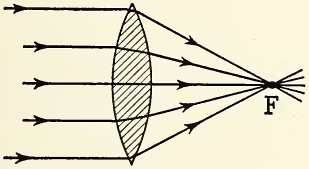

FIG. 8. Converging lens.

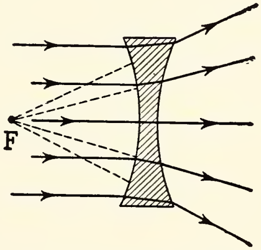

FIG. 9. Diverging lens.

The *principal axis* of a lens is a line through its center perpendicular to the faces. If the light beam is parallel to this principal axis (i.e., a principal bundle of rays) its focal point lies on the axis (*F* in Fig. 8). This point is called the *principal focus* of the lens.

The converging lens is thickest at the center, and the light is bent inward. A diverging lens is thickest at the edge. The continuously varying prisms now have their bases outward, and the light rays are bent outward. The beam is not brought to a focus, but the diverging rays proceed *as if* coming from a common point (*F*). Such a point is called a *virtual* (i.e., unreal) *focus*.

626. Parallel Beam from Lens or Mirror. Thus if the incident beam is parallel to the principal axis of a converging lens the rays are refracted so as to pass through the principal focus. On the other hand, the rays of light in Fig. 8 may be considered as being propagated in the opposite direction. If the source is placed at the principal focus the diverging rays are bent inward and emerge from the lens as a parallel beam.

Likewise, if the light source is placed at *F* in Fig. 4, the rays will be traced in the opposite direction and will be reflected as a parallel beam. Examples are the electric flashlight and the automobile headlight.

Excepting the brief reference in this concluding section only parallel light beams have been considered as falling upon our mirrors and lenses in this chapter. Actually in most instances the bundle of rays will be coming from a nearby source and therefore diverging. A more general discussion of mirrors and of refraction and of lenses is given in succeeding chapters.

CHAPTER 48

MIRRORS

627. Parallax. A single ray (AB) gives the direction of a source of light but not its position. Given two diverging rays, we can find their common intersection and hence the position of the light source. (It is for this reason that we were given two eyes [Fig. 3].) This is called *parallax.*

Examples of parallax: The surveyor uses parallax to measure the distance to a distant object (A); he first carefully measures a *base line* BC, and then, sighting with his theodolite, he measures the angle between the lines of sight AB and AC. . . The range finder, used in warfare, consists of two telescopes set at a fixed distance apart (the base line); it is an arrangement for determining the angle of parallax between the two points of view; it is calibrated to read the range directly. . . The astronomer can determine the distance to the sun by observing its position simultaneously from two different observatories and determining the parallax. He can also measure the distance to some of the nearer stars by observing their parallax as seen from opposite sides of the earth's orbit; in this case the base line is the diameter of the orbit (186,000,000 miles).[1]

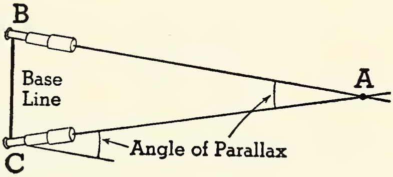

FIG. 1. Parallax.

In our study we shall always need *two* rays to locate the position of an image.

628. Images with a Plane Mirror. (1) *Ray Method.* When the light rays diverging from a point source S strike a plane mirror (Fig. 2a) they continue to diverge after reflection and appear to come from a point S' behind the mirror. This is the image; it is a *virtual* image (not a *real* image) because the rays never actually pass through this point.

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[1] This is not to be confused with the aberration of light (§613), which depends upon the velocity of the earth, not upon the orbit diameter.

It is easy to show (1) that the image is located upon a perpendicular dropped to the plane of the mirror and (2) that the image distance q (behind the mirror) equals the object distance p (in front of it). To locate the image we need two rays. Let us take one ray AB, reflected directly back from the mirror, and any other ray CD. The image lies at the intersection of these reflected rays. Since it lies on the extension of AB the first part of our proposition is proved at once. Now since the vertical angles DCN and $S'CM$ are equal (by geometry) and the angles of incidence and reflection, SCN and DCN, are equal (law of reflection) $\angle SCN = \angle S'CM$. Hence,

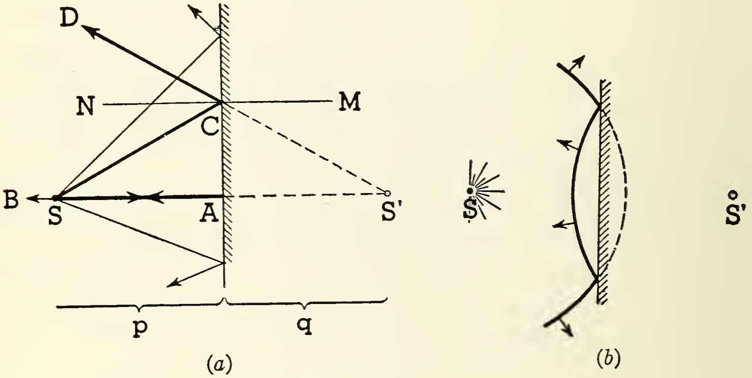

FIG. 2. Image in plane mirror.

$$\angle ACS = \angle ACS'.$$

The angles SAC and $S'AC$ are right angles; the side AC is common to the triangles SAC and $S'AC$; hence these triangles are equal and the corresponding sides SA ($= p$) and $S'A$ ($= q$) are equal.

We could have used any other rays in the proof; *all* reflected rays appear to come from point S'.[1]

(2) *Wave Method.* It is instructive to consider this image formation by the wave method. Think of a wave leaving the

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[1] When the position of a mirror on a wall is such that a perpendicular dropped from my eye to the plane of the mirror (the wall) does not strike the mirror, I cannot see my own image. However, my image, upon this perpendicular, is visible to anyone else who places his eye within the reflected beam.

point S (Fig. 2*b*) moving out in all directions as the surface of an expanding sphere. The figure represents the wave front shortly after it has reached the mirror. A part of the wave has been turned back; this reflected wave front is exactly the reverse of the dotted wave front (the wave as it would have been in the absence of the mirror). S', the apparent source of this wave (i.e., the virtual image), is therefore as far behind the mirror as S is in front.

For this plane mirror $p = q$. Therefore, assuming eq. (1) (p. 566) as applying: $O = I$; the image and object are of equal size. We have seen that *real* images (of real objects) are *inverted.* Here the image is virtual and erect. Virtual images (of real objects) are always erect.

629. Curved Mirrors. In Chapter 47 we saw how a parallel beam of light was converged by a concave mirror. The beam there used was parallel to the principal axis, and the point of convergence was called the *principal focus* of the mirror. The distance from the principal focus to the mirror is called the **focal length** of the mirror.

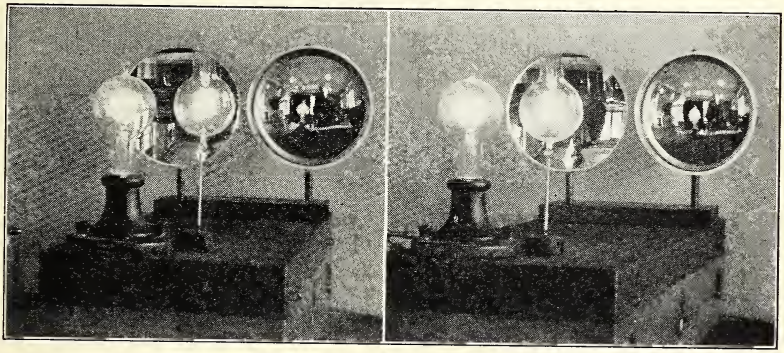
Left eye Right eye

FIG. 3. Concave and convex mirrors. (Stereoscopic views.) Images behind mirror move *with* the eye; before mirror *opposite to* the eye (parallax).

This concave spherical mirror is a small portion of a spherical surface (interior); the center of this sphere is the *center of curvature* of the mirror; its radius is the *radius of curvature.* The outer spherical surface forms a *convex* mirror; a convex mirror diverges (instead of converging) the light beam; the *center* of curvature is behind the mirror (radius of curvature negative). Occasionally mirrors are used with curvature in only one plane. They are small portions of a cylindrical surface. A cylindrical mirror converges a beam in one (say vertical) plane and leaves it unchanged in the perpendicular (horizontal) plane.

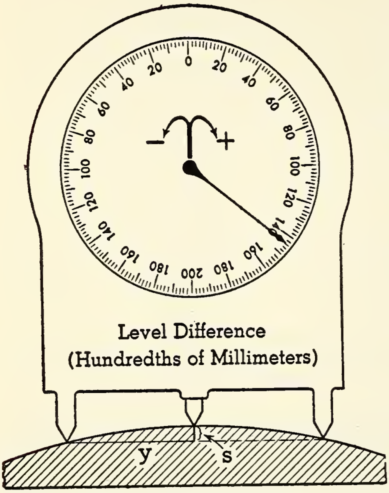

FIG. 4. Spherometer.

630. The Spherometer. The curvature of mirrors can be determined most accurately by optical means. But a very convenient method is by direct measurement, using the spherometer. Figure 4 shows the type of spherometer used by opticians. The middle of the three legs is movable and is attached to a pointer and scale. When the three legs rest on the spherical surface the displacement of the middle leg above or below the others is the sagitta of an arc, and the distance between the two outer legs is the chord ($2y$). Hence by the sagitta equation (§720):

$$r = \frac{y^2}{2s}.$$

In a spherical mirror the curvature is the same measured along any direction in the surface. In a cylindrical mirror the radius, measured in one direction, is the radius of the cylinder; along the perpendicular direction the sagitta is zero and the radius infinite.

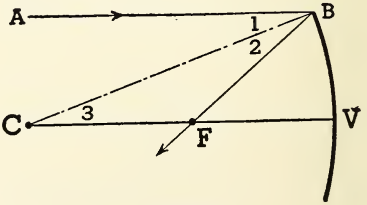

FIG. 5

631. Focal Length and Radius of Curvature. It is easily shown that the focal length of a spherical mirror is half the radius of curvature. In Fig. 5, C is the center of curvature, F the principal focus of the mirror. CB is perpendicular to the mirror. Consider the ray ABF; by the laws of reflection, its angle of incidence (1) equals its angle of reflection (2). But angle (1) equals angle (3) (opposite interior angles with parallel sides). Hence (2) equals (3), and the triangle CBF is isosceles. Therefore the length CF equals BF.

So far the proof is rigorous. Now if the mirror is small[1] BF is approximately equal to VF (the focal length) and hence the principal focus is located — rather exactly for small mirrors — midway between the mirror surface and the center of curvature.

632. Image of an Object at Infinity. If an object is a great distance from the mirror, the beam of rays which reaches the mirror from any given point in the object is parallel. Light from a distant point on the principal axis forms a beam parallel to this axis (principal bundle) and is converged to the principal focus. From any other point in the distant object the beam is itself parallel but is oblique to the principal axis; such a bundle of rays is brought to a focus above, below, or to one side of the principal focus in the focal plane of the mirror. The **focal plane** is the plane through the principal focus perpendicular to the principal axis. From each individual point in the distant object light is focused to a single point in this plane and a real image is formed in this plane.

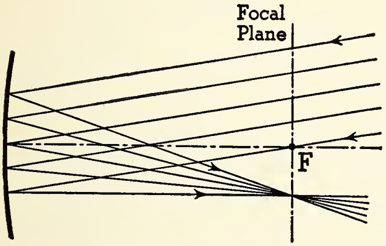

FIG. 6. Focal plane.

633. Conjugate Foci. Let us now consider the images of less distant objects. As an object is brought nearer, the light rays strike the mirror with greater and greater divergence and are converged by the (concave) mirror with more and more difficulty and the focus recedes farther and farther from the mirror. When the object reaches the principal focus, the reflected beam is no longer convergent but parallel. With an object closer than this, the reflected rays remain divergent, and they have no real focus; they have a virtual focus (as in a plane mirror) behind the mirror.

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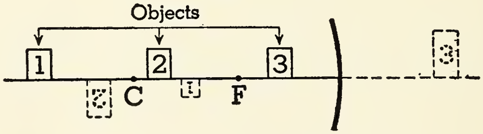
[1] For our proof to hold, the essential requirement is that the mirror be only *a small part of a sphere,* making the angle BCV small. The angle which the mirror makes at the center (twice BCV) is called the *angular aperture* of the mirror. For spherical mirrors of large angular aperture the various rays parallel to the principal axis do not all come together at a single point and the mirror has no exactly defined principal focus. (See §638.)

The two related points, the point in the object from which a beam diverges and the point in the image at which it converges, are called conjugate foci.[1] A point at infinity has its conjugate focus in the focal plane; the center of curvature is conjugate to itself; points closer than the principal focus are conjugate to virtual focal points behind the mirror. Table 48 shows the relative positions of object and image.

TABLE 48

CONJUGATE POINTS

Concave Mirror



Object between	$\infty - C$	$C - F$	F − mirror
Image between	$F - C$	$C - \infty$	Behind mirror (virtual)
Magnification:	Image reduced	Image enlarged	Image enlarged

Convex Mirror

For all objects a virtual image is formed behind the mirror and within the (virtual) principal focus. Image reduced.

634. Graphical Construction of Image. A formula will be derived by which the conjugate positions of object and image can be determined exactly, but it is instructive to locate the position of the image graphically. The mirror with its principal axis is drawn, and the center of curvature and principal focus are indicated; then the two rays required to locate the image of some point in the object are drawn. It is convenient to select two rays from the following:

(1) The ray parallel to the axis; this is reflected through the principal focus.

(2) The ray through the principal focus; this is reflected parallel to the axis.

(3) The ray through the center of curvature; this is reflected back on itself.

(4) The ray to the vertex; this is reflected at equal angle with the axis.

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[1] Conjugate means related (Latin: wedded); focus means converging point.

All four of these rays coming from the tip of the object are shown in Fig. 7*a*. The construction here is for an object lying beyond the principal focus. The intersection of the rays after reflection gives the corresponding point of the image. Rays from any other point on the object can be treated similarly, or, usually, the position of one point in the image having been located, the rest of it is simply drawn in. (The student can trace out the rays for the tail of the arrow.) The image in this case is inverted.

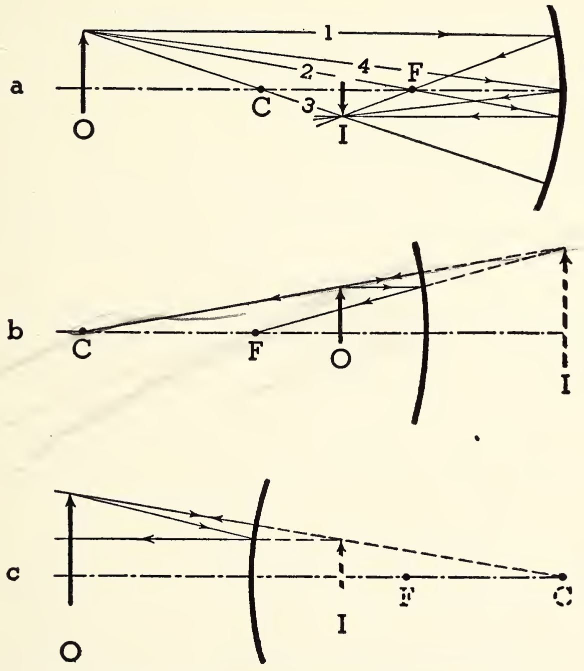


Fig. 7. Image formation in mirrors.

When the object lies within the principal focus (Fig. 7*b*) the reflected rays diverge. We extend them to get their virtual intersection. (We have chosen to use here rays 1 and 3, the incident ray 3 being on the line *from* the center of curvature but not actually passing through it because the object is nearer the mirror.)

Figure 7*c* shows the construction for the convex mirror. We have used rays 2 and 3. (The student can draw in the others.)

The rays in this case do not of course pass through the center of curvature and principal focus (which are behind the mirror) but lie along the lines passing through these points. The image then is virtual and erect.

635. Fundamental Equation of the Mirror. In Fig. 8 we find two pairs of similar right triangles: $\triangle AVB$ is similar to $\triangle EVD$ (because the two angles with the principal axis are equal), and $\triangle ACB$ is similar to $\triangle ECD$ (vertical angles are equal). Since corresponding sides are proportional we have from the first pair:

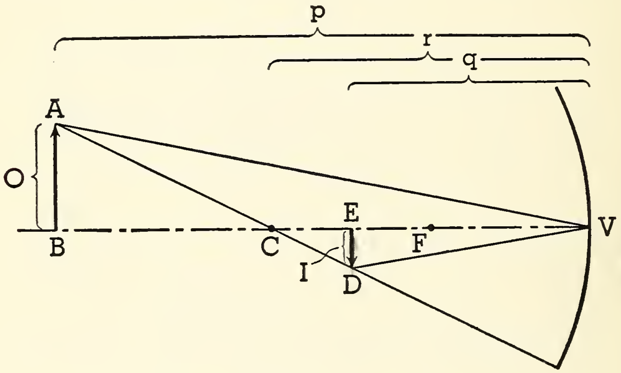


FIG. 8. $1/p + 1/q = 2/r$.

$$\frac{O}{I} = \frac{p}{q}. \qquad (a)$$

The sizes of object and image are proportional to their distances. (This is an example of the general law. See Table 48.)

Now in the second pair of triangles $BC = p - r$ and $CE = r - q$. Hence since these triangles are similar we have

$$\frac{O}{I} = \frac{p - r}{r - q}. \qquad (b)$$

From (*a*) and (*b*)

$$\frac{p}{q} = \frac{p - r}{r - q}$$

which may be written in the form[1]

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r} \tag{4}$$

or

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}. \tag{4'}$$

where f is the focal length $r/2$.

636. Signs in the Mirror Formula. This is the general formula for the mirror and applies to virtual images as well as real, to the convex mirror as well as concave. But in applying it, it is necessary to distinguish between positive and negative values of q, r, and f. In the derivation all distances in front of the mirror have been regarded as positive. Hence p will always be positive; q will be positive for real images but negative for virtual ones. The center of curvature and principal focus both lie behind the mirror if it is convex; hence for convex mirrors r and f must be taken as negative.

Example. A man holds a convex mirror 10 in. from his face. If the radius of curvature of the mirror is 20 in. describe the image.

Here $p = 10$ in., $f = -10$ in. Hence we have

$$\frac{1}{10} + \frac{1}{q} = -\frac{1}{10},$$

or

$$\frac{1}{q} = -\frac{1}{10} - \frac{1}{10} = -\frac{1}{5}, \quad \text{and} \quad q = -5 \text{ in.}$$

Hence the image is 5 in. behind the mirror. Being virtual the image is erect. Since it is half as far from the mirror, the image is half as large as the object.

637. Shaping Spherical Mirrors. If two pieces of glass are ground one on the other, with a combined back-and-forth and rotary motion, the two surfaces become slightly spherical, the one concave and the other convex. (This is because only spherical surfaces can remain in contact for both displacement and rotation.) In this way large mirrors of small curvature can be shaped. The mirror is first ground into shape in this manner with coarse abrasive (Carborundum powder); then, continuing the combined back-and-forth and rotary motion, the scratches are ground out with finer and finer abrasives and the surface finally polished with rouge. The glass surface is then tested and if satisfactory is silvered. With a little patience the student can make the 6-in. mirror of a highly respectable telescope.[2]

---

[1] Clearing of fractions gives $qr + pr = 2\,pq$; dividing both sides by pqr gives eq. (4).

[2] The materials required for such a mirror can be obtained, together with instructions, for about $5.00. See advertisements in the *Scientific American.*

Grinding machines may be used. Substantially in this way the largest telescope mirrors are ground. Lens surfaces, for example in eyeglasses, are ground by machine in somewhat the same way, but against specially shaped iron grinding forms.

638. Defects of Mirrors. Although only two rays need be used to determine graphically the position of the image of a given point on the object, it is assumed, of course, that all other rays coming to the mirror from this point will converge here. They must do so if there is to be a perfect image. As a matter of fact, they never do this perfectly, even when the surface is perfectly spherical. For this there are three principal reasons:

(1) *Spherical aberration.* When a light beam parallel to the principal axis falls on a large spherical mirror, only the rays near the center converge exactly at the principal focus. In our derivation of the focal length (§631) an approximation was made which can be justified only for a small mirror (with diameter small in comparison with the radius of curvature). For a mirror of large " angular aperture " (i.e., forming a considerable portion of a hemisphere) the reflected rays form a cusp as shown in Fig. 9 (left). (The reader has seen this reflection pattern, the " caustic curve," when light is reflected on the surface of a glass of milk.) The spherical mirror, though easy to make, has not exactly the proper form to bring a parallel beam to exact focus. The defect is called *spherical aberration.* It becomes important for mirrors of large aperture; it is eliminated by making the mirror in the form of a paraboloid instead of a sphere. The parabolic mirror eliminates spherical aberration when the object is at infinity

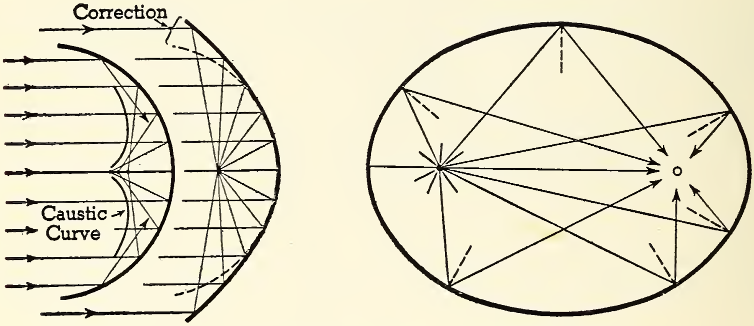


FIG. 9. Spherical aberration and its elimination.

(incident beam parallel) or for the converse case when the object is at the principal focus (reflected beam parallel). The mirror of the automobile headlight is parabolic, and the lamp filament is placed at the principal focus. . . But no single mirror can eliminate this aberration for objects at all distances. (For an object at the center of curvature the sphere itself is the proper shape — each ray strikes the mirror perpendicularly and is reflected back.) In general, the mirror should be ellipsoidal; when the object is at one focus of the ellipse, the image is at the other. (A circle is an ellipse with coincident foci; a parabola is an ellipse with one focus at infinity.)

(2) *Astigmatism for oblique rays.* We have said that a parallel bundle of rays oblique to the principal axis is focused in the focal plane. But, even for mirrors corrected for spherical aberration, this is not exactly true. Indeed, when the angle of obliquity is large the rays are not brought to a point focus. The defect is called astigmatism for oblique rays.[1]

(3) The largest astronomical telescopes have concave mirrors. The mirrors are parabolic, and the star image always falls very close to the principal axis. Therefore spherical aberration and astigmatism are eliminated. But even here perfect point images cannot be obtained, and the reason for this lies in the nature of light itself. Light is a wave motion; it does not actually travel in straight rays, and there is always a certain amount of *diffraction.* This diffraction is most important for narrow beams. This is one of the reasons why astronomers seek larger and larger telescopes. The enormous 200-inch telescope at present (1940) under construction has a diameter of some 10,000,000 light waves!

QUESTIONS

1. One often misjudges the distance of a horizontal wire, but not a vertical one. Why?

2. A man can see his whole figure in a 3-ft. plane mirror placed at the proper height on the wall. Explain.

3. A plane mirror is placed in the center of the wall of a room. If *A* stands in a further corner of the room, where must *B* stand to see *A*'s image? Where will the image be?

---

[1] Stigma (Greek) means spot; *a*-stigmatic means "not spotted." In optics it refers to focusing of light not in a focal "spot" but in a pair of focal lines. (Observe these line foci when a point source of light is placed some distance to one side of the principal axis.) Astigmatism is also caused by distorted lenses or mirrors (see Fig. 50–7).

4. A silvered ball is often used as a garden ornament. Where does it form the image of the sun?

5. Can a concave or convex mirror be used as a " burning glass "?

6. In the " solar stove " a large semicylindrical mirror focuses the sun's rays upon a long water pipe. Where should this pipe be placed? Is it necessary to rotate the mirror? Are all the rays focused onto the pipe?

7. Where is the image in a concave mirror when an object is slightly beyond the center of curvature? slightly closer than the principal focus? a rather long distance away?

8. Compare the positions of images of lamp bulb and man in the concave mirror of Fig. 3.

9. For objects located closer than the center of curvature of a concave mirror the image: (*a*) is always virtual; (*b*) is always real; (*c*) may be either.

10. In a convex mirror the image always lies between: (*a*) the principal focus and the mirror; (*b*) the center of curvature and the principal focus; (*c*) the center of curvature and infinity.

Vocabulary: Parallax, virtual (real) image, principal axis (bundle, focus), focal length, center of curvature, spherometer, conjugate foci, spherical aberration, astigmatism (for oblique rays), diffraction.

PROBLEMS

1. A plane mirror is placed 3 ft. behind a 50-candlepower lamp, in a black-walled room. What is the intensity of illumination 6 ft. in front of the lamp?

2. Two opposite walls of a 12-ft. room are mirrors. George faces one mirror and Henry the other, each 3 ft. away. How far away will George see the image of Henry's face? Will he see several images?

3. Two adjacent walls of a room are mirrors. If a man stands in the center of the room, where will his image in one of these mirrors be? The second mirror will form an image of this image. Where will this be? Draw figure.

4. The radius of curvature of a concave mirror is 3 in. An object is 6 in. from the mirror. Draw the figure to scale, and find the position of the image.

5. Find the position of the image in problem 4 from formula.

6. Compute the position of the image of an object 1 in. from the mirror of problem 4. Will it be erect or inverted? Find the magnification. Make the graphical construction.

7. A small lamp is placed 1 yd. from a 9-in. (focal-length) concave mirror, 6 in. above the principal axis. Where will its image fall?

8. A concave shaving mirror has a radius of curvature of 30 in. Describe the image as to position and size when one's face is 10 in. in front of the mirror.

9. A man holding a 10-in. (focal-length) mirror 15 in. from his face cannot see his image clearly. Explain. (Find position of image.)

10. Two concave mirrors are placed some distance apart facing each other. A small source is placed at the principal focus of the first. The light goes to this mirror, is reflected to the second, and then focused. Where will this focus be? Draw figure.

11. The radius of curvature of a convex mirror is 4 in.; an object is 4 in. away. Locate the image (*a*) by formula; (*b*) by graphical construction.

12. Find the image of an object which is 40 cm from a convex mirror, radius of curvature 10 cm.

13. Where is my image when I stand 10 ft. from a 10-in. silvered globe? How tall is the image if I am 6 ft. tall?

14. Show that the principal focus and center of curvature of a plane mirror are at infinity. Use eq. (4) to show that the image is behind the mirror the same distance that the object is in front.

15. The normal eye cannot be focused upon an object closer than 10 in. (*a*) How close can I use a concave shaving mirror of focal length 10 in.? (*Hint:* $p - q = 10$.)[1]

16. Where must a concave mirror with a 3-ft. radius of curvature be placed to produce an image twice the size of the object?

[1] Involves quadratic equation. Interpret both solutions.

CHAPTER 49

REFRACTION

639. Relative and Absolute Index of Refraction. When light passes from one medium into another, as for example from water into glass, the angle of incidence (i) and the angle of refraction (r) are related thus:

$$\frac{\sin i}{\sin r} = \text{a constant } (n). \qquad (3)$$

This is Snell's Law. The constant depends upon the nature of the two media and is called the index of refraction of the second relative to the first. For example, the index of refraction of common glass is about 1.5 relative to air (or vacuum), 1.125 relative to water, very nearly unity relative to Canada balsam. The light beam is refracted toward the perpendicular in passing from air into glass; it is refracted less in passing from water into glass; it passes from Canada balsam into glass almost without deviation. The index of refraction of common glass relative to carbon bisulfide is less than unity; passing into glass from carbon bisulfide a ray is bent *away from* the perpendicular (i [and $\sin i$] less than r [and $\sin r$]).

In the majority of instances we are interested in the passage of light into the substance from *air* (or the reverse). It is usually not necessary to distinguish between the refracting properties of air and of empty space. The index of refraction of a substance relative to a vacuum (or practically to air) is called its **absolute index of refraction.** Absolute indices of refraction for several substances are given in Table 49.

TABLE 49

ABSOLUTE REFRACTIVE INDEX (FOR YELLOW LIGHT)

Air	1.0003	Crown glass	1.50±
Water	1.33	Flint glass	1.65±
Canada balsam	1.53	Calcite (ordinary)*	1.66
Carbon bisulfide	1.63	Calcite (extraordinary)*	1.48

* Ordinary and extraordinary rays; see Chapter 54.

For all ordinary transparent substances the absolute indices of refraction are greater than unity. Conversely, index of refraction of vacuum relative to a material medium is less than unity; the ray emerging from a material body is bent *away from* the perpendicular.

The relative indices of refraction can be obtained from these absolute indices. It can be shown that the index of refraction of a substance *A* relative to some other substance *B* is equal to the absolute index of *A* divided by that of *B*. (This was shown experimentally by Snell, and we shall presently show that it is a consequence of the wave theory.) For example, the absolute index of common glass is $\frac{3}{2}$, of water $\frac{4}{3}$; for light passing from water into glass the (relative) index of refraction is $\frac{3}{2} \div \frac{4}{3} = \frac{9}{8}$ (or 1.125 as given above).

Examples. 1. Find the angle of refraction for a light beam striking a smooth water surface at grazing incidence.

$$n = 1.33,\ i = 90°;\quad \sin i = 1$$

$$\sin r = \frac{\sin i}{n} = \frac{1}{1.33} = 0.75,$$

and, from Table 52 (p. 673), $r = 49°$.

Conversely a ray projected outward with an angle of incidence (in the water) of 49° emerges from the water at a grazing angle.

2. Suppose that Fig. 47–7 represents a crown-glass prism immersed in water. If the ray passing out through the right face has an angle of incidence (i') of 30° find the angle of emergence (the angle of refraction r' in the water).

Refractive index of water relative to glass: $n' = \frac{1.33}{1.50} = 0.89.$

$$i = 30°;\quad \sin i' = 0.50.$$

Therefore

$$\sin r' = \frac{\sin i'}{n'} = \frac{0.50}{0.89} = 0.56,$$

and, from Table 52,

$$r' = 34°.$$

640. Examples of Refraction. A drawing-room stunt is to place a coin at the bottom of a glass just hidden from sight by the rim; how can the coin be seen without moving the eye? We fill the glass with water and the line of sight is bent over the edge (as *ABC* in Fig. 1). . . A crooked stick of the shape *ABC* would appear straight (we could sight along it). On the other hand, a straight stick thrust obliquely into the water appears bent upward. . . A submerged object is always below where it appears to be; a boy spearing fish learns to aim below where the fish appears to be. As light rays from a submerged object leave the water the angle of parallax is increased in the ratio 4 : 3. (For small angles the sine is proportional to the angle.) Hence we underestimate the depth of a pool in the ratio of 3 : 4.

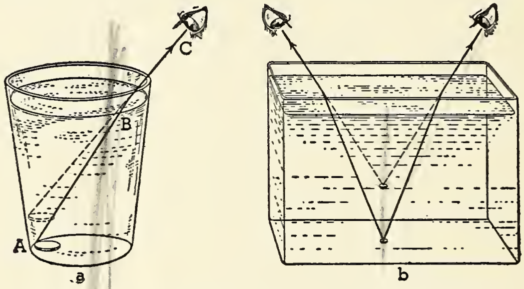


FIG. 1. Refraction; apparent depth decreased.

When a beam of light strikes the water at grazing incidence, $i = 90°$, $\sin i$ is unity; therefore, as in the preceding example, $\sin r_{\text{max.}} = 1/n = 3/4$, and the maximum angle for a ray entering water is 49° (Table 52). A " fish-eye view " of the world above the surface of the water is included in a cone making 49° with the vertical — the fish sees the sun rise 49° to the east and set 49° to the west. . . To a much smaller extent our own atmosphere displaces the position of bodies in the sky; the sun is really already below the horizon when we see it begin to set, the light rays being bent into view by refraction as they enter the atmosphere.

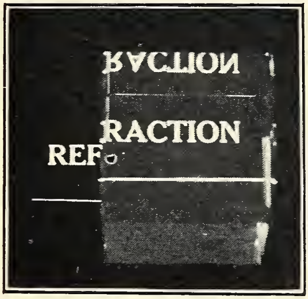


FIG. 2. Refraction in glass cube. (Total reflection from further face.)

Whenever light is refracted at a boundary some of it is also reflected (*partial reflection*). This reflection can be seen, rather faintly, from the inner two oblique rays of Fig. 4. It is this reflection of light, together with the refraction, which enables us to detect transparent objects. One can dilute carbon bisulfide with another liquid until the refractive index is practically equal to that of glass (relative index unity). A piece of glass immersed in this liquid is very nearly invisible. (If the indices could be exactly matched for all colors it would be completely invisible.) Canada balsam has nearly the same index as glass; hence this gum is used to cement together the parts of a compound lens; this reduces the reflection at the junction.

641. Refraction and the Velocity of Propagation. The speed of light is less in glass or water than in free space.

Light is electromagnetic radiation, an "oscillating tendency for electric charges to move" (§554). When this light wave passes from free space into matter the electrons bound in the atoms are set into synchronous vibration and the speed of light is decreased, its wave-length shortened. We can compare this (though the analogy is not perfect) to a wave passing along a string coming suddenly to a part of the string loaded with beads. This velocity change accounts for the refraction of the light beam.

Figure 3 represents light passing from free space into an optically denser medium (such as glass). *AB* is the wave front of the oncoming wave, angle *i* [or (*i*)] the angle of incidence. The light wave is retarded in the glass. Hence while the Huygens wavelet travels from *B* to *D* in the vacuum it travels the smaller distance *AC* in the glass. Wavelets from other points in the wave front travel intermediate distances. Then *CD* is the wave front of the refracted ray. *AC* is the direction of the ray; the ray is bent toward the normal.

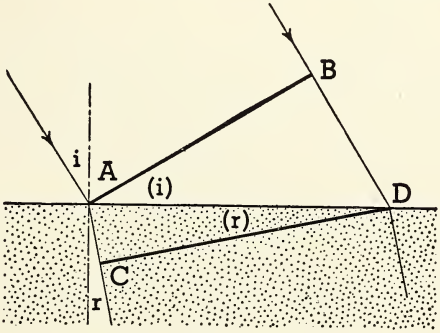


Fig. 3. Index of refraction.

We can now derive Snell's Law of refraction and give a somewhat more concrete meaning to the index of refraction (n).

In Fig. 3 the angle (r) equals r and hence is the angle of refraction. $\operatorname{Sin} i = \dfrac{BD}{AD}$; $\sin r = \dfrac{AC}{AD}$; hence,

$$\frac{\sin i}{\sin r} = \frac{\dfrac{BD}{AD}}{\dfrac{AC}{AD}} = \frac{BD}{AC}.$$

But, since the light travels the distances BD and AC in the same time, BD and AC are proportional to the speeds (v) of light in vacuum and in glass:

$$\frac{BD}{AC} = \frac{v_{\text{vacuum}}}{v_{\text{glass}}}.$$

This ratio is evidently a constant. Hence we have derived Snell's Law:

$$\frac{\sin i}{\sin r} = \text{constant } (n).$$

The constant is here the absolute index of refraction. *Thus we interpret the absolute index of refraction of any substance as the ratio of the speed of light in vacuum to its speed in the medium.*

In 1851 Foucault measured the speed of light in water and found it to be $\frac{3}{4}$ as great as in free space. (The index of refraction of water is 1.33 or $\frac{4}{3}$.) In glass, with index of refraction 1.5, the speed of light is $\frac{2}{3}$ as great as in free space.

If the first medium in Fig. 3 is a material substance (as, for example, water) instead of a vacuum this proof proceeds in the same way. It is thus shown that the *relative index of refraction* of two substances is the ratio of the light velocities in them. Now let n_w and n_g represent the absolute indices of water and glass and let $n_{w \rightarrow g}$ represent the index of glass relative to water; let v_w, v_g, and v_v represent the speed of light in water, glass, and vacuum. Then

$$n_{w \rightarrow g} = \frac{v_w}{v_g} = \frac{v_v/v_g}{v_v/v_w} = \frac{n_g}{n_w}.$$

which is our rule for finding the relative index of refraction.

642. Total Reflection. We have shown in a numerical example that light entering water at grazing incidence has an angle of refraction of 49°. Consider rays projected outward from the surface of water. The angle of refraction (r') in the air is now greater than the angle of incidence (i') in the water.

When i' is 49° the ray emerges at grazing angle ($r' = 90°$, $\sin r' = 1$). What will become of a ray striking upward at the surface at a greater angle than this? Since the sine of an angle cannot exceed unity there can be no emergent ray; the ray is *totally reflected.*

When light is directed from an optically denser to a less dense medium total reflection always occurs when the angle of incidence (i') exceeds a certain value, called the *critical angle.* The critical angle is thus the greatest angle of incidence (i') in the denser medium corresponding to an angle of refraction (r') of 90° in the other medium. ($\sin r' = 1$.) Suppose that the second medium is empty space (or air). Then we have by Snell's Law:

$$\frac{\sin i'_{\text{critical}}}{1} = n',$$

Where n' represents the index of refraction *of empty space relative to the denser medium.* It is better, however, to remember the relation as

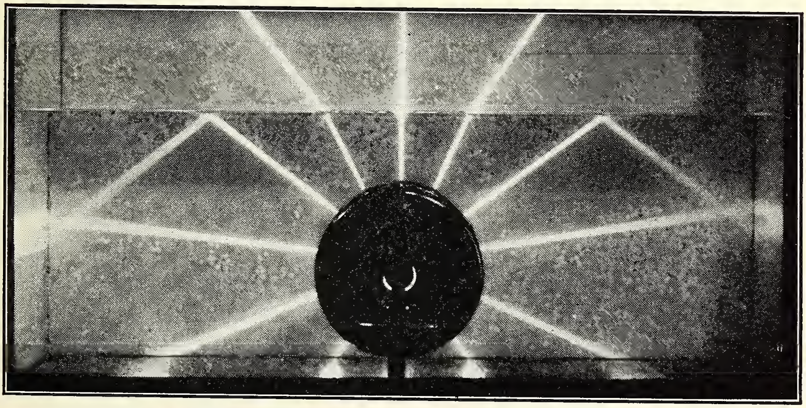


FIG. 4. Refraction and total reflection in tank of water.

$$\sin i'_{\text{critical}} = \frac{1}{n},$$

where n is the absolute index of refraction (the index of the medium relative to vacuum). (The same relation holds at a boundary between two material media with n now the refractive index of the denser substance relative to the other.)

For crown glass $n_{\text{abs}} = \frac{3}{2}$ and the critical angle (against air) is 42° ($\sin 42^\circ = \frac{2}{3}$, approximately). We have previously shown that the critical angle of water is 49° (§639).

643. Examples of Total Reflection. A silver mirror reflects only about 90 per cent of the light. Naturally a perfect reflector finds wide application in optical instruments. Usually this takes the form of a " 45° prism." Rays entering the short face of the prism perpendicularly strike the back face at 45° (greater than the critical angle of glass) and are totally reflected. . . An interesting demonstration of total reflection is the "piping" of light through a glass or quartz rod. (Quartz is preferable to glass because of its negligible absorption.) The light entering one end and striking the sides of the rod at greater than the critical angle cannot emerge. It passes through the rod, even around slight curves, and out the further end. . . The mirage seen on the desert and frequently on a hot road is due to total reflection as light from the horizon strikes at grazing incidence the hot stratum of air just above the ground.

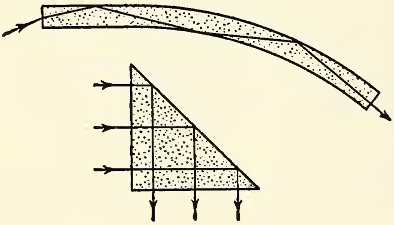


FIG. 5. Total reflection in bent rod and prism.

644. Reflection, Transmission, Absorption. When light falls upon a surface it may be reflected (as from silver or chalk), transmitted (as through glass or water) or absorbed (as by a black surface). Except for the case mentioned above there is never complete reflection. By metals, light is usually largely reflected, partially absorbed; it is partially transmitted through very thin metallic films. (Gold leaf, for example, transmits a little green light.) From a silver mirror 90 per cent of the light falling perpendicularly on the surface is reflected; the rest penetrates the surface and is absorbed.

When light falls perpendicularly upon a transparent medium like glass or water, most of it passes into the medium but about 5 per cent is reflected. (The reflection is the same whether the light passes from lighter to denser medium, or vice versa.) As the angle of incidence increases a larger proportion of the light is reflected. But only for a beam striking a surface of smaller refracting index at greater than the critical angle is there total reflection.

645. Dispersion. Ordinary white light is a mixture of all the spectrum colors. We have shown that refraction is due to the change in speed of light. But when white light enters a dense medium the retardation (i.e., the index of refraction) is not quite the same for the various component colors. (For example, for crown glass the index varies from 1.50 for red light to 1.51 for violet.) In consequence, the different colors, red, yellow, green, blue, indigo, violet, are deviated differently by a prism, red the least, violet the most. This breaking of white light into components is called *dispersion.* Dispersion always accompanies refraction, and Newton supposed that the two were strictly proportional. This is not so; for example, flint glass, though only 30 per cent more refractive than crown glass, disperses the light nearly twice as much. In this fact lies the secret of the achromatic lens (§652).

QUESTIONS

1. When is light reflected in transparent media? When is the reflection complete?

2. A crown-glass prism is placed in carbon bisulfide; show that a ray is deviated *toward* the edge of the prism. Describe a lens which, placed in this liquid, will be converging.

3. A cubical glass vessel is used as an aquarium. Can a light placed at one side of the vessel (outside) be seen through the front face?

4. Grasp a glass of water, not too tightly, with dry fingers. Looking down at the water surface, the lower fingers, below the water level, cannot be seen through the glass. They can be seen if they are wet and make intimate contact with the glass. Explain.

5. Explain why there cannot be total reflection from either face of a window pane.

6. " A fish sees the setting sun at an angle of 49°." Will this be changed if the water surface is covered by a glass plate?

7. Why is the surface of ground glass white? It becomes fairly transparent when wet; explain.

8. Explain why a grease spot decreases the reflection and increases the transmission of a piece of paper.

9. The index of refraction of water relative to air is equal to: (*a*) light velocity in water divided by that in air; (*b*) the absolute index of water divided by that of air; (*c*) the sine of the critical angle of water.

10. A light beam passing into the front face of an aquarium at any angle cannot be totally reflected at the back because: (*a*) the index of refraction of

water is less than that of glass; (*b*) the angle of emergence (r') equals the angle of incidence (i); (*c*) the critical angle is too great.

Vocabulary: Absolute (relative) index of refraction, total reflection, dispersion.

PROBLEMS

1. What is the speed of light in water, crown glass, flint glass?

2. In Fig. **47–5**, the angle of incidence for beam 1 is 30°; what is the angle of refraction?

3. A light beam strikes a flint glass plate at an angle of 45°; through what angle is it deviated as it enters the glass?

4. Make a table for crown glass like Table 47 (p. 578).

10 **5.** A crown-glass prism has a refracting angle of 30°. Find the deviation of a ray which enters the prism perpendicular to the face.

6. A beam of light passes from air through a glass plate into water. The angle of incidence as the beam enters the plate is 60°. Find the direction of the beam in the glass and in the water. (For common glass, $n = 1.5$.)

7. Find the critical angle of flint glass.

8. A ray enters perpendicularly one face of a 60° (equilateral) crown-glass prism. From which face does it emerge? Find the angle of deviation of the beam.

CHAPTER 50

LENSES

646. Lens Types. Several different types of simple lenses are shown in Fig. 1. Converging lenses (thickest in the center) may be double-convex, plano-convex, or concavo-convex; diverging lenses (thinnest in the center) may be double-concave, plano-concave or concavo-convex. In the converging lens a beam of parallel light is brought to a point focus (Fig. 47–8); in the diverging lens the beam is diverged and leaves as if coming from a center behind the lens (Fig. 47–9).

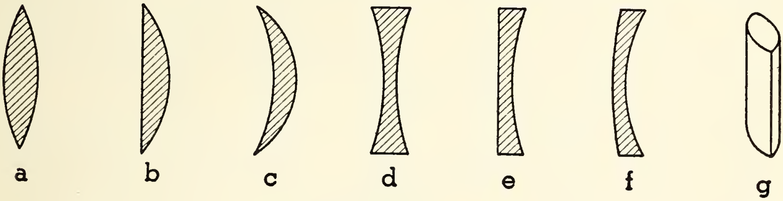


FIG. 1. Types of lenses.

In these ordinary lenses the faces are spherical in shape. In the cylindrical lens (*g*) the light beam is focused only in one plane and gives a line (rather than a point) image.

647. Conjugate Foci. The focal plane is a plane through the principal focus perpendicular to the principal axis. We have seen that after passing through a converging lens a principal bundle of rays (parallel to the principal axis) converges to the principal focus; other parallel beams (oblique to the principal axis) are focused in the focal plane. Hence the image of a distant object lies in this plane.

If the object now approaches the lens the image recedes, just as with the mirror. The difference is that for the converging lens the real image is formed on the opposite side from the object. We have the same general object-image formula:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$

but in using this formula we must adopt a different rule of signs.

648. Graphical Construction. Let us first construct the image graphically. We need two rays; we chose (Fig. 2):

(1) A ray parallel to the axis which is deviated to pass through the principal focus F.

(2) A ray passing through the center of the lens which is undeviated.

Another convenient ray (shown dotted) is

(3) A ray through F' which leaves the lens parallel to the axis.

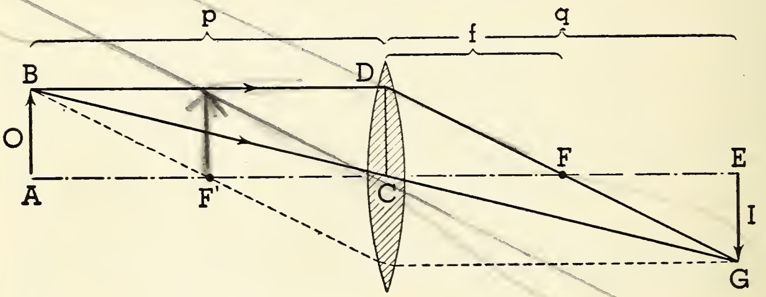
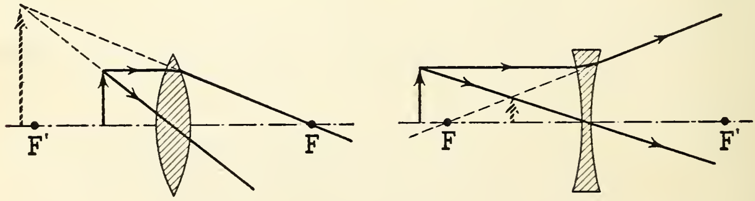


FIG. 2. Graphical construction of image.

A lens has two principal foci: F, the image focus for a principal bundle of rays passing through the lens from left to right; and F', for similar rays passing from right to left. On the other hand, F' is the *object* focus for rays passing to the right; that is to say, rays from F' leave the lens parallel to the principal axis. For thin lenses (which are the only ones which we shall discuss) F and F' are equally distant from the lens (the focal length).

Figure 3*a* shows the construction for an object closer than the principal focus. The emergent rays diverge and have a virtual focus behind the lens (i.e., on the object side). An object closer than the principal focus has a virtual image.



(*a*) (*b*)

FIG. 3. Virtual images: converging and diverging lenses.

Figure 3*b* shows the construction for a diverging lens. The principal focus (F) is virtual, and for any real object the diverging lens always gives a virtual image.

Observe: Virtual images are erect; real images inverted.

649. The General Lens Formula. In Fig. 2 there are two pairs of similar triangles: $\triangle ABC$ and $\triangle EGC$; $\triangle CDF$ and $\triangle EGF$.

From the first pair we have:

$$\frac{O}{I} = \frac{p}{q}. \tag{a}$$

This is the general formula for object-image size. Since $DC = O$, we have from the second pair

$$\frac{O}{I} = \frac{f}{q - f}. \tag{b}$$

Equating (a) and (b),

$$\frac{p}{q} = \frac{f}{q - f},$$

which can be rearranged to give

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}. \tag{5}$$

Rule of signs. Since the rays of light *pass through* a lens, a real image lies on the opposite side of the lens from the object. Hence, in eq. (5) f and q are considered as positive *when they are measured away from the object.* f is negative for diverging lenses (3b); q is negative for virtual images (3a and 3b).

Examples. 1. (The magnifying glass.) Find the position of the image when the object is placed 0.9 in. from a converging lens of 1-in. focal length.

$$\frac{10}{9} + \frac{1}{q} = 1; \frac{1}{q} = 1 - \frac{10}{9} = -\frac{1}{9}; \; q = -9 \text{ in.}$$

(Image virtual, 9 in. on object side of lens.)

2. Prove that the image in a diverging lens of any real object is virtual. f is negative.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f(-)}; \quad \therefore \quad \frac{1}{q} = \frac{1}{f(-)} - \frac{1}{p}.$$

Since both quantities on the right are negative q is negative.

650.* The Wave Method; Focal Length and Surface Curvature. We shall understand the action of the lens more clearly if we study the warping of the wave fronts instead of the bending of the rays. When a beam passes through a converging lens, the central portion of the wave front is retarded in the glass, while the outer portion, moving mostly through air, overtakes it. Figure 4 illustrates the convergence of a parallel beam to the principal focus. The plane wave (1, 2, 3) is shown entering the left face of the lens. The central rays pass through the glass from 2 to 2′ while the outer rays are passing through air from 1 to 1′ and 3 to 3′. (1′, 2′, 3′) is the emergent wave front which comes to a focus at *F*.

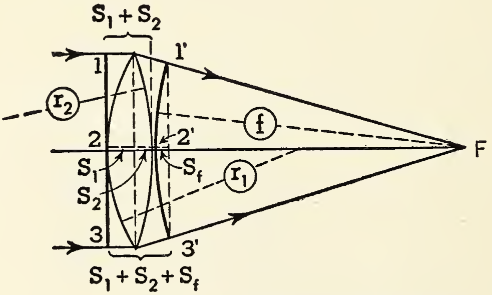


FIG. 4. Focal length and surface curvature.

We shall now find the relation between the focal length and the radii of curvature of the two lens faces. Let us represent the radii of curvature of the three arcs in Fig. 4 by r_1, r_2, and f. r_1 is the radius of curvature of the left lens face; r_2, of the right face; and f is the radius of curvature of the emergent wave front and hence the focal length of the lens. Also s_1, s_2, s_f are the sagittae of these three arcs.

Now the wave travels a distance $s_1 + s_2$ through glass while it travels (approximately) $s_1 + s_2 + s_f$ through air. Hence

$$\frac{\text{Speed in air}}{\text{Speed in glass}} = \frac{s_1 + s_2 + s_f}{s_1 + s_2} = n \text{ (index of refraction),}$$

or

$$1 + \frac{s_f}{s_1 + s_2} = n$$

or

$$s_f = (n - 1)\ (s_1 + s_2).$$

But by the sagitta equation (§720) $s = y^2/2\,r$. Thus these three sagittae are proportional to the reciprocals of their radii ($1/f$, $1/r_1$, $1/r_2$), and these reciprocals can be substituted[1] for the sagittae

---

[1] If y is the half-chord (half-diameter of the lens) the equation becomes

$$\frac{y^2}{2\,f} = (n - 1)\left(\frac{y^2}{2\,r_1} - \frac{y^2}{2\,r_2}\right),$$

and dividing by $y^2/2$ gives eq. (6).

on both sides of the equation, giving

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{6}$$

This shows that the focal length depends on both radii and also upon the amount by which the refractive index exceeds unity.

Example. Find the focal length of a plano-convex lens: the radius of curvature of the convex face is 10 cm, and the index of refraction is 1.5. Here $r_1 = 10$; r_2 is infinite.

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10} + \frac{1}{\infty}\right) = \frac{1}{20}.$$
$$f = 20 \text{ cm}.$$

651.* Converging Power. A strong lens has a short focal length. The reciprocal of the focal length is a measure of the "converging power" of the lens. When the focal length is expressed in *meters* its reciprocal gives the *power* of the lens in *diopters*. For example, a converging lens with a 2-meter focal length has a power of ½ diopter; a diverging lens with 25-centimeter focus has a power of −4 diopters.

This is a very convenient measure because reciprocal lengths appear throughout the fundamental optical formulas. The converging power of a lens pair (close together) is simply the sum of their separate powers. A +2 lens and a +3 lens make a compound lens of +5 diopters; a +3 lens and a −1 lens make a +2-diopter combination (half-meter focal length). Indeed a single lens can be considered as having a converging power given by the sum of the powers of its two faces: $(n - 1)(1/r_1)$ and $(n - 1)(1/r_2)$.

To appreciate the significance of converging power consider the bending of the wave fronts. Let us call $1/q$ the *convergence* (C_q) of the emergent wave front (§300). (If $q = \frac{1}{2}$ meter the convergence is 2; if $q = \infty$ the convergence is zero; if the wave is *di*verging the *con*vergence is negative.) The wave from the source is diverging; its convergence (C_p) is $-1/p$. We may now rewrite eq. (5) as

$$C_q - C_p = \text{Power of lens}, \tag{5'}$$

or, in words, *the convergence of the emergent beam exceeds that of the incident beam by the converging power of the lens.* Convergence and converging power are usually expressed in reciprocal meters (diopters).

Example. An opera glass has two lenses: +10 diopters (objective) and −25 diopters (eyepiece) placed 5 cm apart. Light from a distant object passes through the pair; where is the final image formed? (Distances and focal lengths and eq. 5 can be used instead of convergence and powers and eq. 5′; this latter method seems more physical, less mathematical.)

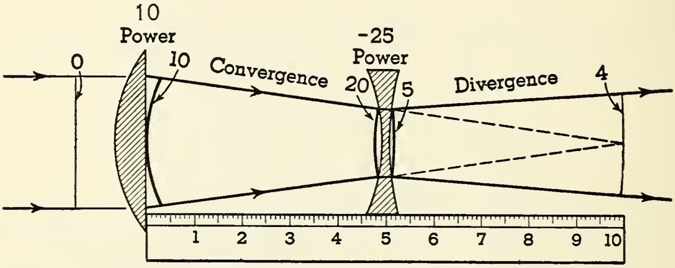


FIG. 5. Opera glass; illustrating convergence and dioptric power.

1. Image by the objective (first lens). $\text{Convergence}_p = 0$; therefore from (5′): Convergence_q = Converging power of lens (10 diopters). Hence were it not for the eyepiece the image would be 1/10 meter = 10 cm from objective (principal focus). (This was obvious without computation.)

2. Image by the eyepiece. The rays reaching the eyepiece are convergent![1] $C_p = 100/5 = 20$; power of lens $= -25$; hence $C_q = -5$. The rays leaving the eyepiece have a *divergence* of 5 diopters; their virtual focus is $\frac{1}{5}$ meter (20 cm) *behind* the lens.

652. Defects of Lenses. *Chromatic Aberration.* The index of refraction depends on the color (§645), and a lens has a shorter

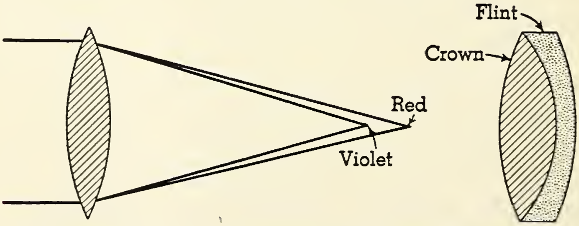


FIG. 6. Chromatic aberration: achromatic lens.

focal length for violet light than for red. Focus the image of a point of light on a screen; when the blue and violet are in focus there is a halo of red which has not yet converged; when the red is in focus there is a halo of violet. This is a serious defect. It is corrected in the achromatic lens. The achromatic lens is in reality a pair of lenses, a strong crown glass converging lens and a weaker flint glass diverging lens (a converging combination). The flint (though of less power) is chosen to produce the same dispersion in the opposite direction; it " undisperses " the light without completely " unrefracting " it. Flint glass is notable for its high dispersion. The lenses of optical instruments are usually such achromatic pairs, cemented together as a single lens.

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[1] One can speak here of a virtual object (p negative)! This is an exception to our general rule that the incident rays are diverging.

Spherical Aberration and Astigmatism for Oblique Rays. Lenses, like mirrors, have astigmatism for oblique rays and spherical aberration. The student has observed that our lens derivations have been only approximations. There is little error for small lenses (diameter small in comparison to focal length), more for larger ones. Chromatic aberration, spherical aberration, astigmatism are all reduced by stopping down the size of the lens with a suitable diaphragm. In a camera there is an adjustable diaphragm by means of which the effective aperture can be varied.

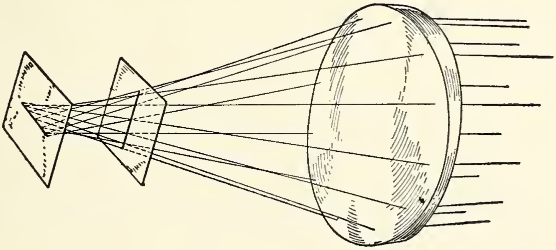

Fig. 7. Astigmatism: due to non-spherical lens (as here) or to oblique rays.

However, stopping down a lens means a loss of light-gathering power (also a decrease in the ultimate attainable resolution, §697). It is not practicable to give lenses elliptical or parabolic surfaces, and it is not necessary. The many surfaces within a compound lens allow a great latitude in design and permit the mathematical optician to devise lenses which are highly corrected.

Camera lenses are rated according to their "*f*-numbers," which give the ratio of the focal length to the diameter of the greatest permissible aperture. In a small box camera the lens is corrected only for chromatic aberration; the largest stop is *f*-16. In a medium-priced camera the "anastigmat" lens (corrected for spherical aberration and astigmatism) may be *f*-8; in the most powerful lenses used in the moving-picture camera the rating is less than *f*-2 — the diameter of the aperture is half as great as the focal length! The small lenses in the microscope objective present a somewhat different problem, and here even greater angular apertures are obtained (§661).

QUESTIONS

1. What kind of lens is a drinking glass full of water? A hollow glass sphere under water?

2. Why is astigmatism for oblique rays not important in the telescope but important in a camera?

3. A lamp is 1 meter from a 2-diopter lens. What is the convergence of the beam just before and after it reaches the lens?

4. How can a prism be made which will refract a beam of light without showing dispersion? Which will produce dispersion without appreciable refraction?

5. What are the principal defects found in lenses?

6. When the object distance from the lens is very slightly greater than the focal length, the image is: (*a*) inverted and enlarged; (*b*) real and reduced; (*c*) virtual and enlarged.

7. When a converging light beam falls on a converging lens the image: (*a*) is necessarily real; (*b*) is necessarily virtual; (*c*) may be either.

Vocabulary: Plano-convex (etc.), power (of lens), diopter, convergence, chromatic aberration, *f*-number, anastigmat.

PROBLEMS

1. Three small light bulbs are placed 60, 40, 20 cm, respectively, from a converging lens of focal length 30 cm. Describe the images as to position and size.

2. Draw the graphical constructions in problem 1.

3. A light source is placed 30 cm from a diverging lens of 30-cm focal length. Where is the image? Make the graphical construction.

4. An object is 3 in. from a lens of 1-in. focal length. (*a*) Draw the construction carefully to scale, and find the position of the image; (*b*) find the position by formula.

5. A camera lens has a focal length of 6 in. An object is 6 ft. away. How far behind the focal plane will its image be focused?

6. The lens of a projection lantern has a 7-in. focal length; the lantern slide is placed 1 in. beyond the focal plane. Where should the screen be located? What is the magnification?

7. How far from a 2-diopter lens should a lamp be placed to throw an image on a screen 5 meters away? Compare size of object and image.

8. The lens in Fig. 7 has a 5-cm focal length in one plane, 7-cm in the other. Suppose that a needle is placed 6 cm from the lens; where will its image lie when it is held: (*a*) vertical; (*b*) horizontal?

9. A watch glass with radius of curvature of 15 in. is filled with water. If it is used as a lens, where will it focus the sun's rays?

10. A plano-convex lens (crown glass) is 10 cm in diameter and is 1 cm thicker at the center than at the edge. Find the radius of curvature and the focal length (sagitta formula).

11. Find the focal length of a lens (crown glass) if each face (convex) has a radius of curvature of 20 cm.

12. Find the focal length when the lens in problem 11 is placed in water.

13. Find the power in diopters of two lenses, each with 10-cm focal length, when they are placed in contact.

14. In the achromatic doublet, Fig. 6, suppose that the crown glass lens has a power of 2 diopters and the flint lens −1.5 diopters. What is the focal length of the doublet?

15. A 2-diopter and a −10-diopter lens are 40 cm apart. A parallel beam enters the first lens. What is its convergence (*a*) as it leaves this lens; (*b*) as it enters the second lens; (*c*) as it leaves the second lens?

16. Two 10-diopter lenses are mounted 30 cm apart. Where will this lens pair focus the image of the sun?

17. Find the position of the image in problem 16 if the distance between the lenses is reduced (*a*) to 20 cm; (*b*) to 19 cm.

18. Two converging lenses with 10-in. focal length are placed 40 in. apart. A light source is placed 20 in. beyond one lens. Where will the rays be focused?

19. A converging and a diverging lens each have a 10-cm focal length and are mounted 2 cm apart. Find the focus for parallel light (*a*) when the light enters the converging lens and (*b*) when it enters the diverging lens.

20. Three converging lenses, each 2 diopters, are placed 1 meter apart. Find the convergence (or divergence) of the beam before and behind each lens and the final position of the image under the following conditions: (*a*) when a parallel beam enters the first lens; (*b*) when the object is 2 meters from the first lens; (*c*) when a plane mirror is 50 cm beyond final lens, object as in *b*; (*d*) when a 6-diopter diverging lens is substituted for the mirror in *c*.

CHAPTER 51

OPTICAL INSTRUMENTS

653. The Basic Arrangements. There are three simple arrangements of object, lens, and image which will enter into the optical instruments described in this chapter (Fig. 1).

(1) When the object is a great distance from the lens the image is formed in (or for nearer objects slightly beyond) the focal plane. (Camera.)

(2) When the object is slightly beyond the principal focus the image (real) is formed a great distance away. (Projection lantern.)

(3) When the object is slightly inside the principal focus the image is virtual and comparatively far away on the object side of the lens. (Magnifying glass.)

Figure 1 should be studied carefully in connection with the work of the chapter. It compares the several instruments studied. The object and image of the right-hand lens (objective) are labeled *O*, *I*; the focal plane of this lens is indicated by a broken line. The focal plane of the eyepiece is indicated by a wavy line. The image *I* of the first lens is the object of this lens; its virtual image is indicated by the broken arrow.

654. Camera. In the camera, light from the object passes through the lens and is focused on the photographic film. It sensitizes the silver salts of the film so that when the film is " developed " and " fixed " we have the " negative," with black particles of finely divided silver giving a permanent registration of the image.

In taking his picture the photographer controls two principal factors:

(1) Focus. Images of all distant objects are formed in the focal plane; simple cameras of the box type have a fixed focus. But for closer objects the image will fall more or less behind the focal plane (Fig. 1), and all except the cheapest cameras have provision for focusing.

SIMPLE SYSTEMS

I. Camera (or Eye): Object at great distance; image in or slightly beyond focal plane.

II. Magnifying Glass (Simple Microscope): Object slightly within focal plane; virtual image 10 inches or more away.

III. Projection Lantern: Illuminated slide slightly beyond focal plane; image at great distance.

COMPOUND SYSTEMS

IV. Compound Microscope: Combination of III and II.

V. Telescope: Combination of I and II.

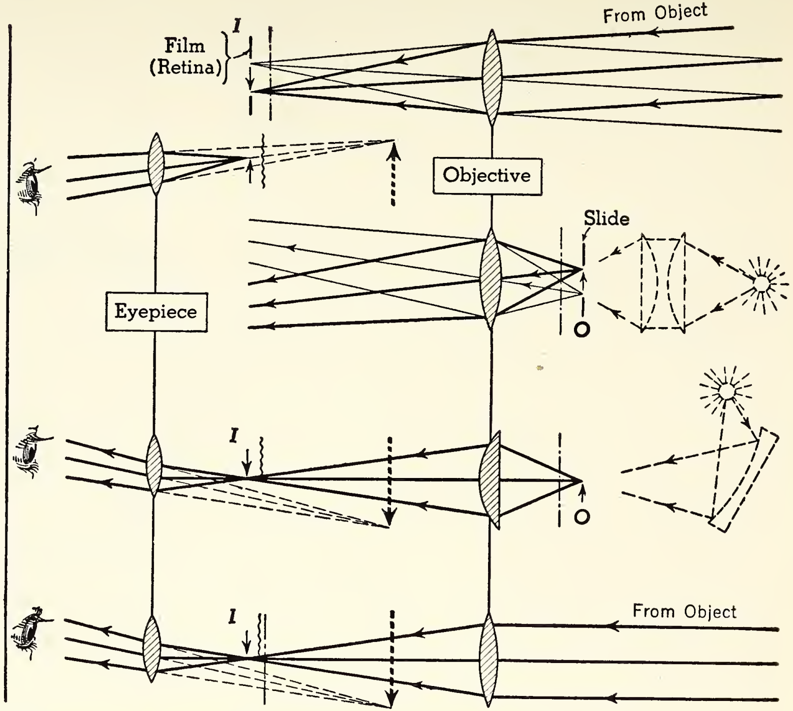

Fig. 1. Fundamental optical systems. Focal plane of objective: —·—·—; of eyepiece ～～～

(2) Exposure. The amount of light striking the film depends upon (*a*) the aperture and (*b*) the time of exposure. The aperture can be adjusted by a diaphragm. The time required for exposures varies inversely as the area (or the square of the diameter) of the aperture. The aperture is measured in terms of the "*f*-number" (§652). In bright sunlight a snapshot can be taken with an *f*-16 opening (the aperture in most box cameras).

The student should by all means take and develop some pictures — if he never has done so. He can borrow a camera, and the materials for development cost only a few cents. It is rather good fun.

655. The Eye. The eye resembles the camera, the retina taking the place of the photographic film. The eyeball is approximately spherical and filled on both sides of the *crystalline lens* with liquids of about the same index of refraction as water (aqueous humor and vitreous humor). The lens has a stratified structure with an index of refraction which gradually increases toward the center (n = 1.4 at the center). The front surface (cornea) of the eyeball is transparent; as in the camera, a diaphragm (iris diaphragm) constricts the aperture (pupil). Light entering the pupil is focused on the retina, refracted largely by the curved surface of the cornea itself, partly by the crystalline lens. From the retina nerves carry the sensation of light to the brain.

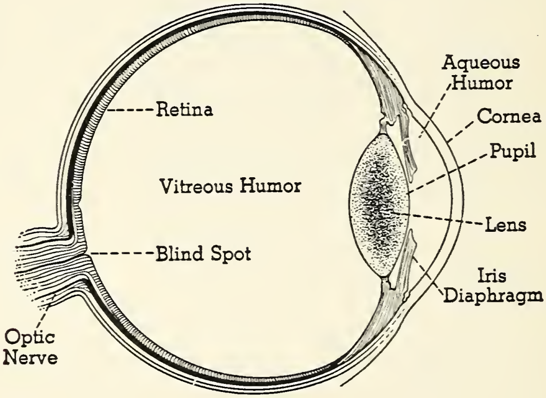

FIG. 2. The human eye.

At rest the normal eye is focused on infinity. Closer objects are brought into focus by muscles which increase the convexity of the crystalline lens (accommodation). The normal eye can focus clearly on objects as close as 10 inches; this is called the *distance of clearest vision.*

656. Defects of the Eye. The most common eye defects are: near-sightedness, far-sightedness, astigmatism, and lack of accommodation. In the near-sighted eye the focal plane of the cornea and lens (relaxed) falls in front of the retina. Near-by objects are seen *without* accommodation; distant objects cannot be seen clearly. Evidently the lens system is too strong; near-sightedness (myopia) is remedied by diverging lenses. . . In far-sightedness (hyperopia) the focal plane of the resting eye lies beyond the retina; distant objects are seen only with accommodation, near-by objects cannot be brought into sharp focus. Hyperopia is remedied with converging lenses. . . Astigmatism is caused by a lack of symmetry in the lens (Fig. 50–7); for example, the lens may be more strongly converging in a horizontal than in a vertical plane: if horizontal lines are seen sharply, vertical lines will be blurred. Astigmatism is remedied by cylindrical lenses (in this example, converging the light in the vertical but not in the horizontal plane).

In addition to these three defects, in middle age the eye muscles weaken and the power of accommodation is largely lost; the remedy here is bifocal glasses.

657. Projection Lantern. "When the object (the slide) is placed slightly beyond the principal focus of the objective lens the image is formed a great distance away (on the screen)." This we have given as the arrangement of the projection lantern. But provision must also be made for sending rays from the light source into the objective lens from all parts of the slide; this requires the *condensing lens*, which is illustrated (dashed) in Fig. 1.

The motion-picture projector has the same optical arrangement as the projection lantern; the difference is only a mechanical one; a view must be shown, the light shut off, a new view substituted in exactly the same place, the light readmitted — all within a twenty-fourth of a second. This is very simple in principle, but that it is done successfully in practice, without flicker, nearly 90,000 times an hour, is certainly one of the triumphs of modern mechanisms.

In the sound film the record of the sound wave is carried in a narrow strip either of variable density or of variable width along one edge of the film. Light passes through this strip to a photoelectric cell. So the fluctuating illumination is converted into electric current, then amplified, and it then produces sound in a loud-speaker.

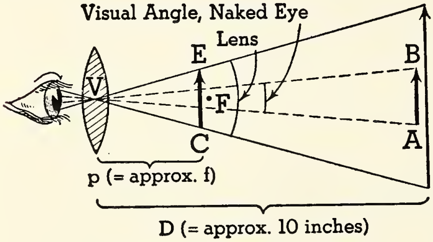

FIG. 3. Simple magnifier.

658. The Magnifying Glass (Simple Microscope). The nearer we place an object to the eye the larger it appears. But the normal eye cannot focus clearly on objects nearer than 10 inches (D, the distance of clearest vision). The magnifying glass allows us to bring the object closer (Fig. 3). When used properly the lens is held close to the eye. The object is placed just inside the principal focus, giving an image 10 inches or more away, where it can be clearly seen. The angle which the object subtends at the eye without the glass is AVB; with the glass it is CVE. Since the arc length BA (or EC) is the same in both cases, these angles are inversely proportional (Fig. 4, p. 678) to the distances (D and p). Hence since p is approximately the focal length we have[1]

$$\text{Magnifying power} = \frac{\angle CVE}{\angle BVA} = \frac{D}{p} = \text{(approx.)}\ \frac{10 \text{ inches}}{\text{Focal length}}.$$

659. Compound Microscope. The compound microscope was first described by Galileo (1610). In optical arrangement it is a combination of the projection lantern and the magnifying glass. (Verify this in Fig. 1.) The object is placed just beyond the principal focus of the objective lens (Fig. 1) so as to give an enlarged image just inside the focal plane of the eyepiece. This gives the first stage of magnification. p is approximately equal to the focal length of the objective, q is approximately equal to the length of the microscope barrel (L). The magnification here is given by:

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[1] The exact expression is $D/f + 1$. A magnifier with 1-in. focal length magnifies about 10 times by the approximate formula, more exactly, 11 times. In metric units $D = 25$ cm for the normal eye.

$$\frac{I}{O} = \frac{q}{p} = \text{(approx.)}\ \frac{L}{f_o}.$$

This image is now viewed with the eyepiece (a magnifying glass) where it is magnified again. The magnifying power of the eyepiece is D/f_e (where f_e is the focal length of the eyepiece). Multiplying, we have for the magnifying power of the compound microscope

$$\text{Magnifying power} = \frac{D \cdot L}{f_e \cdot f_o}\ \text{(approx.)}.$$

A microscope of large magnifying power has a long barrel and short-focus lenses for objective and eyepiece.

The object may be illuminated from above. More often it is a transparent film or tissue and the light is sent through it from the condensing mirror or condensing lens below the stage of the microscope. (Compare the projection lantern.)

Example. Find the approximate magnifying power of a microscope with 20-cm barrel when fitted with a 2-mm focal-length objective and an eyepiece with 20-fold magnification.

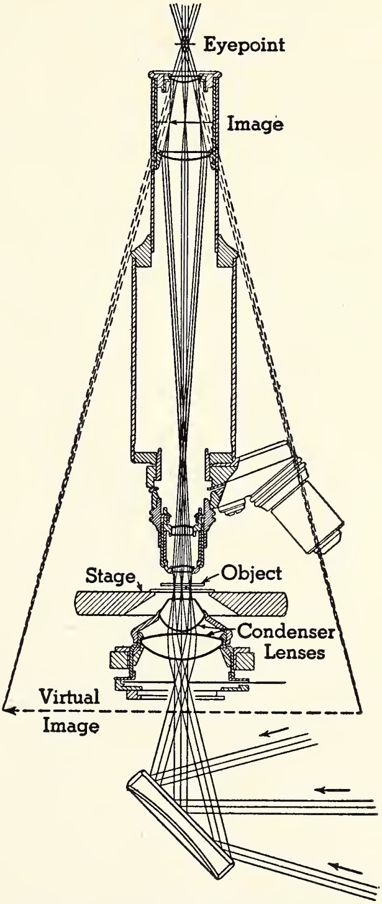


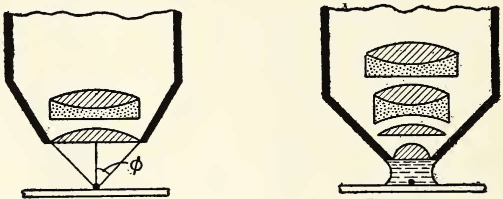
FIG. 4. Compound microscope.

$$\text{Magnifying power of objective} = \frac{L}{f_o} = \frac{20}{0.2} = 100.$$

$$\text{Total magnifying power} = 100 \times 20 = 2000.$$

660. Resolving Power of Microscope. The normal eye can distinguish two light beams as separate when the angle between them, as they enter the eye, is greater than (approximately) 1 minute. This is the " resolving power " of the eye. Two light sources a mile away and less than a foot and a half apart will appear as one; at the distance of clearest vision (10 inches) the limit of resolution is about 0.1 millimeter.

It might be supposed that the microscope in the preceding example, since it enlarges things 2000-fold, would enable us to see things 2000 times smaller, would allow us to distinguish objects 1/20,000 millimeter apart. But this is not so, for the following reason: Light is a wave motion, always with more or less diffraction, never giving exact images of the object. The smaller the aperture of the objective and the narrower the light beam, the more serious will this diffraction be (§697). The " resolving power " will depend on the angle of the aperture 2 ϕ (Fig. 5). But whatever the aperture there comes a limit to the resolution when the size of the object becomes comparable with the wave-length (λ) (about 5/10,000 millimeter for green light). With suitable aperture an experienced microscopist can recognize objects as distinct at half this separation. The limit of resolution of a microscope is usually taken as



Microscope Objective

Immersion Lens

FIG. 5. Microscope objectives.

$$\text{Least distinguishable interval} = \frac{\frac{1}{2}\lambda}{\sin\phi}.$$

where 2 ϕ is the angle subtended by the lens.

661. High-Resolution Microscopes. The wave-length of light is shortened when it enters a liquid.[1] Because of this, resolving power is increased in the *immersion* lens. The space between the specimen and the immersion objective is filled with a liquid with substantially the same refractive index as the lens. In a liquid of refractive index n the wave-length is $\lambda = \lambda_{\text{air}}/n$; therefore the resolving power of the immersion lens is

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[1] $\frac{\lambda_{\text{liq.}}}{\lambda_{\text{air}}} = \frac{v_{\text{liq.}}}{v_{\text{air}}} = \frac{1}{n}$; the first equality (proportion) follows from the fundamental wave motion equation (eq. 2, p. 300); the second equation is the definition of n. Therefore $\lambda_{\text{liq.}} = \frac{\lambda_{\text{air}}}{n}$.

$$\text{Least distinguishable interval} = \frac{\frac{1}{2}\lambda_{\text{air}}}{n \sin \phi}.$$

The microscopist calls $n \sin \phi$ the *numerical aperture* of the lens; for an immersion lens it may be 1.4; for a dry front lens it is necessarily less than 1.

A further extension of the range of visibility has been achieved by using ultra-violet light with wave-length less than 3/10,000 millimeter. Since glass is opaque to such wave-lengths the lenses are made of fused quartz. The image may be viewed on a fluorescent screen (§682), but it is usually photographed. Bacteria have been detected which are beyond the range of the ordinary microscope. Under favorable circumstances structure with dimensions of about 1/5000 millimeter can be observed.

662. The Ultra-Microscope. These are the smallest things that the eye can see as distinct images with shape. But much smaller particles still scatter light and can be observed as mere points of light in the ultra-microscope. Here an intense beam of light passes transversely across the field of the microscope. If no scattering particles are present, no light enters the lens. Each particle, scattering the light, appears as a star against a dark background. Each star is scuttling around in continued, erratic motion because of its Brownian movement (§215). Smoke particles in the air and colloidal particles in a liquid can be seen. Colloidal particles of gold have been observed as small as 17 angstroms in diameter — so small that each contains only 100 gold atoms. If they were 100 times larger in diameter and contained 1,000,000 times as many atoms they would still be formless points of light!

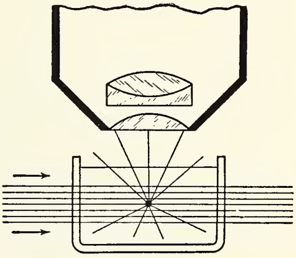


FIG. 6. Ultra-microscope.

663. Astronomical Telescope (Refractor). The image of the distant object is formed in (or for less distant objects slightly beyond) the focal plane; this image is viewed through the eyepiece (combination of arrangements 1 and 2). This is represented in Fig. 7; here the two dots represent the principal foci of the objective (with long focal length) and eyepiece; refer also to Fig. 1.

The magnifying power of the telescope is given by

$$\text{Magnifying power} = \frac{f_o}{f_e}.$$

Proof: The magnifying power is the ratio of the visual angle (θ) subtended by the image as seen in the eyepiece to the visual angle which would have been subtended by the object without the aid of the telescope. This latter angle is θ' which equals θ''. These angles vary inversely as the distances (Fig. 4, p. 678) which differ only slightly from f_e and f_o. Therefore

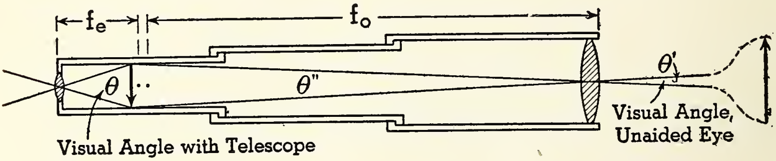


FIG. 7. Magnifying power of telescope. $\theta : \theta' = f_o : f_e$.

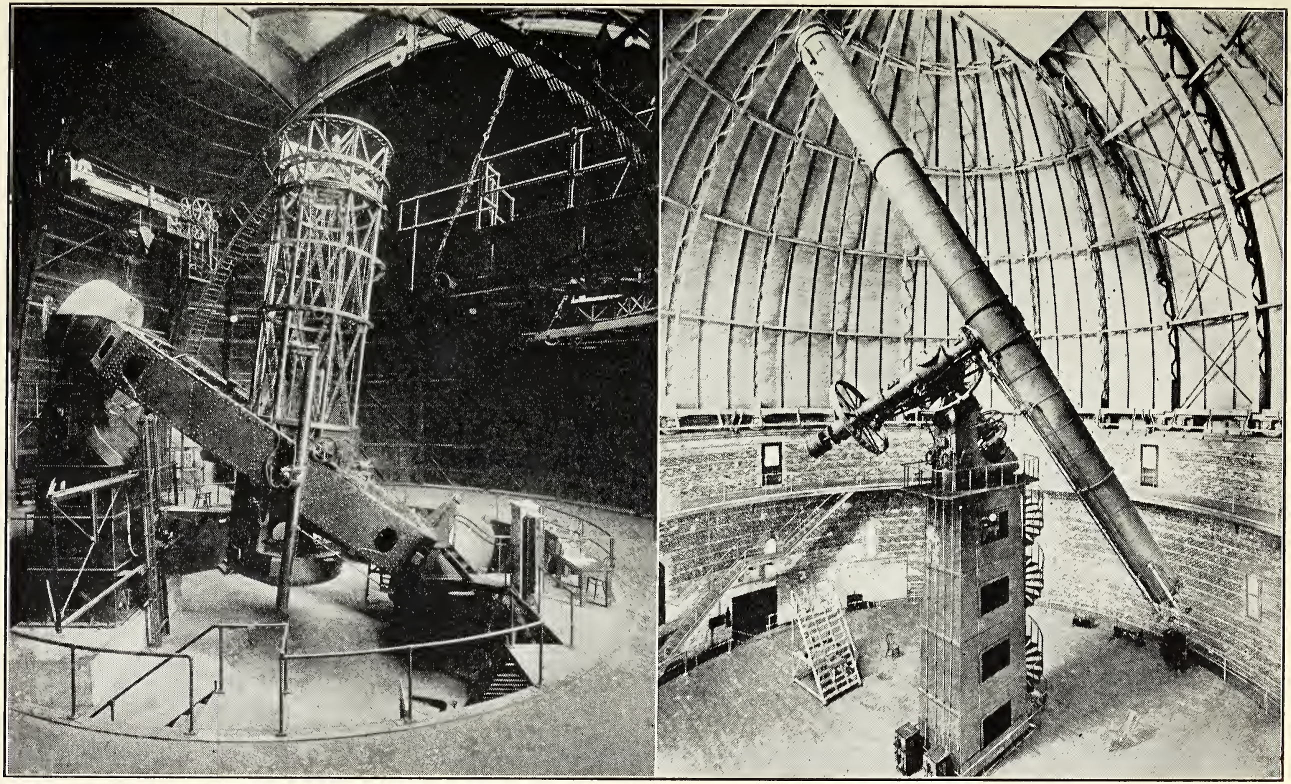
$$\text{Magnifying power} = \frac{\theta}{\theta''} = (\text{approx.})\,\frac{f_o}{f_e}.$$

In contrast to the microscope, the objective in a telescope has a long focal length.

664. Erecting Telescopes. It is evident in the above figure that the image in the simple (astronomical) telescope is inverted. There are three ways in which erect images can be obtained:

(1) Erecting lens. Instead of going directly to the eyepiece the rays from the objective, after the image is formed, pass through another lens, giving a real reinverted image. This is now viewed by the eyepiece. This method has the disadvantage of making the telescope unduly long.

(2) The opera glass. The eyepiece is a *diverging* lens placed in the beam where it is still converging to form the image. (Fig. 50–5.) Such an eyepiece reinverts the image, giving an erect image.



100-inch reflector (Mt. Wilson) 40-inch refractor (Yerkes)

Fig. 8. Astronomical telescopes.

(3) Binoculars. Here, with the aid of several 45° totally reflecting prisms, the light beam is folded back on itself so as greatly to shorten the length of the telescope. In the course of these reflections the image is reinverted.

665. Reflecting Telescope. In the largest astronomical telescopes the objective is a concave mirror instead of a lens. After being polished to spherical shape the mirror is "parabolized" to eliminate spherical aberration. After striking the mirror the rays from the distant object converge toward the principal focus; but, since this would be an inconvenient place for the eyepiece, in one way or another these rays must be deflected to one side. They are observed there with the eyepiece.

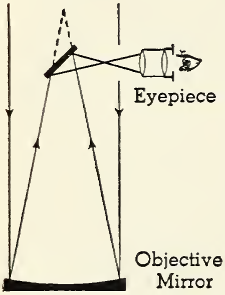


Fig. 9. Reflecting telescope.

Newton discovered chromatic aberration in lenses and despaired of correcting it; this led him to the invention of the reflecting telescope.

666. History of the Telescope. Before the time of Galileo the astronomers sighted upon the stars with sharp eyes. The most famous of early observatories was that of Tycho Brahe, named Uraniborg ("city of the heavens"), on the island of Hveen (Denmark). Seven years after Brahe's death the refracting telescope was invented in Holland; the following year Galileo (1609) devised a simple telescope, and then a better one, and then one that magnified 33 times. With this he scanned the heavens, discovered the moons of Jupiter, the crescent shape of Venus, the mountains on the moon — all this with no correction for chromatic aberration. A century passed before achromatic lenses were invented. In the meantime Newton had built his 6-inch reflector; the possibilities of this type of telescope were shown by Sir William Herschel, musician turned astronomer, who in 1789 built a great reflector 4 feet in diameter. Today the largest refractors have about 40-inch aperture; a 200-inch reflector is under construction for the Mount Palomar (California) Observatory.

667.* Resolving Power of Telescope. The larger the lens or mirror the greater the light-gathering power and the greater the number of stars that can be seen. But, equally important to the astronomer, the greater the diameter the greater the resolution. The resolving power of a telescope depends upon the ratio between the diameter (d) of the objective and the wave-length of light. The least angle at which two point sources can be distinguished as separate is (approximately)

$$\text{Least distinguishable angle} = \frac{\lambda}{d} \text{ radians.}$$

Example. Find the resolving power of a telescope with an objective 20 cm in diameter (consider $\lambda = 5600 \times 10^{-8}$ cm, Fig. 52–9).

$$\text{Least distinguishable angle} = \frac{\lambda}{d} = \frac{5600 \times 10^{-8}}{20} = 2.8 \times 10^{-6} \text{ radian.}$$

This equals approximately 0.01 minute.

Note that the radius of this objective was 100 mm; hence *the resolving power of an objective with* 1-*mm radius is* 1 *minute of arc.* Using the results of this example we may express the resolving power for the telescope as follows: The least distinguishable angle (in minutes of arc) equals approximately the reciprocal of the radius of the objective in millimeters.

For the 200-in. (2.5-meter radius) reflector the theoretical resolving power is 1/2500 minute — 1/40 second of arc. This is 2500 times greater than the resolution of the eye. Assuming no atmospheric disturbance, this mirror should distinguish two light points 0.2 mm apart and a mile away!

QUESTIONS

1. Is it necessary to change the length of a telescope, as we look first at the moon, then across a lake, then across the room?

2. Where is the object normally situated in the simple microscope, projection lantern, compound microscope, telescope?

3. The object positions in Fig. 1 are the usual ones. Is the camera or telescope ever used with object closer than image?

4. Where must a cross-hair be placed in a compound microscope in order that it be seen coinciding (without parallax) with the image? Can the cross-hair be used in a telescope? in an opera glass?

5. Upon what factors does the resolving power of a telescope depend?

6. What kind of instrument would be used to observe the oil drops in the Millikan oil-drop experiment? to read a thermometer carefully? to observe the deflection of a galvanometer (scale and mirror)?

7. The condensing lens in a projection lantern should be so placed that it focuses the light from the source: (*a*) on the screen; (*b*) on the slide; (*c*) into the projecting lens.

8. The resolving power of a microscope is limited because: (*a*) we cannot distinguish objects much smaller than a wave-length of light; (*b*) the best of microscopes are far from perfect; (*c*) there are spherical aberration and astigmatism.

Vocabulary: Objective, eyepiece, condensing lens, myopia, hyperopia, astigmatism, magnifying (resolving) power.

PROBLEMS

1. Find the magnifying power of: (*a*) simple microscope, 1-in. focus; (*b*) compound microscope, objective 4-mm focus, eyepiece 25-mm focus, barrel length 25 cm; (*c*) astronomical telescope, objective 50-cm focus, eyepiece 2-cm focus.

2. Approximately what visual angle does a man subtend at 100 yd.? Compare his image in a telescope: focal length of objective 18 in., of eyepiece 0.75 in.

3. The planet Jupiter subtends a visual angle of 50″. Will it appear as a point to the naked eye? How large did it appear to Galileo (§666)? How large (what angle does it subtend) in the Yerkes telescope with 62-ft. objective and 1-in. eyepiece?

4. A human hair has a diameter of about $\frac{1}{500}$ in. What angle does it subtend at the distance of clearest vision? What angle does it subtend in a 6-in. microscope with a 4-mm objective and 2-cm eyepiece?

5. What is the overall length when focused on a distant object of (*a*) a telescope with objective 10-cm focus, eyepiece 4-cm focus; (*b*) same with erecting lens with 4-cm focus; (*c*) opera glass with 10-cm objective and 4-cm (diverging) eyepiece (Fig. 50–5)? (*Hint:* With erecting lens $p = q = 2f$.)

6. A snapshot can be taken with f-16 diaphragm opening and $\frac{1}{25}$-sec. exposure. How much time is required for another picture with 6 times greater intensity of illumination and an opening of f-8?

7. What is the maximum value of the relative index of refraction of the crystalline lens (relative to the aqueous humor)?

8. Assuming the pupil of the eye opened to a diameter of 4 mm, what is the theoretical resolving power? Compare with the value given in the text.

CHAPTER 52

DISPERSION

668. The Cause of Dispersion. In explaining refraction, electrons in the transparent medium have been compared to beads on a string. Actually, except in metallic conductors, the electrons are not free but are attached to atoms; a better comparison would be to consider the beads as attached by small springs giving each of them a natural period of vibration. A much better analogy is to consider each atom as a minute radio station with not one but many natural periods. The effect of the atoms upon a passing wave (i.e., the index of refraction) will depend on the frequency of the light, becoming greater as this comes more nearly into resonance with one of the natural frequencies of the atomic electrons. Thus the index of refraction varies somewhat with the frequency. This results in dispersion — the breaking of light into its component frequencies as a beam passes through a prism. Usually the natural frequencies of the electrons in the atoms are higher than those of the light wave, and so the violet (high-frequency) light wave is retarded and refracted more than the red.[1] In flint glass and carbon bisulfide there are resonance frequencies which lie in the ultra-violet not far from the visible part of the spectrum and the dispersion is great. Flint glass prisms are used in spectrometers.

669. The Spectrometer. The spectroscope is used for observing the spectrum; the spectrograph for photographing it. Both are classed as spectrometers.

A spectrograph consists of three parts:

(1) The collimator in which the beam, diverging from the slit, passes through a lens and is made parallel.

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[1] Exceptions are found in certain blue and violet dyes, which have natural frequencies and selective absorption in the red-orange-yellow parts of the spectrum. Fuchsin (magenta), for example, refracts yellow more than green. This is known as *anomalous dispersion*. It is of little practical importance because the colors showing such dispersion are also rather strongly absorbed.

(2) The prism which bends the parallel light beam; if a single color is present it gives a single deviated parallel beam; if several colors it gives several beams, deviating the violet most, red least.

(3) The camera where the lens brings each of these parallel beams to a focus on the photographic film. In the spectroscope, a telescope is substituted for the camera.

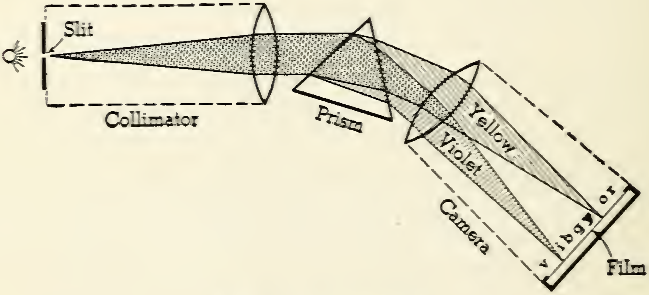
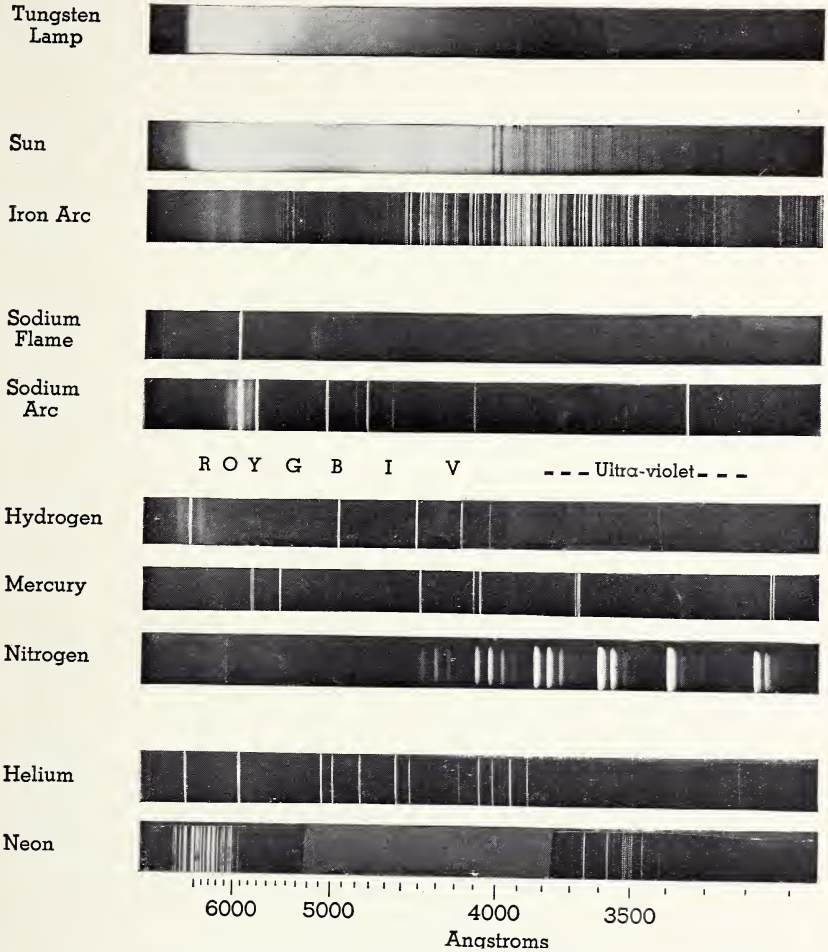


FIG. 1. Spectrograph.

670. Emission Spectra. The spectrum colors, in order of their refraction, are red, orange, yellow, green, blue, indigo, and violet. The student should observe several characteristic spectra. He may well start with the "sodium flame," the yellow light observed when common salt (sodium chloride) is placed in a flame. The light comes from the sodium atoms; for many purposes it can be considered as a pure color, a single frequency. This light passing through the spectroscope is bent as a single beam (upper beam in Fig. 1) and is converged as a single line — the image of the slit. This single yellow line which we see is the spectrum of the sodium flame.[1] The greenish-blue light of the mercury-vapor lamp consists of several colors — two slightly different yellows, green, blue, violet. Each color is deviated differently by the prism and each gives its own slit image — two close yellow lines, one green, one blue, two violet lines — these are the strongest lines in the visible spectrum of

[1] Light of a single frequency is *monochromatic*. Sodium light is nearly but not quite monochromatic. Actually two frequencies are present, differing by one part in a thousand and giving a double line; unless the slit is very narrow and the resolving power of the spectrometer fairly high these two lines will overlap and appear as one.

FIG. 2. SPECTRA
Visible and near ultra-violet.



The solar spectrum, extending with decreasing intensity to about 2950 angstroms, has thousands of absorption lines. Many are due to iron. The strong pair is due to ionized calcium. In our picture few lines show in the visible region because of overexposure. . . Only the most easily excited lines appear in flame spectra. . . The hydrogen spectrum shows the famous Balmer series due to the hydrogen *atom*. Corresponding absorption lines appear in the solar spectrum. . . Bands, as shown for nitrogen, are characteristic of *molecular* spectra. . . Helium was discovered by the appearance of the yellow line (λ 5875) in the chromosphere of the sun (eclipse of 1878).

The intensity minima in the green of the continuous spectra are due to the decreased sensitivity of panchromatic film in this region.

mercury. A neon lamp gives a great many red and yellow lines; iron vapor gives thousands of lines, spread from red to violet and continuing into the ultra-violet. These are examples of *bright-line spectra*. Such spectra are obtained when a source emits one, several, or many distinct wave-lengths.

In contrast with these spectra, the spectrum from an incandescent lamp is a continuous band of color, from red to violet. Such a *continuous spectrum* shows that all frequencies within this range are emitted. Any heated solid or liquid gives a continuous spectrum. Observe the spectrum from the white-hot carbon of the carbon arc or from the yellow flame of an ordinary gas burner (due not to the gas itself but to incandescent particles of smoke).

Gases and vapors give discrete (line) spectra which are characteristic of the emitting atom or molecule; incandescent solids and liquids give continuous spectra. The free atom or molecule in the gas is not influenced by neighbors and can " express its own individuality " — the electrons in these minute radio stations emit their own characteristic frequencies; the atoms in the solid disturb each other so that characteristic frequencies are impossible. The range of the continuous spectrum depends on the temperature (§258), very little upon the nature of the element.

671. Analysis of Line Spectra. Nothing in modern physics is more interesting than the story of the deciphering of the characteristic spectra of gases. The spectrum is the voice of the atom, in which it has revealed itself to the physicist. The lines can be arranged in definite series. In all spectra, even the most complex, definite relations exist between the emitted frequencies. These relations at first seemed no more understandable than Egyptian hieroglyphics once were. We should expect fundamental frequencies and overtones with double, triple, etc., frequencies, but this was not the relation found. The atom could not be explained away simply as a minute radio station. The ideas of light and matter, of electrons and protons, of space and time, had to be fundamentally changed before this " voice of the atom " could be interpreted.

This long story begins with Newton. We find him, while still a student at Cambridge, breaking white light into its component colors; he recombined these spectrum colors to give white light, showing that white light is only the combination of all colors. A hundred and fifty years later Fraunhofer detected the absorption lines in the solar spectrum; by 1860 chemists were identifying elements by their spectra and the astronomers were studying the spectra of the stars. Ångström (in Sweden) tabulated wave-lengths, all measured to 10^{-8} centimeter (angstrom unit); wave-lengths were measured with greater precision by Rowland in America. Balmer and Rydberg and others discovered the relations between spectrum lines. By the end of the last century a vast amount of data had accumulated and important relations were recognized, but it was not until 1913 that any steps were made towards interpretation. In that year Niels Bohr proposed his theory of the atom, basing it on the Quantum Theory; this proved to be the Rosetta stone for the hieroglyphics. This and the New Quantum Mechanics (introduced 12 years later) led to general interpretation of spectra and, along with this, rapid progress in all branches of spectroscopic investigation. Spectrum lines have been measured and classified — not only visible spectra but in the infra-red, ultra-violet, far ultra-violet, x-ray, and gamma-ray regions. From all this has come our clearest picture of the world of the atom.

672. Wave-Lengths of Visible Light. We shall find in the next chapter how wave-lengths are measured. Roughly, the range to which the human eye is sensitive extends from about 0.0004 to 0.0007 millimeter. The color depends on the wave-length as shown in Table 50. Color is a psychological effect, and its classification is necessarily somewhat arbitrary.

TABLE 50

WAVE-LENGTHS IN ANGSTROMS

7200		6400		5900		5800		5000		4600		4400		4000
	Red		Orange		Yellow		Green		Blue		Indigo		Violet	

673. Ultra-Violet and Infra-Red. The wave-length of visible light is several thousands of times shorter than the most minute radio wave ever obtained and some thousands of times longer than typical x-rays. Electromagnetic radiation exists throughout this whole range. In the spectrum of the sun or arc-light we can detect invisible radiation with wave-length shorter than violet (ultra-violet) and longer than red (infra-red). The ultra-violet light, though invisible to the eye, affects the photographic plate, causes fluorescence in certain substances (§682), induces many chemical reactions, produces photosynthesis in the growing plant, and causes sunburn. This radiation is largely absorbed by glass; we do not get sunburned on the glassed-in sun porch.

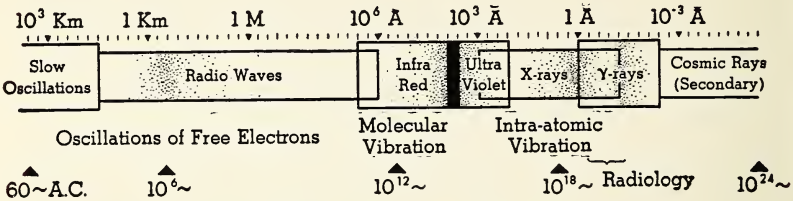


FIG. 3. The electromagnetic spectrum.

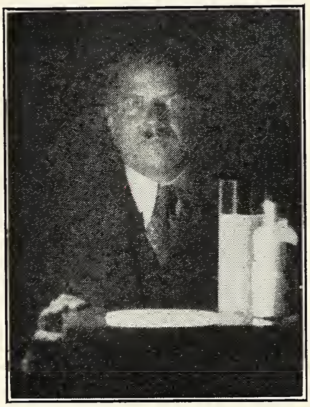


FIG. 4. Ultra-violet light. Eyes and objects on table fluoresce, collar reflects, flesh largely absorbs.

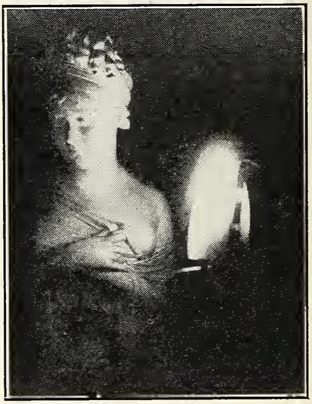


FIG. 5. Infra-red light. (Taken by radiation from non-luminous flatiron.)

Compared with the ultra-violet, infra-red radiation is very impotent. It causes no fluorescence and has little chemical action, no photosynthesis; it can be detected photographically only near the visible (on special plates). We can detect infra-red radiation by observing the heating when it is absorbed — a method applicable to any form of radiation but very insensitive. We are warmed by the radiant heat (infra-red) from the stove.

The larger part of the sun's radiation is in the infra-red and plays its role in heating our atmosphere.

674. Absorption Spectra. When white light enters colored glass, certain colors are absorbed. When the transmitted light is examined in the spectroscope one finds an absorption band extending throughout a considerable part of the spectrum; with purple glass, yellow and green are missing; with red glass all the shorter wave-lengths are absorbed. Such *absorption bands* are characteristic of liquids and solids.

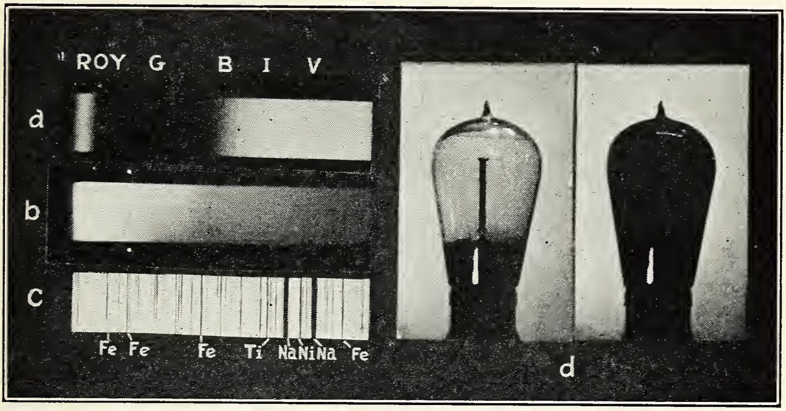


FIG. 6. Absorption spectra. (*a*) Band absorption in purple glass. (*b*) Line absorption: light from incandescent lamp through sodium vapor; (absorption line marked by white dots). (*c*) Solar spectrum in yellow (great dispersion showing sodium doublet). (*d*) Lamp bulb containing sodium vapor, *left* in white light, *right* in sodium light.

In gases a more interesting kind of absorption occurs. If we examine the spectrum after white light has passed through sodium vapor or chlorine vapor or iodine vapor we find that certain lines have been removed from the continuous spectrum. (Observe the effect with iodine; here there are hundreds of absorbed lines so close together that in a small spectroscope they appear as black bands.) These are the same frequencies which the vapor would emit if excited. In sodium vapor we find the continuous spectrum broken by a black line, really a doublet, in the yellow, where the emission spectrum showed a bright doublet. Such *absorption lines* are characteristic of gases. The atom is now acting as a "radio receiving station," absorbing those frequencies for which it is "tuned." However, not all but only the more fundamental emission lines appear in the absorption spectrum (Chapter 55).

675. The Rainbow. The most impressive example of dispersion found in nature is the rainbow. This is due to refraction and reflection of light from the spherical drops of rain. Take a round flask full of water and, holding it at arm's length, somewhat above the level of the eye, allow a horizontal beam of light to pass over your shoulder and fall on it. When the angle of elevation is between 40° and 42° there appears on the bottom surface, almost at grazing incidence, a spot of colored light, changing from violet to red as the angle is increased. The path of the light, twice refracted and once reflected, is shown in Fig. 7. Light is refracted and reflected at all angles, but for these particular angles the parallel beam (of proper color) returns as a parallel beam to the eye; other colors are scattered (the theory is developed in textbooks on optics).[1]

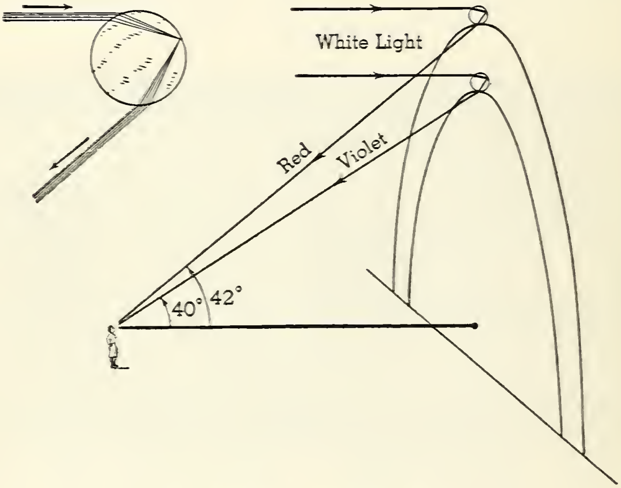


FIG. 7. The rainbow.

When the horizontal rays of the setting sun fall upon the drops of rain, each spherical drop has its brilliant-colored spot when viewed at an angle between 40° and 42°. Thus looking into the rain we see a violet arc making an angle of 40° with the direction of the sun's rays, outside of that a larger red arc, and within this two-degree interval the other colors of the spectrum. This is the primary rainbow. The larger secondary rainbow, often seen, is caused by light *twice* reflected in the drops. Halos around the sun and moon and sundogs have a somewhat similar explanation; they are caused by refraction and reflection, not by spherical drops of rain, but by minute ice crystals (hexagonal prisms) in the upper atmosphere.

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[1] This experiment was first performed by Descartes, philosopher, mathematician, and scientist of the Dutch school, preceding Newton. The "Cartesian diver" (§37) and "Cartesian coordinates" are named for him.

676. **Fraunhofer Lines.** In 1817 Fraunhofer observed that the spectrum of sunlight, which to casual observation appears to be a continuous spectrum, is in fact interrupted by many black lines. He listed the stronger ones; the larger spectrographs show that there are thousands of them. These are absorption lines. Each of them corresponds to an emission line of some element. We find here the close doublet of sodium, several lines of calcium and of hydrogen, hundreds of lines of iron vapor, and so on — lines for practically all the common elements found on the earth. Above the molten surface of the sun an atmosphere of hot vapors extends for several thousand miles. The incandescent surface emits a continuous spectrum; as this light passes out through the solar atmosphere these characteristic lines are absorbed by the vapors present. Thus the spectrometer analyzes the chemical nature of the gases around the sun. Fifty-seven of the elements of the earth have been identified in the solar atmosphere. One element was recognized on the sun before it was discovered by the chemist (helium — meaning " the sun ").

677. **Stellar Spectra; Doppler Effect.** The spectrograph is one of the most important tools of the astronomer. It is as important in interpreting the riddle of the heavens as it is for the riddle of the atom. The astronomer substitutes his huge telescope for our collimator. As he points his telescope-spectrograph at the sky he finds some nebulae emitting bright-line spectra — they are gases not yet congealed into suns. He finds stars emitting continuous spectra — some redder and therefore colder, some bluer and therefore hotter, than our sun — and from these spectra he deduces the age of the star. He observes the absorption lines, often much like the Fraunhofer lines of the sun; but in some of the younger stars only the lines of hydrogen or helium appear.

From the stellar spectrum the astronomer also determines the speed of approach or recession. When a star approaches, the frequencies in the spectrum are increased because of the Doppler effect (§325); the spectrum lines are shifted slightly to the violet. When a star recedes the shift is to the red. For all the stars in our galaxy, sisters to our sun, the velocities are small in comparison with the speed of light and the shift is rather small. But astronomers have found in the nebulae extraordinary Doppler shifts, where blue changes to green, green changes to yellow, yellow to red. Reaching out with his telescope to other galaxies (spiral nebulae) millions of " light-years " away, the astronomer finds these all receding from us, very rapidly, with speeds proportional to their distances away! The most distant nebulae have a speed of 20,000 miles a second! This has led to the conception of the " Expanding (or Exploding) Universe."

In general, the telescope reveals to us where the stars are; the spectrograph tells what they are. Because of the close union between the two sciences, this branch of astronomy is called astrophysics.

COLOR

678. Hue, Saturation, Brilliance. Colors differ in *hue, saturation*, and *brilliance*. The hues are the colors of the spectrum, with the addition of purple (a mixture of red and violet). As we find them in nature colors are never pure — they are usually more or less diluted with white light; pink instead of red, sky-blue instead of blue. This is a difference in *saturation;* the pure colors are saturated; pink is an unsaturated red. There is also a certain amount of general absorption; that is, there is black mixed with the hue: compare brick-red with crimson, brown with yellow, dark green with green. This is a difference in *brilliance*. For example, we pass from white, through the grays (hueless, completely unsaturated), to black by decrease in brilliance.

679. Mixing Colors. The eye does not resolve colors into their spectral frequencies. Sodium light (λ 5890) gives the sensation of yellow, but exactly the same sensation is given by a mixture of red and green light in proper proportions. An unsaturated green is obtained by a mixture of yellow and violet or of orange and blue; or a light blue can be obtained by a mixture of green and violet (Fig. 8). The white light of the sun is a mixture of all colors, but the same white can be obtained by a mixture of only two *complementary colors:* yellow and indigo, orange and blue, red and bluish green (opposite on the Newtonian color circle). For green there is no complementary hue in the spectrum; purple is the complementary hue and this is not obtained as a single frequency but is a mixture of red and violet.

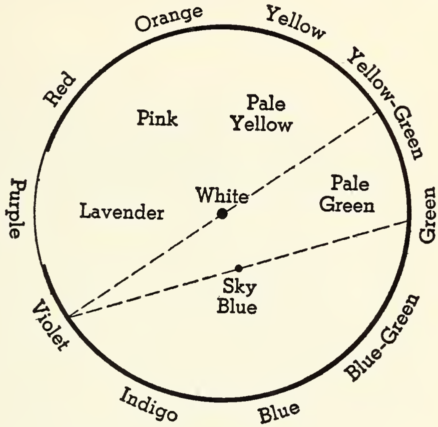


FIG. 8. Newton color circle.

In Fig. 8 the pure spectral hues and purple are represented about the circumference, unsaturated colors within the circle, white at the center. The white content of the color (lack of saturation) gradually increases toward the center. Complementary colors are opposite each other. Such a color circle (originally suggested by Newton) shows in a general way the results of mixing colors; for exact quantitative representation the shape of the figure must be somewhat altered.

680. Sensitivity of the Eye. Contrast this recognition of hue with the perception of pitch. It is not true that the pitch sensation of middle D is obtained by superposition of middle C and middle E. Nor do we obtain the sensation of pure noise by the superposition of a single low-frequency pitch with some complementary high-frequency pitch. The ear more or less successfully resolves the sound into its components. In the ear there are many thousand cells probably each with its own characteristic pitch sensation. In the eye there are (according to the celebrated Helmholtz-Young theory) only three primary color sensations — red, green, violet — which when more or less equally excited give the sensation of white, and for unequal excitations give the various hues.

The sensitivity of the eye for different parts of the spectrum is shown in Fig. 9. The eye is most sensitive to yellowish green, where all three of the primary stimuli are rather strongly excited. In this part of the spectrum it has been found that 1 candlepower is equivalent to 1/50 watt of radiant energy (mechanical equivalent of light).

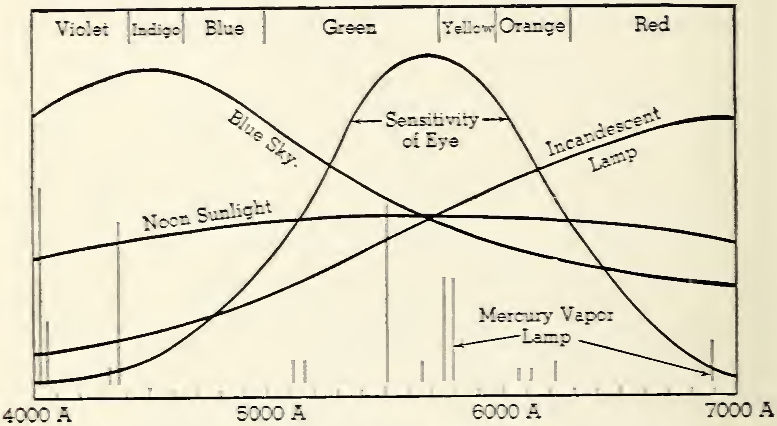


FIG. 9. Color sensitivity of eye and distribution of energy of different light sources. (Distribution curves are on different scales and are not to be compared as to intensity.)

681. Selective Reflection. Apples are red, violets are blue, because of selective absorption and reflection. From a white object all wave-lengths are diffusely reflected, from red only the longer ones, from blue the shorter ones. The other colors are absorbed. The light falling on a red barn penetrates a short distance into the surface; the shorter-waved violet, blue, green light is absorbed; the longer wave-lengths are scattered by the red pigment and re-emerge as the fairly brilliant, nearly saturated red which we see.

The " natural " color of things depends also on the light falling on them; a green flower and a white flower are the same color in green light — both reflect the light; and in green light a red rose would appear black.

The mixing of pigments presents a somewhat different problem from the mixing of colored light. The colors of pigments are due to the *subtraction* of color. If orange light and complementary blue light are mixed in proper proportion the result is white; but if an orange pigment which reflects only red, orange, yellow and yellowish green (absorbing all others) and a blue pigment which reflects only indigo, blue, and bluish green are mixed the mixture will absorb all colors except possibly some of the green, and the pigment is dark green or black. One must know the absorption characteristics of the pigments to predict the exact color of a mixture.

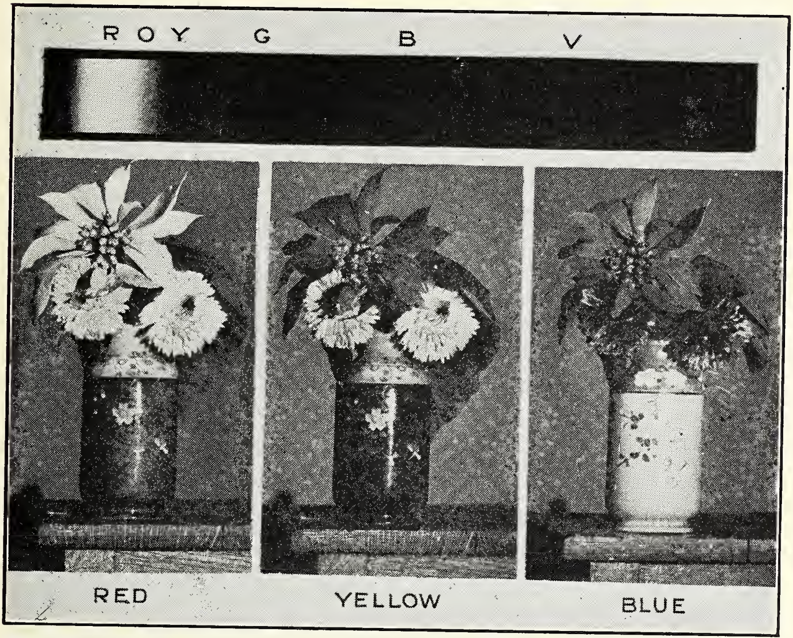


FIG. 10. Selective reflection. Red and golden flowers in blue vase, photographed with lights of different colors. *Above:* reflection spectrum of red flower (poinsettia).

682. Fluorescence and Phosphorescence. Selective reflection accounts for the colors of most things about us; these objects can have only colors contained in the light falling on them. But this is not true for everything. Certain substances absorb the impinging light and then at once re-emit a different color; the phenomenon is known as *fluorescence*. The greenish surface tinge of engine oil is due to fluorescence; the oil is luminous even in ultra-violet light — ultra-violet light is absorbed and visible light is re-emitted. Quite a number of substances show this effect; it is most striking in a solution of fluorescein.

In some substances the re-emission of light continues long after the exciting light is removed; the phenomenon is then called *phosphorescence.*

It is a significant fact that in practically all instances of fluorescence and phosphorescence the emitted light is of *longer* wavelength than that of the exciting light which is absorbed (Stokes' Law of Fluorescence). Neither the engine oil nor the fluorescein shows its greenish fluorescence in the yellow light of a sodium flame; both fluoresce in white light or in blue or violet or ultra-violet light. Visible fluorescence may be excited by ultra-violet light or x-rays but not by infra-red or radio waves.

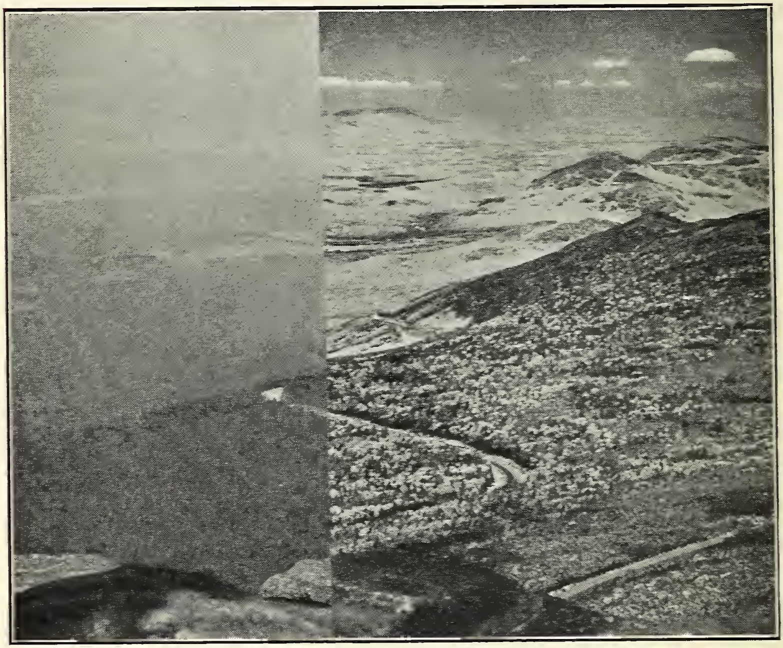


FIG. 11. Landscape by visible and infra-red light. Left side taken with ordinary film, right with infra-red film and filter Observe that the sky does not scatter infra-red.

Quite recently fluorescence has been put to practical use in illumination; the student has probably seen fluorescent luminaires in store windows. The lamp is a neon-mercury vapor arc, in a fluorescent tube. The arc emits strong ultra-violet radiation which excites the tube to emit visible light of a quality very much like daylight.

683. The Greenhouse Effect. A greenhouse or a sun porch is kept warm by the trapping of solar radiation. Glass transmits visible and near-visible light, but absorbs the further infra-red and ultra-violet. Visible light from the sun passes into the greenhouse and is absorbed by foliage and soil. These surfaces reradiate, but the radiation is now in the far infra-red and does not pass through the glass windows. In much the same way the earth is blanketed by its atmosphere which is transparent to most of the solar radiation, but absorbs the heat radiated from the earth. This has a fundamental effect on climate. Not only does it raise the mean temperature, but this heating of the atmosphere from below causes convection currents, and so, indirectly, causes winds and rain (§253).

684. Scattering of Light. Light is scattered by small particles (§662), by chance density variations in the atmosphere, and to a certain extent by the individual molecules of the air. The scattering is greatest for the short wave-lengths. (Long water waves pass around an obstacle; shorter waves are to a greater extent scattered or reflected.) Scattered sunlight gives us the blue of the sky; the residual, unscattered rays which we see from the setting sun are largely red and yellow. A distant landscape is obscured by a blue haze of scattered light. Hence distant views are photographed using red filters (to eliminate the shorter

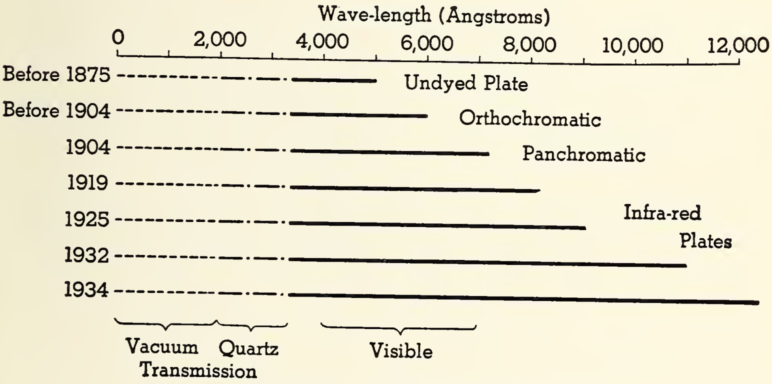


FIG. 12. Sensitivity of photographic plates.

wave-lengths) and red-sensitive (panchromatic) film. Extremely long-range photography has been made possible by the perfection of a film sensitive to the infra-red.

QUESTIONS

1. Under what conditions are continuous and line emission spectra obtained? Line and band absorption spectra?

2. How is a spectroscope changed to a spectrograph?

3. How is electromagnetic radiation of wave-length 10 meters excited and how detected? 1/1000 mm? 1/2000 mm? 1/100,000,000 mm?

4. What is the range of the visible spectrum? There are approximately how many wave-lengths of blue light in a millimeter?

5. If the sodium lines of a star are found to have wave-lengths of 5880 and 5886 instead of 5890 and 5896, is the star approaching or receding? How does its speed compare with the speed of light?

6. White light can be produced by the combination of yellow and indigo. Compare the appearance of a rose in this light and in sunlight.

7. Is it by a fortunate coincidence that sunlight happens to be "white"? Is artificial light "white"?

8. Viewing the reflection of a lamp filament from a piece of red glass (held obliquely) one sees a white image and, slightly displaced, a red image. Explain.

9. Infra-red radiation far removed from visible can be detected: (*a*) by fluorescence; (*b*) by a radiometer; (*c*) by photographic film.

10. The sky is blue because: (*a*) short waves are scattered more than long; (*b*) the atmosphere absorbs long wave-lengths; (*c*) the eye is most sensitive to blue.

Vocabulary: Spectrometer, emission (absorption) spectra, continuous (line) spectra, ultra-violet, infra-red, fluorescence, phosphorescence, Fraunhofer lines, hue, saturation, brilliance, complementary colors, scattering.

CHAPTER 53

INTERFERENCE AND DIFFRACTION

685. Young. The phenomenon of interference was discovered by Thomas Young in 1801.

Young was one of the most remarkable men that ever lived. He was a precocious youth; he read fluently at the age of 2, read Latin at 6, Greek soon after; at 10 he studied Hebrew, Chaldee, Arabic, Syriac, Persian, and modern languages. He entered medical school in 1792 at the age of 19; a year later he had investigated the crystalline lens and explained accommodation of the eye. For this he was elected to the Royal Society. In 1799 he started medical practice and got his M.D. degree nine years later. In 1801 he was Professor of Natural Philosophy in the Royal Institution; there he performed his famous interference experiment and resurrected the wave theory of light. Others had thought of kinetic energy as a " living force " — Young first called it " energy." He became Superintendent of the Royal Almanac, Insurance Actuary, and Professor of Medicine. He wrote biographical articles and medical books, discovered astigmatism, and explained dispersion and capillary action. The astronomer knows of " Young's Theory of the Tides "; the psychologist knows of " Young's Theory of Color Perception "; the physics student knows of Young's Modulus; the archaeologist knows of Young's work on the Rosetta stone. For, when that most famous of Egyptian relics was discovered, it was Thomas Young who first deciphered it! And withal (says his biographer) he played the flute, sang, and danced!

686. Young's Interference Experiment. Suppose that we let light from a distant slit (A) pass through two adjacent slits B and C and then onto a screen. If light traveled in straight lines we should get two narrow streaks of light on the screen. Instead we observe a pattern consisting of a number of equally spaced light and dark fringes. The closer the slits B and C are together the farther apart are these fringes. (For clarity the distance between the slits and the separation of the fringes is greatly exaggerated; Fig. 4 gives an actual photograph of the effect.)

This phenomenon is explained by interference. The light waves from B and C reach point L in phase, since this point is equally distant from the two slits. Hence, here there is light (constructive interference). Now moving down the screen we come to a point D which is a half wave-length farther from B than C. The waves reach D out of phase, here there is destructive interference, darkness. Continuing down the screen we reach a point (L') which is 1 wave-length farther from B than C and here the waves meet exactly in phase again. There is constructive interference and an intensity maximum. (There is more or less light at other points where the waves are more or less in phase, complete darkness only where the phases are exactly opposite.) At D' the two patterns differ by 3/2 wave-lengths and there is darkness. At L'' there is a phase difference of 2 wave-lengths and another maximum. And so on. We shall call L', L'', L''' etc., the first-order, second-order, third-order, etc., diffraction images. They correspond to differences in path of 1 wave-length, 2 wave-lengths, 3 wave-lengths, etc.

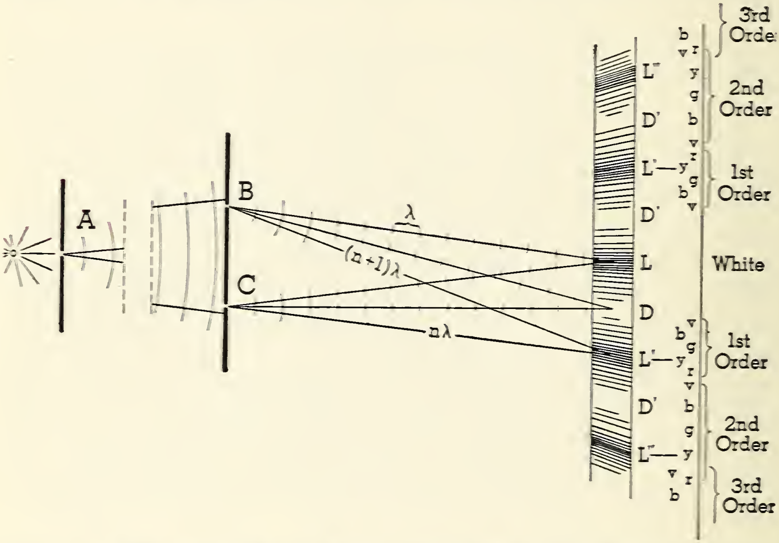


FIG. 1. Young's experiment; with monochromatic and (at extreme right) with white light. Compare Fig. 25–5*b*.

687. Diffraction-Grating Formula. This was Young's experiment; it was convincing evidence for the wave theory because it led directly to a measurement of the wave-length of light. The student can make two parallel slits, less than a millimeter apart, and repeat the experiment. (Cut them with a razor blade in an undeveloped photographic plate.) He must measure the distance (d, Fig. 2) between his slits with a microscope, and measure the displacement (s) of the first-order diffraction image and the distance l' from the slits to the image. Then, as we shall now show

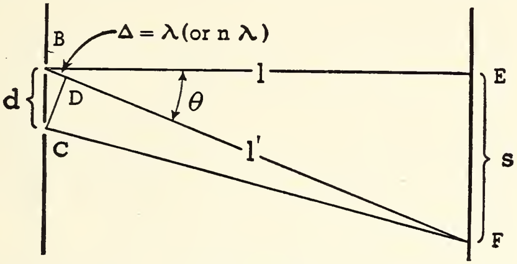


FIG. 2

$$\frac{\lambda}{d} = \frac{s}{l'},$$

and λ can be obtained.

In Fig. 2, F represents some point on the screen, distant BF and CF, respectively, from the two slits. CD is drawn perpendicular to BF; then the right triangles BDC and FEB are similar and the corresponding sides proportional

$$\frac{BD}{d} = \frac{s}{l'}.$$

DF and CF may be regarded as equal; BD is then the path difference.

For the first-order constructive interference this path difference must equal 1 wave-length. Thus for the first-order image

$$\frac{\lambda}{d} = \frac{s}{l'}.$$

Evidently $s/l' = \sin\theta_1$, where θ_1 is the angle of deviation for the first-order image. Thus

$$\frac{\lambda}{d} = \sin\theta_1.$$

For the second-order image the path difference is 2 λ; the angle of deviation (θ_2) is given by

$$\frac{2\,\lambda}{d} = \sin\theta_2.$$

In general for the *n*th order:

$$n\lambda = d\sin\theta_n. \tag{7}$$

Thus the wave-lengths can be determined by measuring the angle of deviation with a pair of slits. But, for precise determination, not two but a large number of equally spaced slits are used — this is the diffraction grating (§690).

688. Interference with White Light. These phenomena are best observed with monochromatic light, in which there is a single definite wave-length and the light and dark fringes occur as shown in Fig. 1. In ordinary white light there are all wave-lengths from 4000 angstroms (violet) to 7000 angstroms (red), and each color has its own diffraction pattern. The zero-order image (L) is white since all colors are in phase here irrespective of wave-length. But the first-order diffracted image occurs for violet at v in Fig. 1, with a path difference of 4000 angstroms and for red almost twice as far out at r with a path difference of 7000 angstroms. So the first-order image has become a first-order spectrum, with violet deviated least, red most. In higher orders the different colors overlap: a retardation of 12,000 angstroms gives a superposition of third-order violet (4000 angstroms) and second-order yellow (6000 angstroms). (See Fig. 6.)

689. Coherent Beams. There are many other ways of getting interference between two light beams, but always the beams must be "related." In Young's experiment both beams came from the same source slit A. If the two adjacent slits were illuminated by light from different sources there would be no interference pattern because there would be no definite phase relation between the light from the two independent sources. The two interfering beams must come from the same source by different paths; two such related beams are said to be *coherent.*

Figure 3 shows several ways in which coherent beams can be obtained; each arrangement gives interference effects similar to those obtained by Young's pair of slits. If a filament is viewed in a mirror at grazing incidence so that the filament and its mirror image appear not more than a millimeter apart, one observes interference bands; the source and its reflection vibrate alike and are coherent. . . Two slightly displaced, coherent images can also be obtained by a " biprism." . . If a lens is cut through the middle and the two halves slightly separated each half will give a separate real image. These two adjacent images are coherent and will exhibit interference effects.

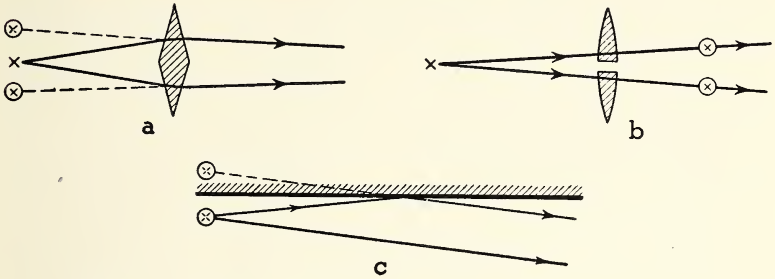


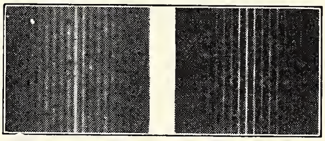
FIG. 3. Coherent images with biprism, split lens, and mirror.

690. Diffraction Grating. In the diffraction grating a large number of slits are ruled on a glass surface.[1] There may be as many as a thousand to the millimeter, equally spaced. When light from a source slit is sent through such a grating the effect is much more striking than in Young's original experiment with two slits, and this for three reasons:

(1) The small interval (d) between the slits means a much larger interval between the different orders and much greater spread of the spectrum within any order.

(2) The large number of slits results in a much greater intensity.

(3) The large number of slits results in destructive interference at all points except those ($L' L''$, Fig. 1) where the interference is *exactly* constructive, giving (with monochromatic light) narrow diffraction lines instead of broad fringes. Thus the grating gives greater resolution (Fig. 4).



(*a*) (*b*)

FIG. 4. Interference pattern with slits $\frac{1}{2}$ mm apart. (*a*) with two slits; (*b*) with ten slits giving sharper lines. (Monochromatic light.)

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[1] This is the *transmission grating*, for use with transmitted light. In the *reflection grating*, used in spectroscopy, the grating is made by ruling parallel lines on a mirror surface.

A parallel beam of light strikes the grating in Fig. 5; light through each slit (a, b, c, etc.) is in phase, and hence (irrespective of wave-length) AB is a wave front. This gives the zero-order beam, transmitted without deviation. Now, considering light of a definite wave-length, if aa' is this wave-length AC is also a wave front. For the retardation bb' is exactly 2 wave-lengths, cc' is exactly 3, and so on, and all phases along AC are alike. This is the first-order diffracted beam. But if aa'' is exactly 2 wave-lengths AD is also a wave front. (The retardation bb'' from the next slit is exactly 4, from the next exactly 6 wave-lengths, and so on.) This is the second-order diffracted beam. So, because of constructive interference of the wavelets along these wave fronts, there result diffracted beams of first, second, and higher order emerging at definite angles from the grating.

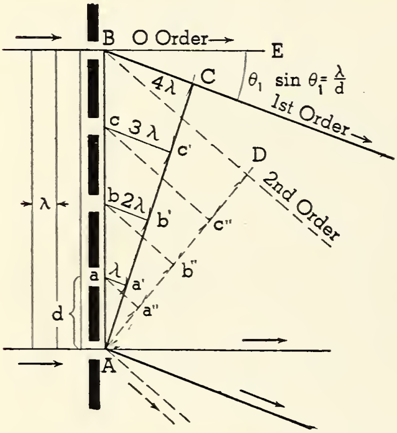

FIG. 5. Theory of diffraction grating.

With only two slits the angles of deviation (CAB for the first order) would not be definite, because even if aa' were only approximately a wave-length there would be approximate agreement in phase of the two wavelets. But here the retardation from the second slit beyond A must be 2 wave-lengths, from the third slit 3, from the hundredth slit 100 wave-lengths, from the ten-thousandth slit 10,000 wave-lengths; and so in a large grating even a small deviation from the correct angle cannot be tolerated. *The resolution increases with the number of slits.*

Angle EBC = angle CAB; this is the angle of deviation of the first-order beam (θ_1). Representing the slit interval by d, we have the relation:

$$\sin\theta_1 = \frac{\lambda}{d},$$

or

$$\lambda = d\sin\theta_1.$$

For the deviation (θ_2) of the second-order beam ($aa'' = 2\,\lambda$) we have

$2\lambda = d \sin \theta_2$; or in general for the nth order:

$$n\lambda = d \sin \theta_n, \tag{7}$$

in agreement with our earlier derivation in §687.

If the light falling on the grating consists of several colors, the red is deviated most, the short-waved violet least. Each order of diffraction gives a spectrum, and the diffraction grating may be used instead of a prism in the spectrometer. Since the light is divided among the several orders, grating spectra are usually less intense than those obtained with prisms. But the grating has several advantages. The spread of the spectrum (dispersion) in the grating spectrograph is usually larger, and the resolution usually greater; moreover, with the grating the wave-lengths are determined directly from the angle of deviation (eq. 7). Both λ and d can be determined with great accuracy, and spectrum wave-lengths are among the most accurately measured quantities in physics.

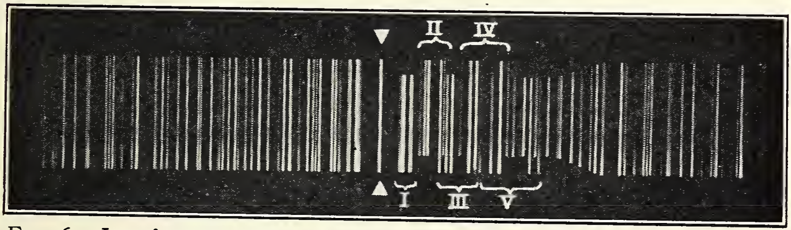

FIG. 6. Interference pattern. Mercury-vapor lamp through grating with 25 lines to the millimeter. Lines have been blocked out to show the early orders.

Figure 6, taken with a very coarse grating, shows many overlapping orders of the spectrum of mercury (yellow, green, blue, violet, ultra-violet lines). Observe that the spread of the spectrum increases with increasing order. The ordinary grating with very much smaller grating space gives much greater dispersion and only a few successive orders.

691. Newton's Rings. Newton himself discovered one example of interference but failed to give it the correct interpretation. Let us take two glass plates, one flat, the other very slightly convex, and place one on the other; the surfaces are in contact at the center and then become gradually farther and farther apart. We illuminate the plates from above with the yellow light from a sodium flame and view the reflection. The whole plate is covered with concentric circles; a large dark spot at the center, surrounded by a broad ring of light, then a dark ring and a light one, and so on, possibly hundreds of rings getting closer and closer together until finally they become indistinguishable to the naked eye (Fig. 8*a*).

These rings are due to the interference of light reflected from the two neighboring surfaces. When the surfaces are $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, etc., wave-lengths apart we get light.[1] (Compare closed organ pipes.) On first consideration one would probably expect these particular intervals to produce destructive interference. When the plates are separated by $\frac{1}{4}$ wave-length the beam reflected from the lower surface travels (down and back) $\frac{1}{2}$ wave-length farther than that reflected from the other. One might expect "hills" of the one to coincide with "valleys" of the other and give destructive interference. But it must be remembered that one reflection takes place from a lighter medium (as the light goes from glass to air), the other from a denser (from air into glass). In one case there is a change of phase in the reflected beam, in the other not. Hence a half-wave retardation (or $\frac{3}{2}$, $\frac{5}{2}$, etc., wave-lengths) is needed to bring the two waves into constructive interference.

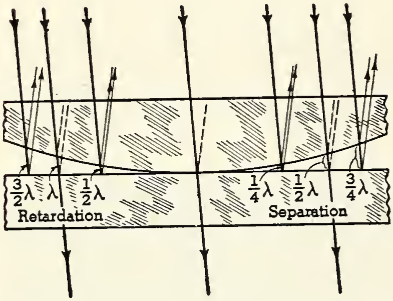

FIG. 7. Theory of Newton's rings. Separation between surfaces indicated at right, retardation (twice the separation) at left. Light is transmitted when the reflection is destructive (represented by broken lines).

If two plane glass surfaces are used and one side wedged up slightly by a piece of tissue paper so that the separation is proportional to the distance we get straight interference bands, equally spaced...This interference phenomenon is used in testing plane surfaces.

Newton's rings can also be observed with white light — a dark spot at the center is bordered by a ring which begins as violet at its inner edge (where the interval equals $\frac{1}{4}\ \lambda_{\text{violet}}$) and send as red on its outer edge, and around this are other rings first of purples, then of more and more complex color mixtures until soon the color mixture has become so complex as to appear as white light and the pattern is no further perceptible.

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[1] This applies to light falling perpendicular to the surface.

692. Colors of Thin Films. In Newton's rings the reflections are from the two surfaces of an air film between glass. Often mica includes air films between the layers which show such interference colors. Or the mica itself, in very thin sheets, only one or two wave-lengths thick, shows similar effects; here the thin film is a denser medium in air. . . The color of an oil film on water is another example of interference of thin films. . . But the best-known example is the soap bubble. Very thin soap films reflect little light; slightly thicker ones reflect violet or blue, still thicker, yellow or red. When the thickness is much greater the colors become confused and the reflected light is white.

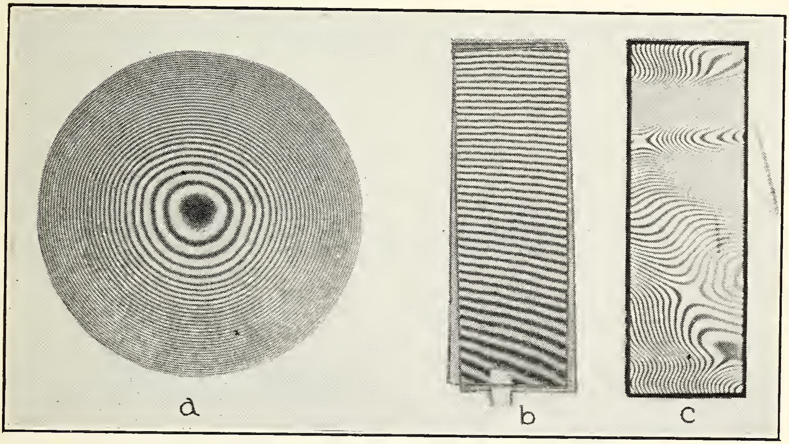


FIG. 8. Interference in films. (*a*) Newton's rings. (*b*) Wedge-shape film between glass plates. (*c*) Interference between opposite sides of microscope slide giving surface contours; relative retardation about 5000 wave-lengths.

693. The Interferometer. The interferometer is a device for measuring wave-lengths with greatest precision and for analyzing the finest structure in small portions of the spectrum. In its simplest form (the *etalon*) it is a plate of glass with exactly parallel sides. This plate is thousands of wave-lengths thick but in principle it is simply " a very thick thin film." And *Michelson's interferometer* is the same in principle. *B* and *C* are silvered mirrors; *A* is a glass plate with a thinly silvered semi-transparent surface. The eye looking into *A* sees *C* and the reflected image of *B* as two parallel surfaces. Light from the source reaches *A* and is partially reflected to *C*, partially transmitted to *B*; at *B* and *C*, the light is reflected, and the eye sees the interference pattern due to the path difference of the two beams. Mirror *C* is movable, and for a strictly monochromatic source the alternations of light and dark can be followed until the path difference (the " thin " film) becomes many centimeters (millions of wave-lengths). In this way the wave-lengths of a spectrum line can be compared (in favorable cases) to one part in several million, more accurately than the platinum-iridium standard of length can be measured. Indeed the wave-length of a certain very sharp line in the cadmium spectrum has been adopted by the International Commission of Weights and Measures as an alternative standard of length.

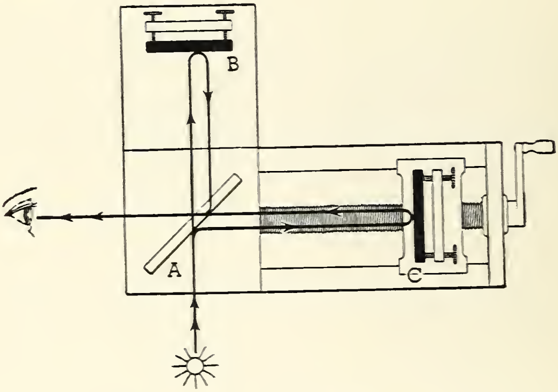


FIG. 9. Michelson interferometer.

The light beams in this interferometer are mutually perpendicular. Using such an instrument floating on a marble slab in a mercury well, Michelson and Morley performed their famous " ether-drift experiment," measuring the relative velocities of light in different directions in the " ether," and demonstrated that the observed velocity of light was a constant, independent of the motion of the observer (§§574, 615).

694. Interference Patterns of X-Rays. A very important illustration of interference by reflection from parallel planes is found in x-rays. X-ray wave-lengths are thousands of times shorter than those of visible light; the parallel planes used are the successive layers of atoms in a crystal.

The regular arrangement of atoms in a crystal places them in parallel planes. Such parallel sets of planes occur with many different orientations. There are horizontal planes of atoms and vertical planes of atoms and many kinds of diagonal planes. (Compare the rows in a well-planted cornfield.) Figure 10*b* shows the arrangement of atoms in a simple cubic crystal (for example, rock salt). One set of these parallel planes is indicated; the x-rays pass through thousands of these parallel layers, and a little radiation is reflected from each. All these reflections will interfere constructively if the relative retardation of the rays from each successive plane is exactly equal to the wave-length. It is the reflection from the particular planes shown which accounts for the uppermost spot of the pattern. Each of the other spots is due to reflection of x-rays of appropriate wave-length from some other set of parallel planes. This is x-ray diffraction. It is quite analogous to interference in a thin film; since the crystal planes are only a few angstroms apart constructive interference now occurs for x-rays and not for visible light.

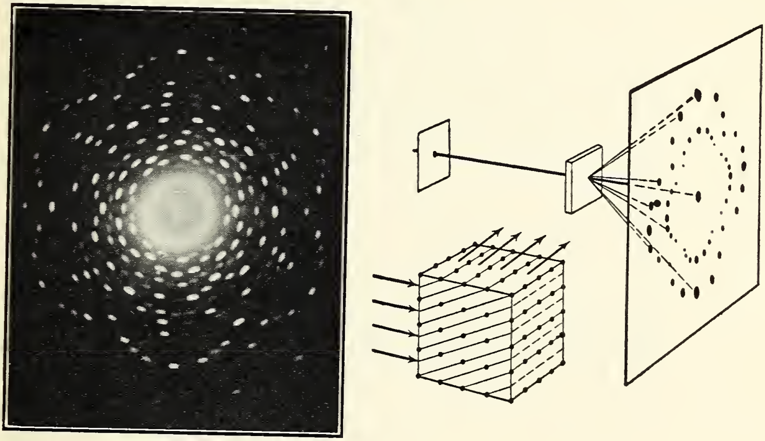

(*a*) (*b*)

FIG. 10. X-ray interference patterns. (*a*) X-rays scattered by quartz crystal. (*b*) Formation of spots by rock salt.

X-ray diffraction was discovered in 1912. The phenomenon was immediately accepted as showing that x-rays were electromagnetic wave motion. The wave-length of the rays commonly employed is less than 1 angstrom.

X-ray diffraction has two important aspects: it is a most important tool in the investigation of crystal structure both in pure science and in industry; by x-ray diffraction x-ray wave-lengths are determined, and these in turn are important in the study of atomic structure.

695. Interference Patterns of Material Particles. The fast-moving electrons in cathode rays can penetrate thin layers of matter, such as thin metal foil or a thin sheet of aluminum. Stimulated by new theories concerning the nature of matter, physicists have studied the scattering of the cathode rays after passing through a thin crystalline plate. Strangely enough, it was found that these were diffracted, giving definite interference patterns very much like x-rays! Somewhat similar results were observed when atoms were reflected from crystals. These discoveries gave convincing support to the New (Wave) Mechanics according to which the motion of every kind of particle — electron, atom, molecule, and even larger particles — is described by the interference of waves (Huygens' Principle). This Wave Mechanics is not to be explained in an elementary physics course. Suffice it to say that these waves, introduced to describe the motion of material particles, are not electromagnetic waves (like light and x-rays and gamma rays).

696. Diffraction. Since light is a wave motion there must of necessity be more or less straggling of the light over into the shadow. Shadow edges cannot be perfectly sharp. Observe the structure in the shadows of the straight edges in Fig. 11. Light penetrates a short distance into the shadow, and beyond the shadow there are several light and dark fringes due to interference of the wavelets at the edge of the beam. When a small circular disc (Fig. 11) is illuminated from a point source of light there is a light spot at the exact center of the shadow; light diffracted around the edge reaches this point all in phase. Light diffracted about the two sides of a needle produces interference fringes, widest where the needle is narrowest. Light passing through a small hole spreads out by diffraction. The hole of the pinhole camera must not be made too small. If the diameter could be made smaller than the wave-length of light the beam would emerge in all directions like a single Huygens wavelet.

There is no sharp distinction between the phenomena of interference and diffraction. Young's experiment is as much an example of diffraction as of interference, and in Fig. 11 the fringes are due to constructive or destructive interferences of many Huygens wavelets.

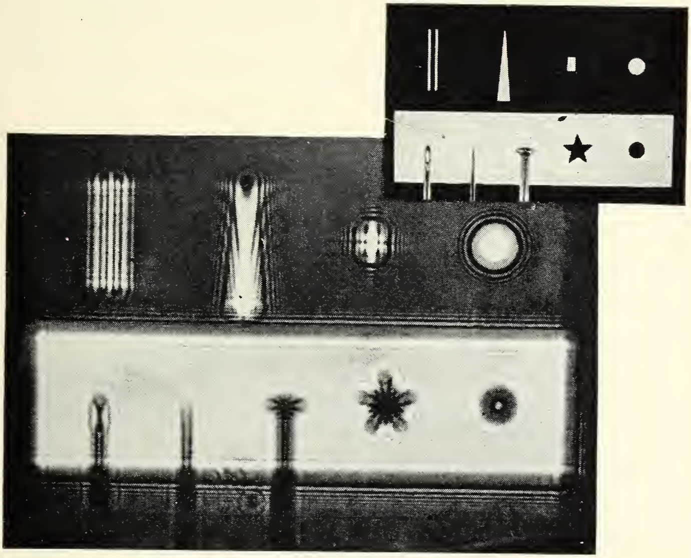


FIG. 11. Diffraction. Shadow picture of objects shown in insert; point source of light about 1 meter away.

Experiment. Cover one end of a long mailing tube (more than a meter long) with tin foil in which you have made a *very small* hole with the point of a needle. Cut a slot at mid-length in the tube, and insert here a pin, screw, or other small object. Illuminate the hole with a 6-volt lamp and observe with a magnifying glass the diffraction in the shadow at the other end. If a film is placed at this end a diffraction picture similar to Fig. 11 will be obtained.

697.* Resolving Power of a Lens. It will now become apparent why the resolving power of a lens depends on the ratio of its radius to the wave-length (§667). For each point in the object there is a minute diffraction pattern in the image, a " circle of confusion," and two points so close together that these circles of confusion seriously overlap cannot be distinguished. At the exact conjugate focus, light from all parts of the lens arrives exactly in phase, but a little to one side the phases are still nearly alike. A little farther away, where the light coming from the upper half of the lens is on the average one-half wave-length out of phase with that from the lower half, there is destructive interference and darkness. This can be taken as the limit of the "circle of confusion." The circle of confusion is, of course, enormously exaggerated in Fig. 12.

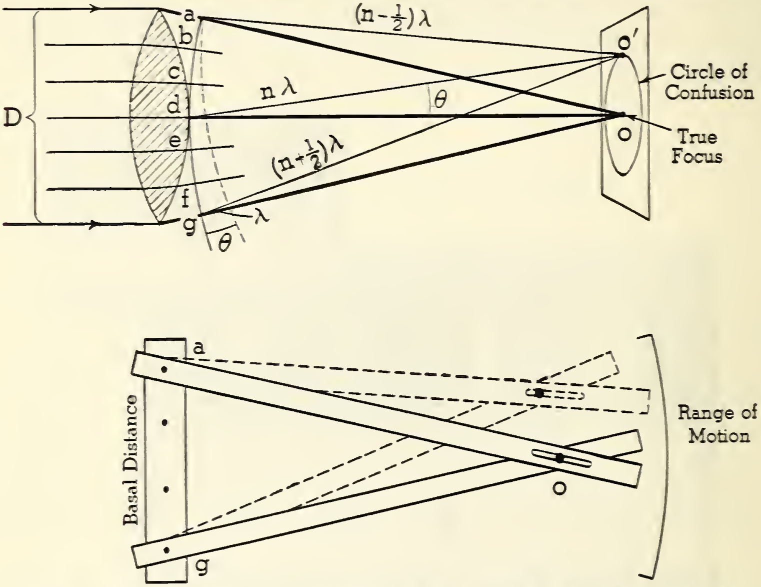
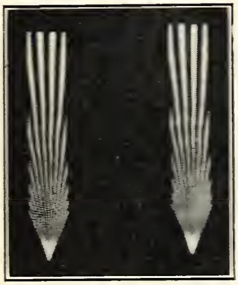


FIG. 12. *Above:* Circle of confusion of lens. *Below:* Model to illustrate resolution of lens.



(*a*) (*b*)

FIG. 13. Resolution: (*a*) taken with *f*-16 stop; (*b*) taken with 1-mm (approx. *f*-200) stop. Observe the "fuzziness" of (*b*).

We can say (approximately at least) that destructive interference will occur at o' when the distance from the upper edge of the wave front is $\frac{1}{2}\lambda$ shorter than from the center. Then the ray from *a* will cancel that from *d*, one from *b* will cancel *e*, and so on. (This is not exactly true for a spherical lens because of the different areas of the successive strips.) Hence from the figure the radius of the circle of confusion subtends an angle of approximately λ/D radians.

There is a simple analogy in mechanics:

Suppose *ao* and *go* to represent two sticks fastened together at *o* and to a base at *a* and *g*. The structure will not be perfectly rigid because the one stick can be slightly compressed, the other stretched. But the rigidity will be the greater, the greater the basal distance *ag*. The effect can be demonstrated by two sticks with a slot at *o* giving a certain freedom of motion. The length of the slot corresponds to λ.

QUESTIONS

1. Why is it not possible to observe interference phenomena between two lamp filaments placed ½ mm apart? Between two slits placed 2 in. apart?

2. Name several ways of getting two closely adjacent, coherent light sources.

3. What happens after light from a point source passes through a single very narrow slit; through two such slits ⅕ mm apart; through a thousand such equally spaced slits? Compare the distance between interference fringes if the slit interval is doubled.

4. Compare the lines in the fifth order of Fig. 6 with Fig. 52–2. The sequence of the lines is reversed. Why? (The line λ 3150 is absorbed by the glass of the grating.)

5. Using white light, only the first few of Newton's rings are seen and these are colored. Explain. What will be the color of the inner and outer edges of the first ring?

6. Two flat glass plates are placed on one another, in contact at one edge, and at the other separated by a piece of paper $\frac{1}{10}$ mm thick. How will " Newton's rings " now appear?

7. With sunlight one gets only a very few Newton rings because: (*a*) the different-ordered spectra overlap; (*b*) the light is too intense; (*c*) the rings get too close together.

8. For measuring the wave-length of a spectrum line a grating rather than a single pair of slits should be used because it gives: (*a*) more light; (*b*) more spread of the spectrum; (*c*) sharper image.

Vocabulary: Coherent beams, diffraction grating, first- (second-, etc.) order image, Newton's rings, interferometer.

PROBLEMS

1. Find the distances between images when sodium light from a distant slit, passing through two slits 1 mm apart, falls on a screen 1 meter away.

2. A grating has 600 lines to the millimeter. What is the angle of deviation in the first, second order (red light)?

3. A window is boarded over with 6-in. planks, with small gaps between boards. A whistle, pitch 6600 vibrations per sec. (wave-length 2 in.), is sounded some distance in front of the window. What will be the direction of the five sound beams (zero, 1st and 2nd order) that enter the room?

4. In Fig. 26–5*b* the regions of constructive interference are photographed black. Indicate the different orders of diffraction. Show by measurement that points in the third-order beam are 3 wave-lengths further from one prong than the other.

5. With a wave-length of 5000 angstroms, how many orders of interference fringes are theoretically possible with two slits 0.01 mm apart?

6. In question 6, how many interference bands will appear across the plate if sodium light is used? (Wave-length approximately 6000 angstroms.)

7. What is the wave-length of sodium light in a soap film (index of refraction 1.33)? Will sodium light be (*a*) reflected or (*b*) transmitted through a soap film 9000 angstroms thick? ($\lambda = 5890$ and 5896 in air.)

8. What different colors are reflected from a thin air film 0.0005 mm thick?

9. Using the sagitta formula, prove that the interval between successive Newton rings varies inversely as the square root of the distance from the center.

CHAPTER 54

POLARIZATION

698. The phenomena of the last chapter are our proof that light is a wave motion; this chapter will prove that the wave is transverse. A transverse wave, unlike a longitudinal one, shows the property of *polarization.* If a long vertical rope is shaken the displacement may be in the east-west plane or in the north-south plane (or of course may have a component in each plane). If it is shaken in random manner it will move "every which way." If, however, we take pains to make it vibrate in a definite plane, for instance north and south, the wave is said to be *plane polarized.*[1] The phenomenon of polarization does not occur in longitudinal waves since for such waves only a single direction of vibration is possible — the direction of propagation. *The fact that light can be polarized proves that its wave motion is transverse.* The electric and magnetic fields are each perpendicular to the direction of propagation.

699. Polarization of Light by Selective Absorption. In any source of light we conceive of the electrons as vibrating in every direction; hence the emitted light beam is not polarized. There are three methods by which the beam can be polarized: by polarized absorption, by reflection, or by double refraction.

Tourmaline is a semi-transparent crystal with the rather unusual property that it transmits light vibrating parallel to the "principal axis" of the crystal and absorbs light vibrating perpendicularly thereto.[2] Consequently if unpolarized light is sent through a tourmaline crystal half of it is absorbed; what emerges is polarized parallel to the principal axis.

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[1] A different kind of regular motion would be to "turn the rope" so that each point in it describes a circle. Such motion is said to be circularly polarized. Such vibration is also found in light; it will not be discussed here, however.

[2] The properties of a simple cubic crystal (such as rock salt represented in Fig. 53–10) are the same in each direction. The crystals which will be mentioned in this chapter appear to be compressed or elongated or in some other way to have properties along one direction different from those in other directions. This unique direction is called the principal axis.

In speaking of the direction of vibration of the electromagnetic wave we refer to the electric, not the magnetic, field.

Recently large sheets of polarizing material ("polaroid") have become available. In a base of cellulose acetate (resembling cellophane) great numbers of small needle-like crystals are embedded, all lying parallel. The crystals absorb one component of polarization, comparatively little of the other. The sheets thus transmit polarized light. They can be made in any size and will undoubtedly result in many practical applications of polarized light.

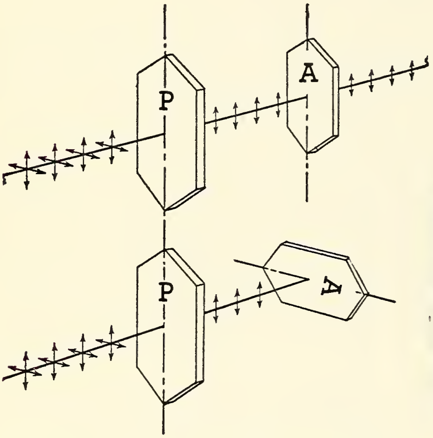

FIG. 1. Polarized transmission. *P:* polarizer; *A:* analyzer.

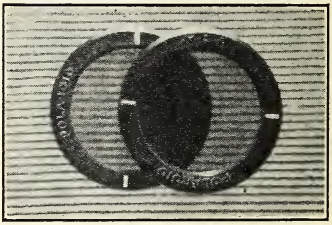

FIG. 2. Polaroid pair. Lower plate transmits vertical, upper plate horizontal vibration.

Experiment. Take two pieces of tourmaline or two sheets of polaroid, place their principal axes parallel, and observe that light is transmitted through the pair (Fig. 1). Place their axes mutually perpendicular, and observe that the pair is now opaque. What light the first crystal lets through, the second blocks out. The first crystal polarizes the light; we analyze the polarization with the second.

700. Polarization by Reflection. "Dig-in component goes in." Let us lay a piece of glass on the table and illuminate it by an oblique ray of light. Most of the light is transmitted, some reflected. When we analyze the reflected light with a piece of tourmaline crystal or a sheet of polaroid we find that it is polarized. When the polarizing axis of the analyzer is horizontal (parallel to the glass surface) the reflected light is transmitted; it is blocked when the axis is in the vertical plane. The unpolarized light of the incident beam can be divided into two components — one vibration in a horizontal, the other in a vertical plane. These are represented by dots and dashes in Fig. 3. (The dash represents vibration in the plane of the paper, the dot perpendicular thereto.) One vibration (•) is exactly parallel to the surface, the other more or less " digs in " to the surface. At the surface only the parallel component is reflected; *the " dig-in " component goes in.* The reflected light is polarized parallel to the surface.

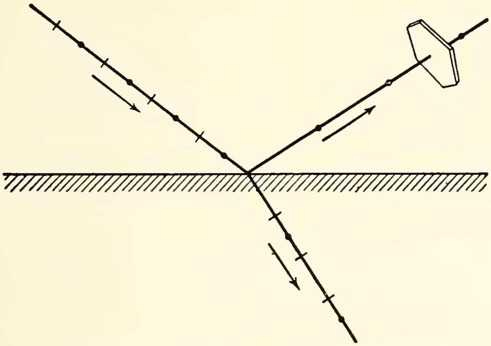

FIG. 3. Polarization by reflection.

The reflected beam is completely polarized — none of the " dig-in " component is reflected — at only one certain angle of incidence, the angle of complete polarization. (This angle is that for which the reflected ray and refracted ray move off in perpendicular directions as in the figure.) As indicated in the figure (by the excess of dashes) the transmitted beam is partly polarized.

Such polarized reflection occurs at the surface of any transparent medium. It does not occur in reflection from a silver mirror — obviously not, since nearly all the light is reflected.

701. Double Refraction. Calcite crystals are rhombohedrons; imagine distorting a simple cubic crystal (like rock salt) by pressing together on two opposite corners (a, b) until the cube has been crushed into the shape of Fig. 4; we shall then have the calcite structure. The principal axis passes between these two opposite corners; along this direction the atoms are more closely spaced than in perpendicular directions. Light travels through the crystal more slowly when the vibration is perpendicular to this axis than when it is parallel thereto. Hence calcite has two indices of refraction: 1.66 (like flint glass) for the perpendicular (⊥) vibration, and 1.49 (like crown glass) for the parallel (||) vibration. Consequently, an unpolarized beam passing into the crystal will be broken into two components, polarized one perpendicular and one parallel to the principal axis, and these components will be refracted by different amounts. This is double refraction.

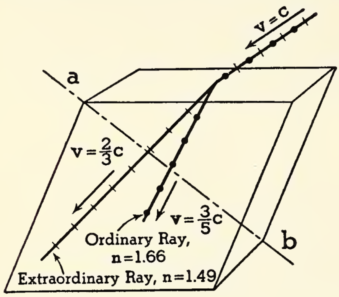

FIG. 4. Double refraction in calcite.

If the incident light ray in Fig. 4 is gradually rotated around to the left the ⊥ component will continue vibrating perpendicularly to the principal axis but the || component can no longer be exactly parallel to the axis; indeed, when the direction of propagation is along the line *ab* both beams vibrate perpendicularly to the axis. Thus the velocity of the first component (the " ordinary ray ") is the same for any direction of propagation, but the velocity of the other (" extraordinary ray ") depends on direction, and the index of refraction varies from 1.49 when the propagation is across the axis to 1.66 when along the axis. The ordinary ray obeys the ordinary law of refraction (Snell's Law), but the extraordinary ray does not. When the angle of incidence

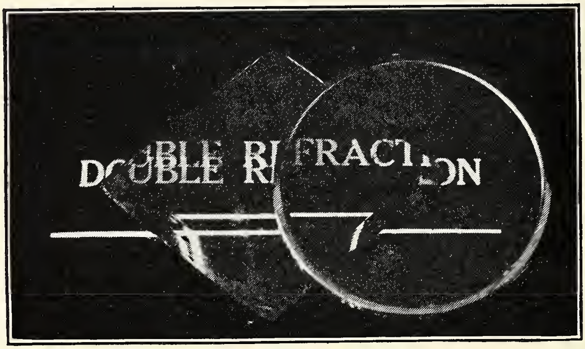

FIG. 5. Double refraction in calcite crystal. (Polaroid at right is set to eliminate extraordinary ray.)

is zero the ordinary ray enters the crystal without deviation but the extraordinary ray is even then deviated! Indeed, owing to the different velocities in different directions the wave front of the extraordinary ray is an ellipsoid instead of a spherical surface.

702. Special Polarizing Prisms. In the Wollaston prism two pieces of calcite are cemented together. In the one the principal axis is horizontal, in the other vertical, as represented by the dots and lines in the figure. When light vibrating in the plane of the paper passes through this Wollaston prism from left to right, it passes from what is (for it) a " denser " to a " rarer " medium. (The velocity increases as the ray changes from " ordinary " to " extraordinary.") Hence at the inner face this ray is bent away from the normal. The situation is just reversed for a ray polarized perpendicular to the page. The Wollaston prism resolves an unpolarized light beam into two oppositely polarized beams and these leave the prism in different directions.

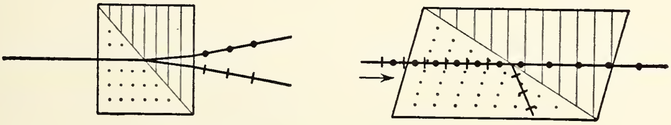

Fig. 6. Wollaston and Nicol prisms.

In the *Nicol prism* the beam strikes the inner face at an angle which for the one beam is greater than its critical angle; this beam is totally reflected. Hence the transmitted beam is completely polarized. Nicol prisms are generally used as polarizers and as analyzers in polariscopes.

703. Rotation of Plane of Polarization. As a beam of polarized light passes through crystalline quartz in the direction of the principal axis the direction of the vibration is gradually rotated. A piece of quartz 4 millimeters thick rotates the plane of polarization of yellow light through 90°. (The rotatory power depends on the wave-length — it is twice as much for violet.) Suppose that such a prism is placed between the two crossed tourmaline crystals in Fig. 1 (lower). The direction of vibration for yellow light will be rotated by 90°, the vertical vibration will be changed to a horizontal vibration as the light passes through the crystal, and the analyzer will now pass this yellow light. Red is rotated less, blue and violet more, and so these colors are more or less blocked by the analyzer.

Certain crystals and many liquids have this property of rotating the plane of polarization. It is of considerable importance in chemical analysis, particularly in the analysis of sugars, some of which rotate the direction of polarization to the right, others to the left.

The phenomena described in this chapter are the simplest out of a great variety of polarization effects. The blue light of the sky, scattered by the air, is polarized. . . Glass under strain acquires a principal axis and polarizes light; this gives an optical means of detecting such strains. . . There are certain liquids with long-chain molecules which line up to give a principal axis and polarize light (" liquid crystals "). . . Many crystals with complicated structure show more complicated effects, often with beautiful colored patterns. These effects are important to the mineralogist and the crystallographer.

The simple effects which have been described indicate the fundamental nature of polarization and show that the light vibration is transverse. This is in agreement with Maxwell's interpretation of light as an electromagnetic wave. Electromagnetic waves can be propagated only in a direction perpendicular to the lines of force.

QUESTIONS

1. Is light polarized when reflected by the moving-picture screen, by a lake in the afternoon, by a silvered mirror? When scattered by the atmosphere, emitted from a lamp filament, emitted as phosphorescent light?

2. Describe several methods of producing polarized light.

3. It has been proposed to eliminate glare in the eyes of the driver by placing polaroid on the automobile windshield. Is the reflection from the pavement polarized? Explain how the problem of blinding headlights could be solved by using polaroid on both headlights and on windshield.

4. The cause of double refraction in a crystal is that the direction of vibration in the crystal affects: (*a*) the velocity of propagation; (*b*) the absorption; (*c*) the reflection.

5. In a Nicol prism half of the light is suppressed by: (*a*) absorption; (*b*) refraction; (*c*) reflection.

Vocabulary: Polarization, principal axis (of crystal), double refraction, Nicol (Wollaston) prism, rotation of polarization.

CHAPTER 55

EMISSION AND ABSORPTION OF LIGHT

Young revived the wave theory of light in 1801. For a hundred years it was unquestioned. Indeed, when Maxwell interpreted light as an electromagnetic wave the position of the wave theory seemed impregnable. But all this dealt with the propagation of light; nothing was known of how light was emitted by one atom or how it was absorbed by another. To be sure, one could think of the light as simply the sloughed-off field from an electron vibrating in the atom — the atom as a minute radio station, to use the language of today — but this led nowhere: the spectrum did not behave as it should. Then in 1900 Max Planck proposed the Quantum Theory to explain the emission of light from incandescent solids. At first this seemed to threaten the wave theory; it appeared simply a return to Newton's corpuscles. But out of it has arisen a wave theory of propagation, not only of light but of every material particle!

704. Light from Incandescent Solids and Liquids. We must discuss thermal radiation once more. (The student should read again §§254–258.) A hot body emits light; it first is a dull red but as the temperature is raised it becomes whiter and whiter. Below about 500° C the emission is invisible radiant heat. Compare the color of a red-hot poker (at say 1000° C) and a carbon-filament lamp (at 1700° C) and a tungsten lamp (at 2500° C) and the sun (at 6000° C). The total emission increases as the fourth power of the temperature (§258), but also with rising temperature the spectral region of maximum brightness moves to shorter and shorter wave-lengths. Figure 1 describes the radiation from a " black body " (§257), but the radiation from other bodies behaves in much the same way. At 3000° the absolute total radiation is 2^4 (16) times greater than at 1500°. (This total radiation is given by the areas under the curves.) At 6000° the radiation is again increased 16-fold. Observe that as the temperature rises the curves move to the left (shorter wave-length, greater frequency). At tungsten-lamp temperature nearly 10 per cent of the emission is in the visible range; at carbon-lamp temperature the relative amount of visible light is less, mostly red and yellow; at the temperature of the sun the maximum of the curve lies in the visible range (5000 angstroms).

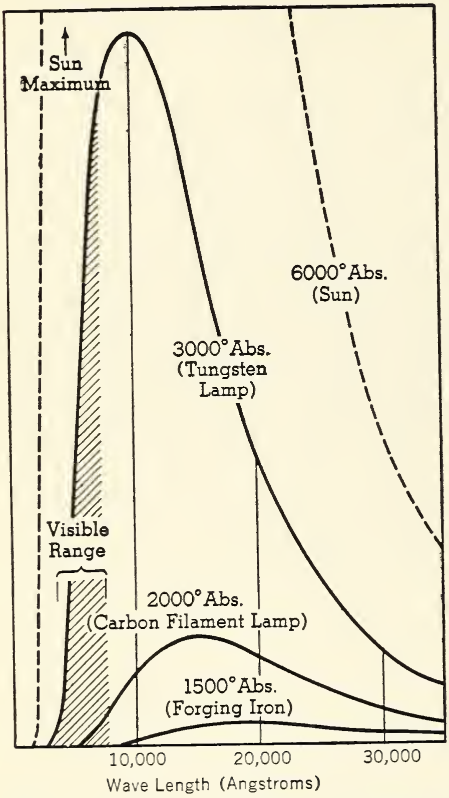

FIG. 1. Black-body radiation. Intensity of emission and wave-length.

705. Quantum of Radiant Energy. This seemed very strange to the theoretical physicists. Why were there no short waves — visible, ultra-violet, x-rays — at low temperatures? We find water waves breaking into shorter waves and then into ripples and finally into heat vibration. Why did not light break up into shorter and shorter wave-lengths? It was to explain this that Planck proposed his quantum hypothesis.

So far as it applies to light, the Quantum Theory may be stated as follows: When electromagnetic radiation is emitted or absorbed by an atom the energy is given to or taken from "the ether" only in prescribed amounts (quanta) which are proportional to the frequency (n). In c.g.s. units the proportionality constant (represented by h) is 6.62×10^{-27}.

Energy radiated or absorbed by one atom = hn.[1]

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[1] It is conventional to represent light frequency by the Greek letter ν (nu), and Planck's Law is generally expressed as $W = h\nu$ ("Energy equals aitch nu.") However, for consistency with our previous usage we shall continue to express frequency by n.

In terms of the new theory light is emitted in " bursts " or " chunks " or *quanta*; the higher the frequency (or shorter the wave-length) the bigger is the quantum. The high frequencies are not emitted from the incandescent solid simply because they demand too much energy; while waiting to accumulate enough energy for an ultra-violet quantum the body radiates several quanta of longer wave-lengths instead. The exact theory led to a complete explanation of the emission of the black bodies.[1]

706. Emission from Gases. Twelve years later the Quantum Theory led Bohr to his model of the atom (§381); this gave the first satisfactory explanation of spectrum lines. According to Bohr's picture the electron in a hydrogen atom is restricted to certain definite states of revolution, orbits with angular momentum equal to $h/2\pi$ or an exact multiple (Fig. 2). The energies of these states are easily computed and vary in quite a simple way.

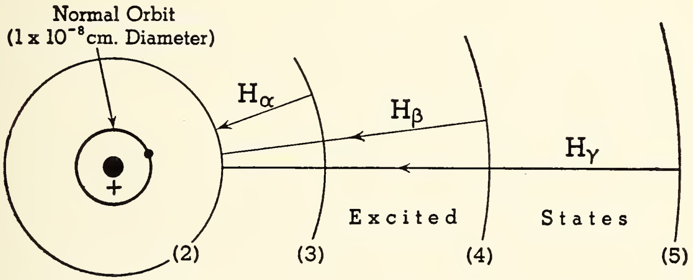

FIG. 2. Normal and excited orbits in hydrogen. (Bohr's model.)

The electron is normally in the innermost orbit. In order to excite light the electron must first be knocked into some outer state. Then as it falls back it emits a quantum of light the energy of which depends on the size of the jump. And according to the Quantum Theory the frequency is proportional to the energy (W):

$$n = \frac{W}{h}.$$

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[1] Here as elsewhere in this book there is no pretense of giving an exact account of the theories of modern physics. The student will find an exposition of the Quantum Theory in textbooks on modern physics.

So corresponding to each different electron jump there is a particular frequency, a particular spectrum line. (The figure shows the transitions for the Balmer series of hydrogen, shown in Fig. 52–2.)

This theory explained exactly the spectrum of hydrogen, and approximately the more complicated spectra of other elements — visible, infra-red, ultra-violet, x-ray.

707. Absorption and Fluorescence. When light passes through a gas certain colors are absorbed. According to the Bohr Theory the energy is absorbed, quantum by quantum, and in the process the electrons in the gas atoms are changed from the state of lowest energy (electron in innermost orbit) to some higher energy state. This is just the opposite of emission, and the frequencies absorbed when the electron leaves the lowest energy state are those which it emits when entering it. But in emission the electron can fall back into *any* state; in absorption it can only leave the normal, lowest energy state (where it is). This explains why only certain emission lines are absorption lines (§674).

In a solid or liquid the orbital motions of electrons are badly disturbed by neighboring atoms. Nevertheless when light is absorbed in a solid or liquid substance some similar process must occur. A light quantum is absorbed and the potential energy of some electron is increased as it is raised to an outer state. Only now the states are not well defined, and there are for solids and liquids absorption *bands* instead of definite absorption *lines*.

Usually after absorption of light the excited electron gives up its energy to the neighboring atoms to be dissipated as heat. It may, however, reradiate part of its energy as it falls back toward its original state. This produces fluorescence if the return is made immediately, phosphorescence if it is delayed. But since the return may be broken into several steps, the re-emitted quantum of energy will usually be smaller than the original. Hence, by Planck's Law the frequency is smaller, the emitted light displaced toward the red. This explains Stokes' Law of Fluorescence (§682).

708. Photoelectric Effect. One of the earliest confirmations of Planck's theory was found in the photoelectric effect. When light releases electrons from a metal surface their speed does not depend on the light intensity. It does depend on the frequency.

The greater the frequency of the light (the shorter the wave-length), the greater the velocity of the emitted electrons (§585). The light is absorbed quantum by quantum. No emission occurs until the quantum (hn) equals the energy necessary to evaporate an electron from the surface (energy of evaporation of the electron). For most metals ultra-violet light is required. According to the theory of Einstein, when the frequency exceeds this threshold value the excess energy appears as kinetic energy of the electron:

$$\tfrac{1}{2}\,mv^2 = hn - \text{Energy of evaporation.}$$

In a series of precise experiments Millikan confirmed this relation.

709. Chemical Action. Light often induces chemical change. Familiar examples are sunburn and the deterioration of chemicals exposed to sunlight. Another example is the photographic film. Light reduces the silver chloride or silver bromide in the emulsion to silver. The unstained film is sensitive to blue, or violet, or ultra violet light, but not to red and yellow. So for every photochemical process there is a limiting wave-length. The light quantum (hn) must be at least as great as the difference in energy of the initial and final chemical states. (See Fig. **52**–12.)

The most important photochemical action is the photosynthesis which occurs in leaves. Here the radiant energy from the sun is converted into chemical energy of the growing plant. Light is absorbed by the green chemical chlorophyll, changes carbon dioxide into starch, and so begins the long chain of reactions which results in the living tissue — ultimately results in you and me.

710. Light Waves and Photons. Have we proved too much? Light is a wave motion: this is proved by all the phenomena of diffraction and interference. Light occurs in quanta: this is proved by all the phenomena of emission and absorption. Light is emitted quantum by quantum from atoms in the sun; quantum by quantum it is absorbed by atoms or molecules on the earth. In emission and absorption light seems to act as a shower of particles — these "light particles" have been called *photons.* But the photons do not travel in straight lines; they seem never to have heard of Newton's First Law of Motion. Where they are to be found and where they are not found, this is determined by

Huygens' Principle; the light wave is the guiding principle for their motion. Perhaps this is the only meaning of a light wave — the guiding principle of a photon. Or perhaps a photon is only one aspect of a wave.

711. Wave Theory of Matter. When Planck proposed his Quantum Theory in 1900 it seemed to be a serious assault upon the wave theory. Probably the shades of Huygens and of Young frowned. Strangely enough the Quantum Theory led to an undreamed-of extension of the wave theory. We have said that photons do not travel in straight lines. Apparently nothing travels exactly in straight lines. Modern physics has shown that in passing through a thin film electrons can be diffracted like x-rays, and hydrogen atoms rebounding from a crystal show interference like light reflected from a diffraction grating (§695). Everything — electrons, atoms, stones — moves just as light moves, really as waves, only approximately, apparently, as rays. Nothing moves according to Newton's laws *exactly*; for everything there is more or less diffraction. Behind everything there is a fundamental Wave Mechanics.

But this carries us very far indeed from the Common Sense Physics with which we began. So short are the wave-lengths that, for bodies larger than electrons, atoms, and molecules, the diffraction is very small indeed. For all practical purposes baseballs and rifle bullets go in definite, more or less parabolic, paths today just as they did in Newton's time. The New Mechanics has revolutionized the picture of the atomic world. But it is not obviously important in everyday life. It is of interest only to physicists and chemists. And to those strange humans who have poetical, philosophical minds and dream dreams — like the Greeks who looked at the stars and asked (unanswerable question) what it all meant.

APPENDIX 1

PROPORTION

If 10 eggs cost 15 cents what will 12 eggs cost?

If 3 men can make 12 boxes in 2 hours, how many boxes can 2 men make in 4 hours?

If 3 men can build a shed in 4 days, how many days will it take 2 men to build it?

712. We must start our study of physics with a very humble branch of mathematics. The concept of proportion occurs not only throughout physics but with surprising frequency in all thinking. "Washington is to his country as a father is to his son." "The relation ('ratio') of a rolling stone to moss is the same as that of a footloose fellow to money." In a certain sense, every simile and metaphor is a proportion. But we are continually using proportion in a more exact sense. The bigger a box the more it will hold. Every line in a map should be in proportion to the original. We shouldn't make mountains out of molehills (either in maps or in life; we should have a proper sense of proportion). We draw a picture of a horse; the picture of the head should be to the horse's head as the picture of the leg is to its leg, as the picture of the tail is to its tail, etc. We are continually using proportion. And an exact understanding of proportion will carry us far into our study of physics.

What is a direct proportion? How is it expressed? How is it solved? What is joint proportion? quadratic proportion? inverse proportion?

DIRECT PROPORTION

If 10 eggs cost 15 cents how much will 12 eggs cost? 30 eggs? 33 eggs?

713. Ratios. By the ratio of A to B we mean simply their quotient. We can write it as $A : B$ or $\frac{A}{B}\cdot$ The ratio of 4 to 8 is $\frac{1}{2}$; the ratio 6 : 2 is the same as the ratio 9 : 3. The ratio of cost to eggs is 3 : 2. What is the ratio of the height of a telephone pole to the height of a man? the ratio of the circumference of a circle to its radius?

714. Direct Proportion. Now if A_1, A_2, A_3, etc., represent a number of quantities of a certain kind (as the number of cents) and if B_1, B_2, B_3, etc., represent corresponding quantities of another kind (as the number of eggs, A_1 being the cost of B_1 eggs), and if the ratio A_1/B_1 equals the ratio A_2/B_2 or A_3/B_3, etc., we say that the A's are proportional to the B's, and the equation

$$\frac{A_1}{B_1} = \frac{A_2}{B_2}$$

expresses the proportion. (Such an equation is usually read "A_1 is to B_1 as A_2 is to B_2.") It may, of course, be written: $A_1 : B_1 = A_2 : B_2$.

It is allowable to exchange means (or extremes) in a proportion. This proportion may be written as

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}.$$

Examples. If 2 eggs cost 4 cents and 3 eggs cost 6 cents and 100 eggs cost 200 cents, and so on, the cost (A) is proportional to the number of eggs (B) . . . The numbers in the first column of Table 51 are proportional to those in the third. . . We are all familiar with the problem " If George can build 5 boxes in 2 hours, how many boxes can he build in 6 hours?" Here the ratio of boxes to time is a *constant* (the number of boxes George can make in an hour).

Loosely speaking, we sometimes say that proportionality means that the first quantity increases " with " the other — exactly, this means that when the one quantity is doubled, tripled, multiplied by a hundred, the other is doubled, tripled, multiplied by a hundred. Are columns I and II or I and V proportional? Is a boy's height proportional to his age?

TABLE 51

I	II	III	IV	V	VI	VII
0	1000	0	Infinity	0	0	20
1	1001	5	12	2	100	20
2	1002	10	6	8	141	20
3	1003	15	4	18	173	20
4	1004	20	3	32	200	20

Now this concept of proportion, sometimes so simple that it seems to be intuitive and independent of formal arithmetic, was the foundation of the earliest physics. It requires no learning to know that, if it takes 3 men to lift a stone, it will take 6 men to lift one with twice the volume. Even a man in the Stone Age could understand that. And so we can see for ourselves many of the proportional relationships of physics. If a man walks at a constant rate, the distance he goes (s) will be proportional to the time (t) — the constant ratio $s : t$ (the number of miles per hour) we call his *speed*. . . Will the strength of a wire be proportional to its length or to its diameter or to its cross-sectional area? . . . Will the weight of a sphere be proportional to its radius, or to its surface area, or to its volume?

715. The Constant of Proportionality. Summarizing: if x and y are two variable quantities with corresponding values, then if the ratio $x/y = c$ (a constant) we say that x is proportional to y. The constant is called the *constant of proportionality*. We may also write this proportion as $x = cy$; this is in fact the way in which it will usually be written.[1] (It is best to read this: x is proportional to y.) So, instead of writing for the circumference (C) of a circle $C/r = 2\pi$, it is usually more convenient to write it as $C = 2\pi r$. 2π is the constant of proportionality. In this case the constant is evidently equal to the circumference of a circle with unit radius. (If $r = 1$, then $C = 2\pi$.)

The constant of proportionality can usually be interpreted and is often so important that it is given a name. In the egg problem $A/B = 1\frac{1}{2}$ cents per egg, and this is called the *price*. We have seen that the ratio of distance traversed to time required is called *speed*.

716. Solving the Proportion. If, in the proportion which may be written as

$$A_1/B_1 = A_2/B_2$$

or

$$A_1 : B_1 = A_2 : B_2$$

or

$$A/B = c$$

or

$$A = cB$$

or

$$A \propto B,$$

we are given the values of A_1, B_1, and A_2, we can evidently solve for B_2. The method in elementary school is to " multiply means (B_1 and A_2) and extremes (A_1 and B_2) " — but this is indeed no more than the following: Write the proportion as $A_1/B_1 = A_2/B_2$, clear of fractions, and solve.

An alternative method is to write the proportion as $A/B =$ Constant, to solve for the constant of proportionality from the known quantities A_1, B_1 and then apply this to find the unknown in $A_2/x =$ Constant.

Example. Let us solve the original problem: if 10 eggs cost 15 cents how much will 12 eggs cost? Cost (A) varies as (B).

1. First method:

$$15 : 10 = x : 12.$$

Multiplying means and extremes, $10\,x = 180$; $x = 18$ cents.
Or in the fractional form: $15/10 = x/12$; $x = 12 \times 15/10 = 18$ cents.

2. Second method: From $A = cB$, find the constant: $15 = c\,10$; $c = 1.5$ cents per egg.

$$\therefore \quad x = 1.5 \times 12 = 18 \text{ cents.}$$

---

[1] This proportion is also sometimes written as $x \propto y$; $\propto$ is the sign of proportionality.

Use each of these methods in the following simple examples:

1. If 3 liters of water weigh 3000 grams, what do 4 liters weigh?

2. If I can walk 10 miles in 3 hours, how far can I walk in 4 hours?

3. If 2 cu. ft. of water weigh 124.8 lb., what do 3 cu. ft. weigh?

4. If the pressure under water is 0.433 lb. per sq. in. at a depth of 1 ft., what will it be at a depth of 10 ft.?

5. If $A = cB$ and if A is 10 when B is 1.932, what will be the value of A when B is doubled?

6. If a train goes 19 miles in 23 min., how far will it go in three times that time? How far in 41 min.?

717. The Proportionality Constants in Right Triangles. In geometry it is shown that if any two triangles have equal angles (hence are similar triangles) the corresponding sides are proportional. That is, if in the one triangle two of the sides (A_1 and B_1) are in the ratio of 1 : 2, a similar ratio holds for A_2 and B_2 in the other:

$$\frac{A_1}{B_1} = \frac{A_2}{B_2}.$$

This is illustrated for right triangles in Fig. 1. In this triangle the side opposite the 30° angle is half as long as the hypotenuse. Of course the constant of proportionality would be different for another angle. One meets right triangles so frequently that this ratio (of opposite side to hypotenuse) has been found and tabulated for all angles. This ratio is called the *sine* of the angle. A few values are given in Table 52.

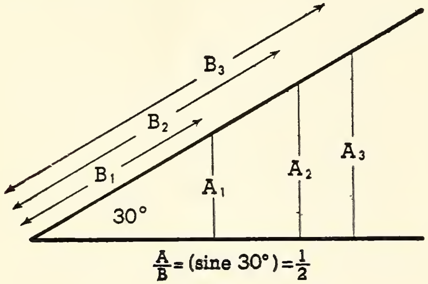


FIG. 1

The ratio of *adjacent* side to hypotenuse is of equal importance. It is called the cosine of the angle. What is the cosine of 30°? (Observe that sine A = cosine $[90° - A]$.) We shall find these ratios useful.

718. Joint Proportionality. A variable quantity A may depend on several independent quantities B, C, etc., in such a way that A is proportional to B as long as C is constant, and to C, when B is constant. Then, if B and C vary together, A is proportional to their product. Such joint proportionality is written as

$$A = cBC,$$

or $$A_1 : B_1C_1 = A_2 : B_2C_2.$$

TABLE 52

Ratios of Opposite (Sine) and Adjacent (Cosine) Sides to Hypotenuse

Angle°	Sine	Cosine	Angle°	Sine	Cosine
0	.000	1.000	46	.719	.695
2	.035	.999	48	.743	.669
4	.070	.998	50	.766	.643
6	.105	.995	52	.788	.616
8	.139	.990	54	.809	.588
10	.174	.985	56	.829	.559
12	.208	.978	57.3*	.842	.540
14	.242	.970	58	.848	.530
15	.259	.966	60	.866	.500
16	.276	.961	62	.883	.470
18	.309	.951	64	.899	.438
20	.342	.940	66	.914	.407
22	.375	.927	68	.927	.375
24	.407	.914	70	.940	.342
26	.438	.899	72	.951	.309
28	.470	.883	74	.961	.276
30	.500	.866	75	.966	.259
32	.530	.848	76	.970	.242
34	.559	.829	78	.978	.208
36	.588	.809	80	.985	.174
38	.616	.788	82	.990	.139
40	.643	.766	84	.995	.105
42	.669	.743	86	.998	.070
44	.695	.719	88	.999	.035
45	.707	.707	90	1.000	.000

* One radian.

Examples. 1. The areas (A) of triangles of constant bases (B) are proportional to the altitudes (H); given constant altitudes, the area is proportional to the base.

$$A = cBH. \quad (c \text{ here is } \tfrac{1}{2}.)$$

What will happen to the area of a triangle if both base and height are doubled?

2. The cost of running incandescent lamps of a given size is evidently proportional to (1) the number of lamps (N) and (2) to the time during which they are lighted (T).

$$\text{Cost} = cNT.$$

The constant here means the cost per lamp per hour. If it costs $1.00 to burn 5 lamps 3 hours, how much will it cost to burn 5 lamps 4 hours; 10 lamps 3 hours; 10 lamps 4 hours; 8 lamps 2 hours?

QUADRATIC PROPORTION

719. Two quantities A and B are often so related that when B varies from 1 to 2 to 3 to 4, A varies, not in these same ratios, but as 1 to 4 (i.e., 2^2) to 9 to 16. *A is proportional to the square of B.*

$$A = cB^2 \quad \text{or} \quad \frac{A_1}{A_2} = \frac{B_1{}^2}{B_2{}^2} \quad \text{or} \quad \frac{A_1}{A_2} = \left(\frac{B_1}{B_2}\right)^2.$$

Examples. 1. It will be found that in Table 51 the fifth column is proportional to the square of the first. Find another example in Table 51.

2. If the radius of a circle is 1 in., its area is 3.1416 sq. in.; if the radius is 2 in. the area is 12.5664 sq. in. (increased in the ratio 1 : 4); for the formula given in geometry is

$$A = \pi r^2 \quad \text{or} \quad \frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2. \quad \text{(See Fig. 3.)}$$

3. If it costs $1.00 to paint a square room with a 12-ft. side, it will cost 4 times as much to paint one with double the side. The cost (C) is proportional to the area, which equals the square of the side (L):

$$\frac{C_1}{C_2} = \left(\frac{L_1}{L_2}\right)^2.$$

How much would it cost to paint one with a 4-ft. side?

720. The Sagitta Formula. Consider, as another example, the way in which a ship sinks below the horizon. One mile away it is only 8 in. below the horizontal, 2 miles away it has sunk 4 times as far, 3 miles 9 times, 10 miles 100 times, 100 miles 10,000 times as far. Evidently the fall is proportional to the square of the distance.

Distance (y)	Fall below Horizon (s)	
(miles)	(inches)	(feet)
1	8	0.66
2	32	
3	72	
10		66
100		6600

We must give here the equation applicable to this horizon problem. Several elements of the circle derive their names from archery. There are the *arc* (*bow*) and the *chord* (cord) and the *sagitta* (arrow). The last (probably unfamiliar to the student) is the line drawn from the center of the arc to its chord. In Fig. 2, y is the half-chord, s the sagitta, r the radius of the circle. In the horizon problem y is the distance to the ship, s its drop below the horizon, and r is 4000 miles, radius of the earth. The relation between these quantities is (approximately)

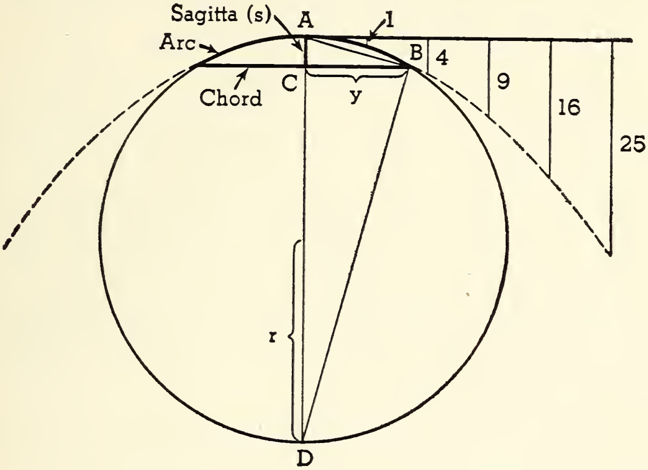


FIG. 2

$$s = \frac{y^2}{2r}.$$

Proof: Triangle ABC is similar to triangle BDC.[1] Hence

$$s : y = y : DC.$$

DC is approximately equal to the diameter; hence we have approximately

$$\frac{s}{y} = \frac{y}{2r}, \quad \text{or} \quad s = \frac{y^2}{2r}.$$

[1] Two right triangles are similar if it can be shown that an angle of one (other than the right angle) equals an angle of the other. . . Two angles are equal if the sides of one are perpendicular to the sides of the other.

$\angle ABC = \angle BDC$, since AB is perpendicular to BD and CB is perpendicular to CD. Therefore $\triangle ABC$ is similar to $\triangle BDC$.

This very nearly settles the question of similar triangles in so far as they occur in the text. Almost without exception these triangles are right triangles in which it can be shown that acute angles are equal, usually because their sides are, as here, mutually perpendicular.

There will be occasional use for this formula. As was seen in the derivation, the relation is only approximately true. The curve for which it is exactly true is the *parabola.* The circle, parabola, and ellipse are related curves and for very small arcs become indistinguishable.

Example. Using the sagitta formula, show that at 90 miles distance the drop in the horizon is approximately 1 mile.

721. Of course we may have cubic or higher proportion. The volume of a sphere is proportional to the cube of the radius. (The formula is $V = \frac{4}{3}\pi r^3$, constant of proportionality $\frac{4}{3}\pi$.) . . . The heat and light radiated from a carbon lamp filament is proportional to the fourth power of the temperature — when the temperature is doubled, 16 times as much energy is radiated in a given time.

Sometimes one quantity varies as the square root or cube root of another. Indeed, if $A \propto B^2$, $B \propto \sqrt{A}$. So the radius is proportional to the square root of the area of the circle; to make the area twice as large the radius must be made 1.41 times as large ($\sqrt{2} = 1.41$). Find two examples in Table 51.

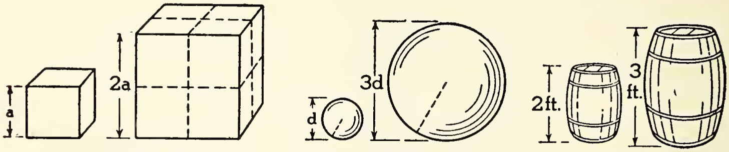
722. Proportion in Similar Figures. If r is the radius of a sphere, D its diameter, C its circumference, A_x the area of the cross-section, A_s the area of its surface, and V the volume, we have

$$\left.\begin{aligned} D &= 2r \\ C &= 2\pi r \end{aligned}\right\} \text{Simple proportion.}$$

$$\left.\begin{aligned} A_x &= \pi r^2 \\ A_s &= 4\pi r^2 \end{aligned}\right\} \text{Quadratic proportion.}$$

$$V = \tfrac{4}{3}\pi r^3 \quad \text{Cubic proportion.}$$

(These geometrical relations should be in the memory — they will be used frequently.) If the radius is doubled, the circumference is doubled, each surface is quadrupled, and the volume is 8 times greater.



Boxes

What is the Surface Area of Each Box?
What is the Volume of Each?

Spheres

Circumference $= 2\pi r$
Surface Area $= 4\pi r^2$
Volume $= \frac{4}{3}\pi r^3$

Barrels

If the Small Barrel will hold 80 Apples, how many will the Large One Hold?

FIG. 3

Such proportional relations hold for any kinds of *similar figures.* Among similar triangles, if one side is doubled, the other sides are doubled, the area is increased 4-fold. *Among similar figures lengths are always proportional to lengths, areas to (lengths)^2 and volumes to (lengths)^3.* For example, if two barrels are of similar proportions but one has half the height of the other, it will have quarter the surface and an eighth the volume. In a large number of *similar* ellipses (all, for instance, with major and minor axes of ratio of 3 : 1) the areas will be proportional to the square of the major axis (or of the minor axis).

One familiar illustration of this is in squares and cubes. A square foot contains 144 sq. in., a cubic foot, 1728 cu. in. How many cubic feet in a cubic yard? There are 10 mm in a centimeter; how many cubic millimeters in a cubic centimeter?

INVERSE PROPORTION

If 3 men can build a house in 4 days, 2 men can build it in how many days? This is an example of inverse proportion.

723. Reciprocals: The fraction $\frac{1}{2}$ is called the inverse or (more usually) the reciprocal of 2; $\frac{1}{3}$ is the reciprocal of 3; 0.1 is the reciprocal of 10; $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$; 1 per cent is the reciprocal of 100. We shall frequently have to do with such reciprocals. If we divide the distance a boy runs by the time, we get his speed, his distance per second: Speed $= \dfrac{\text{Distance}}{\text{Time}}$. For example, the speed of a sprinter is 10 yards per second. The reciprocal of this is the time he takes to go a yard: $\dfrac{\text{Time}}{\text{Distance}} = 0.1$ second per yard. . . Dividing the cost by the number of eggs gives the price (cents per egg); the reciprocal gives the eggs per cent.

724. Inverse Proportion. Frequently one quantity varies as the reciprocal of the other; the quantities are then *inversely proportional.* The quantities 12, 6, 4, 1 are inversely proportional to 1, 2, 3, 12.

If A is inversely proportional to B the proportion can be written

$$A = c\frac{1}{B} \quad \text{or} \quad \frac{A_1}{A_2} = \frac{1/B_1}{1/B_2};$$

or more simply as

$$AB = c \quad \text{or} \quad \frac{A_1}{A_2} = \frac{B_2}{B_1} \quad \text{or} \quad A_1 : A_2 = B_2 : B_1.$$

In an inverse proportion the product (not the ratio) *of the two quantities is constant.* The larger B, the smaller A; when B is doubled, A is halved; when B is quadrupled, A is quartered; if B increases as 3 to 7 then A decreases as 7 to 3.

Examples. The angles in Fig. 4 both cut equal arcs; the angles are then inversely proportional to the lengths of the sides (r_1, r_2). . . A rectangle is to have a given area; then the greater one side the shorter must be the other, for the product is to be constant. . . If triangles are to have a given area, the base must vary inversely as the altitude — if we double the one we must halve the other. . . The size of a piece of pie obviously varies inversely as the number of pieces into which the pie is cut. . . The number of apples which can be put in a barrel varies directly with the volume of the barrel and inversely as the size of the apple. If 300 apples of a given size can be placed in a given barrel, how many in a barrel half again as large? How many in the first barrel if the apples are half again as large? How many of the larger apples in the larger barrel?

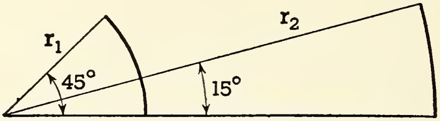


Fig. 4

QUESTIONS

1. Indicate the proportional relationships among the following quantities. (In some cases quantities are independent; in some they merely increase together without a strict proportionality; in some they are very nearly proportional and in others exactly so; indicate these relations.) (*a*) Number of houses and number of people in community; (*b*) number of doctors and number of deaths; (*c*) size of wheat crop and price per bushel; (*d*) size of wheat crop and price of gasoline; (*e*) cost of railway fare and distance; (*f*) price of automobiles and number of sales; (*g*) number of windows required for ventilation and size of the window; (*h*) cost of skyscraper and its height; (*i*) area of equilateral triangle and size of base; (*j*) age of boy and height; (*k*) boy's height and weight; (*l*) length of side and diagonal in a square; (*m*) diameter and radius of a circle.

2. An orange is 2 inches in diameter. Approximately what is its circumference, its surface area, and its volume? (Consider $\pi = 3$ approximately.)

3. Compare the sine and the cosine of 45°. Compare the sine of 30° with the cosine of 60°.

4. Find similar triangles (*a*) in Fig. 4, p. 66, (*b*) in Fig. 10, p. 69 and (*c*) in Fig. 3, p. 166. State why you regard the triangles as similar.

5. Does the word *rate* (as " interest rate," "death rate," "rate of covering distance ") imply a *ratio*? Explain.

6. *Memory:* What is the diameter of the earth? What is the sine of 30°? How many square inches in a square foot? How many cubic inches in a cubic foot? What is the reciprocal of zero?

Vocabulary: Ratio, direct (inverse, quadratic, joint) proportion, constant of proportionality, means, extremes, sine, cosine, reciprocal, sagitta.

PROBLEMS

1. When the sun is directly overhead, how long is the shadow of a 50-ft. pole which inclines 10° from the vertical? Also find the length of shadow for a 5-ft. pole and a 6-ft. pole.

2. A track extends toward the northwest. How far west and how far north is a train after traveling 10 miles?

3. If a 10-ft. pole leans against a building which it touches 8 ft. from the ground, what angle does it make with the ground?

4. If 3 hen eggs, 2 in. long, make a breakfast, how many pigeon eggs $1\frac{1}{4}$ in. long?

5. Old conundrum: If a hen and a half lays an egg and a half in a day and a half how many eggs can a hen lay in a day?

6. If a varies as b and b inversely as c and c as d^2 and d varies inversely as e, how does a depend on e? If e is doubled, what happens to d, c, b, and a?

7. Suppose b to be tripled; how will a change: (1) if a varies as b, (2) if a varies as $1/b$, (3) if $a = cb^2$, (4) if $a_1/a_2 = \sqrt{b_1/b_2}$?

8. If a circular field will pasture 20 cows for 15 days, how many days will one with twice the radius pasture 23 horses, if horses eat half again as much as cows?

9. If pingpong balls are half as big in diameter as tennis balls, how many pingpong balls can be held in a box which holds 100 tennis balls? When two such boxes are filled, one with tennis balls and the other with pingpong balls, which kind of ball has the greater total surface and in what ratio?

10. If 10 men can carry 3000 bricks up 30 ft. in 8 hours, how many hours will it take 5 men to carry 2000 bricks up 40 ft.? How many "brick-feet" can be raised per man per hour?

11. If it costs 20¢ to silver a round mirror of 5-in. diameter, what will it cost to silver one 6 in. in diameter?

12. If silver costs 90¢ per oz. and if silver weighs 6 oz. per cu. in., what will it cost to silver a mirror 10 in. in diameter with a coating 0.001 in. thick?

13. Are the numbers given in Table 9, p. 90, proportional? Is the braking distance of an automobile (Fig. 3, p. 129) proportional to its speed?

APPENDIX 2

WORKING PROBLEMS IN PHYSICS

After a little experience the student will discover that the most direct, perhaps only, way to grasp the significance of a physical law is to work several simple numerical problems. And if he develops good habits of work, these simpler, illustrative problems will be easily, quickly solved.

1. Before working the problems, if formulas are given in the text, jot them down; consider in what kind of problem they can be used. Many numerical examples are worked out in the text; if you are having trouble with problems work these examples out on paper for yourself, and use the book for confirmation.

2. Develop a system. Whenever possible make a rough sketch. If a formula is applicable write it down. List the knowns and unknowns; check the units. Rewrite the formula with numerical values inserted. Multiply out and solve; give the units. Is the answer reasonable?

Work problems on paper punched for looseleaf notebook.

3. A few simple mathematical operations are required in problem solving — little more than handling fractions and transposing simple equations. . . Find the ratio of $\frac{2}{3}$ to $\frac{3}{7}$; of 20 to 0.2. Find x if

$$\frac{1}{x} = \frac{2}{x} + 7; \text{ if } -2 = \sqrt{\frac{2}{x}}. \qquad \text{(Answers: } \tfrac{14}{9};\ 100;\ -\tfrac{1}{7};\ \tfrac{1}{2}.)$$

If you cannot do these you need several hours' mathematical review and you should get this review at once. " Review of Pre-College Mathematics " by Lapp, Knight, and Rietz is recommended.

SQUARES OF NUMBERS 10–99

	0	1	2	3	4	5	6	7	8	9
10	100	121	144	169	196	225	256	289	324	361
20	400	441	484	529	576	625	676	729	784	841
30	900	961	1024	1089	1156	1225	1296	1369	1444	1521
40	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
50	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
60	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
70	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
80	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
90	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801

4. Problems can often be worked mentally calling $\pi = 3$ (instead of 3.1416) and $g = 1000$ (instead of 980), etc. But in written problems keep three " significant " figures: use 3.14 or $3\frac{1}{7}$ for π; for 19,388 write 19,400; for

0.000149273 write 0.000149. Do not make computations unnecessarily difficult by carrying along useless figures.

5. The student should be able to derive square roots. But usually the square root can be found with sufficient accuracy from the preceding table. Examples: the square of 15 is 225; the square root of 625 is 25; the square root of 1.96 is 1.4; the square root of 19.6 is 4.42.

6. Approximation for powers of numbers near unity. *If* e *is a small fraction,* $(1 + e)^n$ = *approximately* $1 + ne$. For example: $(1.1)^2$ = approximately 1.2 (exactly 1.21); $(1.002)^3$ = approx. 1.006; $(0.999)^2 = (1 - 0.001)^2$ = approx. 0.998. $\sqrt{1.01} = (1.01)^{\frac{1}{2}} = 1.005$. What is the square of 1.05; what is the square root? How much is $(1.000,022)^2$? If an error of 1 per cent is made in measuring the side of a square, the area will be in error by 2 per cent.

7. Express very large or very small numbers with powers of 10.

10^3 = a thousand; what are 10^6, 10^9, 10^{12}, 10^5?

$10^{-1} = 0.1$; what are 10^{-3}, 10^{-6}, 10^{-2}?

$6.1 \times 10 = 61$; what are 6.1×10^2, 6.1×10^3, 3.3×10^4, 1.67×10^{-24}?

$6{,}100{,}000 = 6.1 \times 10^?$; $0.03 = 3 \times 10^?$

Express as $10^?$: a million million million, six thousand million, one thousandth, seven billionths.

In multiplying powers of 10, add exponents; when dividing, subtract. Examples.

$$\frac{10^4 \times 10^6 \times 10^{-1}}{10^3 \times 10^2} = \frac{10^9}{10^5} = 10^4 = 10{,}000.$$

$$\frac{3 \times 10^4 \times 8 \times 10^6 \times 10^{-1}}{2 \times 10^3 \times 6 \times 10^2} = \frac{24 \times 10^9}{12 \times 10^5} = 2 \times 10^4 = 20{,}000.$$

APPENDIX 3

SYSTEMS OF MECHANICAL UNITS

In ordinary unit systems time and length are always regarded as two of the fundamental dimensions. In the so-called "absolute systems" the third fundamental dimension is mass; in the "gravitational systems" the third fundamental dimension is force, mass being regarded as a derived dimension. In any case this third fundamental unit is based upon the standard kilogram (or the standard British Imperial Pound); in an absolute system the unit is the *mass* of this standard or a definite fraction thereof; in a gravitational system the unit of force is the *weight* of this standard (or a definite fraction thereof).

In the United States all mass units are defined in terms of the standard kilogram. The pound is legally defined as 453.5924277 grams, which is practically identical with the British Imperial Pound Standard. In the c.g.s. and British (f.p.s.) *absolute* systems of units the gram or pound is taken as the unit of mass. In these systems the force unit is derived; in the absolute c.g.s. system it is the dyne; in the absolute f.p.s. system it is the **poundal,** the force which will give a 1-lb. mass an acceleration of 1 ft. per sec. per sec.

TABLE 53A

	British (F.P.S.)		Metric (C.G.S.)	
	Absolute	Gravitational*	**Absolute***	Gravitational
Time	second	**second**	**second**	second
Length	foot	**foot**	**centimeter**	centimeter
Force	poundal	**pound**	**dyne**	gram
Mass	pound	**slug**	**gram**	small metric slug (approx. 980 grams)
Work	foot-poundal	**foot-pound**	**erg**	gram-centimeter

* Used in this text.

In the corresponding *gravitational* systems the unit of force is the weight of a gram mass (or a pound mass). These force and mass units depend on g and hence vary slightly on the earth's surface.[1] The unit of mass is then the mass which is given unit acceleration by this unit force; in the c.g.s. system it is approximately 980 grams, and this may be called a **small metric slug**; in the f.p.s. system it is approximately 32.2 lb., and this is the ordinary slug. In European countries another gravitational system based upon the meter, kilogram weight, and the second is also used (Table 53B).

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[1] When it is necessary to attribute an absolute value to the gravitational units the **standard gram weight** (or pound weight) is defined as the weight of a gram mass (or pound mass) at a locality where acceleration of gravity is 980.665 cm per sec.^{2} (or 32.1739 ft. per sec.^{2}), this being rather close to the value of g at sea level at a latitude of 45°.

The British (f.p.s.) absolute system exactly parallels the metric absolute system; pounds (mass) and poundals correspond to grams (mass) and dynes. Despite this advantage the system is seldom used in practice and in this book we have preferred to use the British gravitational system from which the familiar engineering units foot-pound and horsepower are derived. Many engineering texts avoid the use of slugs by using in all formulas W/g (weight in pounds $\div g$) in place of m. W/g is evidently identical with the mass measured in slugs.

The c.g.s. absolute unit of work (erg) is inconveniently small for many practical purposes, and as a practical unit the joule (10^7 ergs) is used. A complete absolute system of practical metric units has been devised using as primary units the meter (instead of the centimeter), the kilogram (instead of the gram), and the second. This is the "m.k.s. system." The unit of force has been called the **newton** (the force which accelerates a kilogram mass 1 meter per second per second). It is 10^5 dynes. A newton of force pushing an object through a meter of distance does a joule (10^7 ergs) of work.

TABLE 53B

	M.K.S. Units	
	Absolute	Gravitational
Time	second	second
Length	meter	meter
Mass	kilogram	large metric slug (approx. 9.8 kg)
Force	newton	kilogram weight
Work	joule	kilogram-meter

The m.k.s. system is rapidly gaining popularity. It can be used together with coulombs, amperes, volts, etc., to make a complete practical system of electrical units.

MAGNETIC UNITS

Though we have used a single system of magnetic units it is possible to introduce three different systems: electromagnetic, electrostatic, and practical magnetic units, corresponding to the three systems of electrical units. Magnetic quantities are then defined in terms of electric currents. For example H may be defined, not as force per unit pole (as we have done) but equivalently as

$$H = \frac{F}{il} \qquad \text{(from eq. 14, p. 458)} \qquad (a)$$

or it may be defined from eq. 15′, 15″ or 15‴, for example

$$H = 4\pi n'i. \qquad (b)$$

Observe that the unit of H as defined by this last equation must vary in the same way as does the unit of current (amperes, abamperes, etc.). For H as defined by the first equation the units must change inversely as the current units (i.e., change as do volts, abvolts, etc.). The field defined by equation b is the field as it is *generated* by the current (or poles); this field is expressed in *oersteds.* The other field as defined by equation (a) is the field as it *acts* on currents or poles; this field is expressed in *gauss.* These units have been adopted by the International Commission of Weights and Measures. If one makes this distinction, in our equations 1, 4, 14, 16′, H must be expressed in gauss, in equations 2 and 15, H must be expressed in oersteds. So long as we use only a single system of units (as is customary in physics) there is no occasion to make this distinction and the introduction of two kinds of fields can only be confusing. The student is warned however that in current literature he will find oersteds and gauss used, often indiscriminately.

APPENDIX 4

FUNDAMENTAL PHYSICAL CONSTANTS

Velocity limit	2.99776×10^{10} cm/sec.
Velocity of light	
Ratio between e.m. and e.s. units	
Liter	1000.027 cc
Normal atmosphere	1,013,249 baryes
Gravitational constant	6.664×10^{-8} dyne cm^2/gm
Standard value of *g*	980.665 cm/sec.^{2}
Avogadro's number	6.024×10^{23}
Mass of atom with unit atomic weight	1.660×10^{-24} gm
Absolute zero	-273.18°C
Gas constant	83,136,000 ergs/°C mole
Boltzmann's constant	1.380×10^{-16} ergs/°C molecule
Mechanical equivalent of heat	4.1852×10^{7} ergs/cal.
Electrochemical constant	96,490 coul./gm-comb.-wt.
Electron charge	4.803×10^{-10} statcoulombs
Electron mass	9.11×10^{-28} gm
Electron magnetic moment (magneton)	9.28×10^{-21} ergs/gauss
Planck's constant	6.62×10^{-27} erg sec.

A final digit in small type signifies that there is an uncertainty (probable experimental error) greater than 2 in this place.

ANSWERS

(*Approximate answers in italics*)

Page 31

2. 1033 gm/cm^2; *34* ft **4.** 130 lb/in^2 **6.** 1.70/lb **8.** 16.6 tons **9.** 8.66 lb/in^2; 2500 tons **10.** 2.16 lb/in^2, 11, 200 lb; 1.08 lb/in^2, 4680 lb **11.** 2, 2.5, 3 kg **12.** 110 gm **13.** 3.48 lb **14.** 3.12 lb

Chapter 4

3. 87.3 lb **4.** 1.67 gm/cc **5.** 0.875 gm/cc **6.** 3.33, 0.84 **11.** 3.2 cu ft **12.** 0.64 in **13.** 13 ft **14.** 20 lb **16.** 66 gm

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3. $2\frac{3}{4}$ tons **4.** 33.3 in^2 **5.** Pressure in cylinder approx. 60 cm (Hg) **6.** 2.2 : 1 **7.** 33.65 in (Hg) **8.** *70.1*, *73.1* (Hg) (assuming normal atmosphere) **9.** *22* lb/in^2 **10.** (*a*) 17.3 lb/in^2 less in attic; (*c*) 0

Chapter 5

2. 3.74, 1.97, 1.35 gm/liter **3.** *2.2* **4.** *560*, *126* in^2 **5.** *74.5* cm **6.** 3.2 cm (Hg); 2.8 cc **7.** *34 000* **8.** *8* cu ft **9.** *7* in **10.** 0.32 lb **11.** 34 milligrams **12.** 3700 liters **13.** 11 kg **14.** 280 tons

Chapter 6

1. 3.2 cm **2.** 10 kg/cm^2 **3.** 10 cm **4.** 1600 lb **5.** 0.03% **7.** 1.5 mm **8.** 0.86 in **10.** 7.8 kg **11.** $\frac{1}{4}$ in (diam) **12.** 2.4 gm **13.** 0.4%, 0.028% **14.** $\frac{1}{2}$ in **15.** 4 times as much

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2. 1.8 kg **3.** 20° N of W **4.** *45°* below horiz **5.** 90 lb **6.** *1000* lb, 6° from vertical **7.** 17.4 kg **8.** 100 lb **9.** 43, 87 lb

Chapter 7

1. 53° with shore; 15 min **2.** *21* mi/hr; approx S **3.** 28.3 lb **4.** 38.6, 34.5, 28.3, 20, 10.4 lb **5.** 14.1, 86.6 lb **6.** 1, 2 lb **7.** 106 lb **8.** 125 lb **9.** 30 lb **10.** 300 lb **11.** 40 lb **12.** 11.5, 113 lb **13.** 82° **14.** (*a*) −50, 0; 0, 30; −43.3, 25; 34.6, 20; 17.3, −10 lb; (*b*) 77 lb, $32\frac{1}{2}$° below horiz; (*c*) 77 lb, $147\frac{1}{2}$° around from horiz **16.** 80.6 gm, 3 o'clock **17.** 48.5 gm, 11 o'clock

Chapter 8

1. 9, 12 lb **2.** $1\frac{1}{4}$ in **3.** 55, 35 lb **4.** 10 **6.** 4.05 ft from John **8.** 1 ft from man **10.** 360 gm down; 1360 gm **11.** 133 gm down; 32 cm **12.** 90 lb;

166 lb 33° from vert **13.** 8.66 lb; 31.2 lb 16° from vert **14.** 1000 lb-ft; 57.7 lb **15.** 375 lb; 708 lb 58° from horiz **16.** 300 lb; 484 lb 30° from vert **17.** 28 in from floor **18.** 0.32 mm **19.** 24°

CHAPTER 9

1. 18 ft **3.** 37.5 ft; 7.75 ft/sec **4.** 5.28 min; 2000, 3125 ft **5.** *1* ft/sec^2; *600* yd **6.** *75%* **7.** 88 ft/sec^2 **8.** 2500 meters/sec^2 **9.** $3\frac{1}{5}$ sec; 156 ft **10.** 940 ft **11.** 32.6 km **12.** 163 sec **13.** (*a*) 1, $1\frac{1}{2}$ sec (*b*) 3 sec (*c*) never (*d*) ±34.8, −44.5 ft/sec **15.** 3 sec; 32 ft/sec **16.** 100 sec; 32 ft/sec **17.** 2000 ft out **18.** 5.8, 23 ft **19.** 1000 ft/sec; 62.5 sec; 1732 ft/sec; 20.5 miles **20.** 165, 41 miles **21.** 1.65 sec; 9 ft out **22.** 471 ft/sec

CHAPTER 10

3. 16 ft/sec **4.** $7\frac{1}{2}$ **5.** 360 000 ft/sec^2 4250 ft/sec **6.** 11 000 lb 5 kg wt **7.** 92 lb **8.** 19.6, −98 cm/sec^2 **9.** 1100 kg wt **10.** 3200, 4200, 200 lb **11.** 98 cm/sec^2; 49 cm **12.** 97 000 dynes **13.** 2 ft/sec^2 **14.** 0.47 lb-sec **15.** 2.1 lb-sec **16.** 10 meters/sec; $\pm 2 \times 10^6$ gm-cm/sec; 2×10^6 dyne-sec **17.** 198 meters/sec **18.** 0.19 ft/sec **19.** 8 ft/sec

CHAPTER 11

1. 300, 400 gm-wt **2.** 0.125 **3.** 294 cm/sec^2 **4.** 23 sec **5.** 366 cm/sec^2 **6.** 0.022 **7.** 4.26 ft/sec^2 **8.** 18% **9.** 8.6, 37.4 tons, 0.68 ft/sec^2 **10.** 50, 86.6, 2.6 lb, 15.2 ft/sec^2 **11.** 1 ft/sec^2 **12.** 3950 cm/sec^2 395 000 dynes **13.** 11.1 **14.** 21 mi/hr **15.** 6.25 meters/sec **16.** 8.5 ft/sec **18.** $a = 200$ ft/sec^2, 80° **19.** *1 000 000 : 1* **20.** 9.35 rev/min **21.** 18 **22.** 33.8 lb **23.** *35 000* dynes

CHAPTER 12

3. 650 ft-lb **4.** 3140 joules **5.** 2.9 joules **6.** 7.4 joules **7.** 52 watts, 1.6 sec **8.** nearly 2 min **9.** 1 cm/sec **10.** 16.7 hp **11.** 2150 joules **12.** 3100 joules **13.** *150*, *100* ft-lb **14.** 5000 joules; 976 ft-lb **16.** *2%* **17.** 53 800 ft-lb **18.** 33.2 hp **19.** 34 600 lb **20.** 980 000 dynes, 3.92 joules, 1980 cm/sec **21.** 450 kilojoules **22.** 46 mi/hr **23.** 32 000, 6400, 25 600 ft-lb; 71.5 ft-sec; 6400 ft-lb **24.** 3.67, 8.24 sec; 100 mi/hr

CHAPTER 13

1. 600 ft-lb **2.** 60, $16\frac{2}{3}$ lb **3.** 62.5% **4.** 33 lb **8.** $6\frac{2}{3}$ **10.** 480 gm **12.** 35 : 6 **13.** $3\frac{1}{3}$ **14.** 7, 12.3, 13.9, 4.1; 45.8 rev/sec **15.** 0.524 lb

CHAPTER 14

1. 312 000 ft-lb **2.** 33 000 ft-lb **3.** 51 800 joules; 0.6 watts **4.** 32 ft **5.** 150 ft/sec; 500 ft **6.** 8 ft/sec **7.** −1.3 lb/in^2 **8.** 100 baryes

CHAPTER 15

1. 20 cm/sec **2.** 4, 40 cm/sec^2 **3.** 24 000 dynes; 2400 dynes/cm **4.** 2.5, 1, 0 ft/sec^2 **5.** 100 000 dynes/cm; 1.59 vib/sec **6.** 0.45 sec **7.** 1.57 sec

8. 0.78 sec **9.** 0.49 sec **10.** 23 700 ft/sec^2; 1480 lb **11.** 3.14 sec **13.** 39 sec **14.** 1 : 2.5 : 0.19 **16.** 15 **17.** 0.79 ft/sec; 2.48 ft/sec^2 **18.** 2.16 cm; 34 cm/sec

Chapter 16

1. 1, 0.4, 0.5, 0.33 slug-ft^2 **2.** 6.28, 2.51, 3.14, 2.09 lb-ft **3.** 2 rev **4.** 19.74, 7.90, 9.87, 6.58 ft-lb **5.** 2.81 sec **6.** 3.3 slug-ft^2; 2.4 rad/sec **7.** 0.123 rad/sec^2; 10.1 sec **8.** 0.57°/sec; 2.58° **9.** 696 000 gm-cm^2; 0.163°/sec^2 **10.** 20 rad/sec; 2.5 Brit. units; 50 ft-lb **11.** 78.4 joules; 626 cm/sec **12.** 188, 376 Brit. units; 35 500 ft-lb **13.** 12 560 units; 0.1 rev/sec **14.** 0.60 sec **16.** 20, 34, 18, 16 gm-cm^2 **19.** 200 m gm-cm^2; 0.90 sec

Chapter 17

1. 5.5×10^{-4} dynes **2.** 1.6×10^{-4} dynes **3.** 245 cm/sec^2; 0.27 cm/sec^2 **4.** 0.7% **5.** 490 cm; 10^4 km **6.** 196 cm/sec^2 **7.** 26 900 cm/sec^2 **8.** Sun's attraction is 2.08 times earth's **9.** *6.1* $\times 10^{21}$ tons (metric); *6.1* gm/cc **11.** 2×10^{-5} gm **12.** 2.67×10^{-3} dynes

Chapter 18

1. 12 600 ergs **2.** 3.06 cm **3.** 0.78 gm **4.** 60 ergs **6.** *8.25* $\times 10^{23}$ **7.** 8.1×10^{15}

Chapter 19

5. *0.7* cm; *0.008* in **6.** 0.136, 0.0017 cm; 0.32 cc **7.** 0.17%; 0.17%; 0.51% **8.** 0.1%; 0.2%; 0.3% **9.** 17 cm **10.** 15.6 cc **11.** 12.1 cc

Chapter 20

1. *16* cm (Hg) **2.** 32 lb/in^2 **3.** *1060* ft^3 **4.** 34 cm **5.** 2.14 cc **6.** *20*-fold **7.** 34.0 liters **8.** 0.91 atmospheres **9.** 18.6 atmospheres **11.** 6.2×10^6 baryes; 6.1 atmospheres **12.** 0.48 km/sec; 0.27 km/sec

Chapter 21

1. 30 000, 6 400 000 cal/°C (walls only) **2.** 62.4 **3.** 20°C **4.** 6.7°C **5.** 6.6°C **6.** 8.3 kg **7.** 2.3°C **8.** 8.6 kg; 1.8°C **9.** 11°C **10.** 5.7°C **11.** 4800 Btu/min **12.** 65°F per min **13.** 1 : 1.7 : 1.3 **14.** 1.5 cu ft

Chapter 22

3. 190 kg **4.** 2.7°C/min **6.** *0.36, 0.061*

Chapter 23

3. 13 **6.** 972 **7.** 1114 Btu **8.** 26°C **9.** 0°, 20°, 100°; 12.5, 1000, 963 gm **10.** 0°, 40°, 100° **11.** 0.46c **12.** 180 000 cal **14.** 24% **15.** 69.7 cm Hg **16.** 18.3, 33.1 ft **19.** 97, 170 cm Hg **20.** 53, 21 cm Hg **21.** 120 cm Hg

Chapter 24

2. 4700 cal **4.** 310 Btu **5.** 1170 cal **6.** 975° **7.** 130 000° **8.** 0.12° **9.** 6.3° **10.** 2.9° **11.** 2.1c **12.** Yes **13.** 15, 90 atm; 38%, 49% **14.** 24% **15.** *3200* large cal **16.** 0.3 cal

Chapter 25

1. 10 : 1 **2.** 1 : 4 **3.** 70 meters/sec **4.** $1\frac{1}{4}$ sec **5.** 10, 10 000 000; 31 meters/sec **6.** 3.5 meters **7.** 343, 1330 vib/sec **8.** 1.34, 5.7 meters **9.** 11.3, 10.7 ft **10.** 9 sec **11.** 387 meters/sec

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2. 48 ft; 2 sec **3.** 16 ft; 0.66 sec **4.** 40 cm; 860 vib/sec **6.** 24, 8 in; 550, 1650 vib/sec **7.** 3 vib/sec **8.** 1100 vib/sec **9.** 1750 vib/sec; 9.8 cm **10.** 120 000 vib/sec

Chapter 26

4. C_3, C_4, G_4 **5.** C_2, G_3, E_4 **6.** 42 cm **7.** *4* mm; *4* meters **10.** 3.4

Chapter 27

1. 4 vib/sec **2.** 220 vib/sec **3.** 3, $4\frac{1}{2}$, 5, $7\frac{1}{2}$ kg **4.** 270 cm **5.** 96.5 vib/sec **6.** 33%

Chapter 28

4. 2 cm **5.** *5000* dynes **6.** 9, 0.36, 1, 7.36 dynes **7.** 100 dynes **8.** 550 dynes **9.** 80 dynes **10.** 45 gm-wt **11.** 1.2 cm **12.** 3000, 300 units **14.** 600 000, 1500

Chapter 29

1. 400 gauss **2.** 31.4, 2.2, 3770 gauss **3.** 0.05 gauss **4.** 23, 9.1 gauss **8.** *1200*, *400* dyne-cm **10.** 3 gauss **14.** 2250 gm-cm^2; 12 sec **15.** 200; 4000; 894 units; 0.22, 0.57 gauss

Chapter 30

2. 36 : 1 **4.** 40, 10 dynes **5.** 15 dynes **6.** 25 statcoulombs **7.** 9 lb **8.** 3, 8, 5 dynes **9.** 4, 4, 6.9 dynes **11.** 0.04 dyne **12.** 10, 6 dynes **13.** 0.0025, 0.01 dyne

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3. 5 dynes/statcoulomb **4.** $1\frac{1}{2}$, 4, $2\frac{1}{2}$ dynes/statcoulomb **5.** $12L$, $32R$, $20L$ (dynes) **6.** 100, 100, 1 **7.** 126, 126, 189 lines, 15 units.

Chapter 31

1. $1\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$ units **2.** 3, 2, $1\frac{1}{2}$ units, 15 ergs **3.** 9 ergs, 13.4 cm/sec; 18 ergs, 19 cm/sec **4.** (*a*) 0, 100 statvolts, (*b*) 50 statvolts/cm, 0 **5.** $E = 10.7$ statvolts/cm **6.** 19.0, −8.3, 27.3 statvolts **7.** 50 statvolts **8.** 2.4×10^{-8} dynes, 15 statvolts **9.** 5000 statcoulombs **10.** 300 000 statcoulombs **11.** 10 statfarads, 2 statvolts, *4* ergs **12.** 480 statfarads **13.** −2.1 statvolts, 9.6 meter diameter **14.** 940 statfarads **16.** 6.4×10^8 statfarads

Chapter 33

3. 6.87 ohms, *9* **4.** 12 ohms; 0.5, 0.3, 0.2 amp **5.** 0.12 amp **6.** 12, 0.5 ohms **7.** 1, 24 amp **8.** 1, 2, 3, 6 volts; 12, 6, 4, 2 amp **9.** 2, 2, $\frac{2}{3}$, $1\frac{1}{3}$

amp; 40, 50, 20 volts **11.** 1, 3.27 amp **13.** 1.25 amp; 25, 75, 50 volts **14.** 6, 2, 3 amp **15.** 60 volts; 1, 1.5, 5.5 amp **16.** 2, 1, $\frac{1}{2}$, $\frac{1}{2}$ amp **20.** 2.5 volts **21.** 1.1, 0.55, 0.275, 2.2 amp; 0.9 ohm **22.** 3 ohms; 10 amp; 2, 28 volts

Chapter 34

1. 1.5, 0.3, 1.2 volts **2.** 5.5 volts **3.** 6, 0 volts **4.** 2 amp; 1 volt **5.** 6 amp **6.** 2 amp; 3 volts **7.** 0.4 amp; 2.4 volts **8.** 2 amp; 190 volts **9.** 1.2, 0.6 amp **10.** 0.44 volts **11.** 4.8, 6 amp **12.** 3 volts; $\frac{1}{4}$ ohm; 3 amp **14.** 1.5 amp **15.** $1\frac{1}{2}$, $1\frac{7}{8}$ volts **16.** 5 amp **17.** 40 volts; 5 ohms; 8 amp; 10.8, 37.6 volts **18.** 2.08 amp; 9.8 volts **19.** 1.2 ohms; 5, 2, $\frac{1}{2}$ amp **20.** 6.05, 5.95 volts **21.** 50 amp **22.** 20 amp; 6.2 volts

Chapter 35

1. 1.1 kilowatts **2.** 9, 18, 6, 12 watts **3.** *1.2*, *2.4*, *0.6* kilowatts **4.** 275, 20 watts **5.** 2.2, 0.2, 2.0 kilowatts **6.** 10^8 **7.** *2%* **9.** 3500 cal **10.** 7.2° **11.** 0.86° **12.** *4* min **13.** 5 amp; 1.2 ohm **14.** 2060, 7.2, 1.2 ohms **15.** 1.44, 484 ohms **16.** 50, 200, 600 watts

Chapter 36

1. .00001044, .0003387 gm **2.** .044, .040, .010, .135, .010, .0012 gm **3.** 1.24 gm **4.** 0.9 **5.** 195 000 **6.** 15 hr **7.** 0.81 liter **8.** 240 gm **9.** .0014 gm **10.** .011 – .033c

Chapter 37

6. *600* dynes **7.** *840* dynes **8.** 500 000 dyne-cm **9.** 0.003 dyne-cm **10.** 5 600 000 dynes; 28 000 000 dyne-cm **11.** 31.4 gauss **12.** *1* amp **14.** 628 gauss; 2513 lines **15.** *1*, *0.5*, 0, *1.5*, *6* gauss **16.** 1800 dyne-cm; **17.** *7* mm; *1.2* gauss **18.** 2 gauss; 200 dynes **19.** 4000 units **20.** 24 amp **21.** 50 units; 0.0001 gauss

Chapter 38

1. *10*, *20* units **2.** *1000* units **3.** 10 000 gauss; 40 000 lines **4.** 100 000 lines **5.** *15*, *7500* lines

Chapter 39

4. 20 volts **5.** 0.8 amp **6.** 1 millivolt **7.** $1\frac{1}{4}$ volt **8.** 800 000, 1 600 000 lines; 3 200 000 lines/sec; 160 volts **9.** 1 volt **10.** *0.0002* volt **11.** 9.4×10^7 lines/sec (each wire); 9400 volts **12.** 75 millivolts **13.** *0.3* millivolts **14.** 34.5 kilovolts **15.** 6 volts; 0.1 amp **16.** 3, 0.15 amp **17.** 500 volts **18.** 3.3×10^{-7} dynes

Chapter 40

1. 500, 1000, 1000 ohms **2.** 500 ohms **3.** 0.22 amp **4.** 110, 220, 220 volts

Chapter 42

1. 160 000 cycles **3.** 188 meters

CHAPTER 46

1. 200 cp **2.** 4 : 7 **3.** 120 ft **4.** 14 : 1 **5.** 4 ft-candles; 36, 63 000 lumens **6.** *37* **7.** 1370 lumens, 16 000 000 ft-candles, 1370 lumens **10.** None

CHAPTER 48

1. 1.7 ft-candles **2.** 18 ft **5.** 2 in. **6.** −3 in. **7.** 12 in. from mirror, 2 in. below axis **11.** −1.3 in. **13.** −2.5, 1.5 in. **15.** 3.8 in. **16.** 0.75 or 2.25 ft from object

CHAPTER 49

2. *23°* **3.** *19°* **5.** *18°* **6.** *35°*, *41°* **7.** *37°* **8.** 60°

CHAPTER 50

5. 0.55 in **6.** 4.66 ft; 7-fold **7.** 55.6 cm; 9-fold mag **8.** 30, −42 in **9.** 45 in **10.** 12.5, 25 cm **11.** 20 cm **12.** *80* cm **13.** 20 diopters **14.** 2 meters **15.** 2, 10, 0 diopters **16.** 20 cm* **17.** ∞, −90 cm* **18.** 20 in* **19.** 40, 60 cm* **20.** (*a*) 50 cm; (*b*) 66.6 cm; (*c*) −100; (*d*) ∞*

CHAPTER 51

1. 11, *625*; 25 **2.** In telescope 28° **4.** 5.4° **5.** *14, 30, 6* cm **6.** 1/600 sec

CHAPTER 53

1. 0.6 mm **2.** *23°*, *50°* **3.** 0°, *20°*, *42°* **5.** 20 **6.** *330* **7.** *4400* angstroms, almost completely transmitted **8.** Red (6700) Violet (4000)

APPENDIX 1

1. 8.7 ft **3.** 53° **4.** 12 (i.e. $3 \times 2^3/1\frac{1}{4}^3$) **6.** $a \propto e^2$; halved, quartered, quadrupled, quadrupled **8.** 35 days ($15 \times 2^2 \times \frac{20}{25} \times \frac{2}{3}$) **9.** 800 ($100 \times 2^3$); pingpong twice ($8 \div 2^2$) **10.** 14.2 hr ($8 \times \frac{2000}{3000} \times \frac{40}{30} \times \frac{10}{5}$); 1125 **11.** 29c $\left(20 \times \frac{6^2}{5^2}\right)$ **12.** 42c ($\pi \times 25 \times 0.001 \times 6 \times 90$)

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* Distances measured from last lens.

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Date Due
COLLE

$$P \qquad = \frac{10}{P}$$

magnifn =

$$① \ \frac{I}{O} = \frac{Q}{P} = \frac{L}{Bo}$$

$$② \ \frac{D}{Bl} = \frac{10}{Bl}$$

$$= ① \times ②$$

Tel $\frac{Bo}{Bl}$

A request for ~~Old~~ Old Mill Stream

1. $F = 2TL$

2. $R = \frac{2I}{ndg}$

3. $\Delta L =$ [illegible]

4. $H = mS\Delta t$

5. $H_{in} = H_{out}$

6. $H = \frac{C \cdot A \cdot \Delta t}{L}$

7. $m_w(\Delta t_w) + m_c S_c (\Delta t_w) = m_s S_s \Delta t_w$

8. 4.186 joules = 1 cal.

9. Eff = [illegible]

10

11

12

13

14 $= \frac{1}{n}$

15

16

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